A TEXTBOOK OF PHYSICS

CLASS XI

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Preface to the First Edition

This text book has been designed for the first year students of the Higher Secondary Classes in accordance with the new Physics Curriculum prepared by the National Curriculum Committee in Physics, National Bureau of Curriculum and Text Books, Ministry of Education, Government of Pakistan Islamabad.

Throughout the book an attempt has been made to name the articles according to the titles mentioned in the curriculum and to fulfil the objectives of teaching physics at the higher secondary level as stated in the said curriculum. This attempt has constrained us to squeeze the whole material of the book in 10 chapters.

Physics is a Science based on reason and logic. One cannot learn physics by mere memorizing the laws and principles of physics. In order to learn physics one has to work with it. In other words the students of physics not only be taught physics laws but also be trained to apply their knowledge of physics to solve specific problems and to explain physical phenomena. To attain this objective a lot of solved and unsolved problems and critical questions have been given in every chapter of the text book.

To give an integrated view of the subject matter contained in a chapter to the reader a summary has been given at the end of every chapter.

The first chapter opens with definition of physics, its historical development and its importance to the modern man. It also contains, Allah's injunction to the Muslims to study science. Contribution of Muslim scientists to physical sciences have also been summarized in this chapter. The study of this chapter will enable the young readers to form an idea how physics has evolved and what contributions have been made by the Muslim scientists to modern scientific thought.

The significant figures and degree of accuracy have been discussed from practical point of view in this chapter. A thorough understanding of these will help the students in theory as well as in practical.

The S1 units have been used in the book because it is the one used in engineering work or otherwise in our country as well as worldwise.

Apart from traditional topics in mechanics, waves and Oscillations, modern topics like weightlessness, Artificial gravity, conventional and renewable sources of energy have been added to the text to enable the readers to appreciate space related problems and energy crisis.

The text consists exclusively of material suitable for students whose mathematical preparation gives no further than algebra and the elements of trigonometry. Apart from the definition of a derivative, no calculas is used.

The text is written by a number of university and college teachers whose names appear in the beginning of the book.

Despite our best efforts some errors might have crept in the text, the readers are requested to point out these errors to us so that they are eliminated in the future editions. We cordially invite any suggestions for the improvement of this text because there is always room for improvement. Such suggestions will be gratefully acknowledged in the subsequent editions of the book.

Finally we would like to add that we alone are responsible for the deficiencies and errors from which the book might suffer.

Editors

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بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

1

INTRODUCTION

1.1 DEFINITION OF PHYSICS

What is physics ? *Physics is the study of the properties of matter, energy and of their mutual relationship.* It tries to explain natural phenomena in terms of fundamental principles and laws.

1.2 VARIOUS BRANCHES

The most basic branches of physics are mechanics and field theory. Mechanics deals with the motion of particles under the action of forces. The field theory explains the origin, nature and properties of fields such as gravitational, electromagnetic and nuclear etc.

The earlier divisions of physics were made on the basis of natural phenomena to which the methods of physics had been applied. These branches consisted of classical mechanics such as dynamics, statics, celestial mechanics, hydrodynamics, aerodynamics, ballistics, heat and thermodynamics; statistical mechanics; optics; acoustics, electricity and magnetism. Many of these are now becoming branches of applied physics or technology rather than of pure physics.

Most recently the study of modern physics is classified according to the particular aspect of study of nature. Thus particle physics or high energy physics is concerned with understanding the properties and behaviour of elementary particles. The next branch in this classification is nuclear physics which is related to neutrons and protons forming the nuclei of atoms, their structure properties, energy states; reactions between nuclei and radio-activity. Atomic physics deals with the structure and properties of atoms as determined by the electrons outside the nucleus. Other branches are molecular physics, solid state physics; physics of liquids and gases, plasma physics, statistical mechanics, relativity and quantum mechanics.

Sometimes classification is made on the basis of particular instrument or technique such as x-ray diffraction, neutron diffraction, mass spectroscopy, infra-red spectroscopy and seismology. The low temperature physics is characterized not only by special instruments involved in the production and measurement of low temperature but also by the phenomena of superconductivity and superfluidity which occur only in the temperature range of 0 K to 120 K. Other branches such as astrophysics, geophysics and biophysics are concerned with aspects related to sciences in which physics is applicable.

Theoretical physics is the study of physical phenomena by means of mathematics and includes mathematical parts of all branches of physics, as well as, most of the content of statistical mechanics, quantum mechanics, relativity and field theory. Theoretical and experimental physics are concerned with the understanding of phenomena of nature and in this respect they are related to each other.

The aim of physics is the construction of a unified theoretical model in mathematical terms. It emphasizes accurate instrumentation, precision in measurement and expressing the results in mathematical form.

1.3 HISTORICAL DEVELOPMENTS

Physics is one of the oldest and most highly organised of all the sciences. In this scientific age, a serious student would naturally try to find out what physics has contributed to this world and how it has grown. This brief historical survey will provide an opportunity to appreciate the importance and the growth of this important branch of science.

The contribution of physics to the cultural and economic welfare of mankind has been very great indeed in times of both peace and war throughout the ages. Too often the students of science and technology overlook the fact that physics is just as much concerned with people as it is with things. Discoveries of natural laws have been made by dedicated efforts of many scientists. It is, therefore, appropriate that the historical outline be developed around the names and periods of famous physicists.

This historical survey may be divided into five periods which are Ancient physics (3000 BC to 600 AD), physics of Islamic Era (600 to 1500), the New Awakening in physics (1500 to 1700), Classical physics (1700 to 1890) and Modern physics (1890 to the present).

Ancient Physics (3000BC-600AD)

Historical records show that the Ancient Babylonians and Egyptians as early as 3000 BC were familiar with some of the fundamental principles of physics especially those dealing with the measurement of land and astronomy. The pyramids of Egypt are a witness to the civil engineering achievement of those days.

It is certain that the Chinese were familiar with the magnetic compass earlier than 1000 BC. From approximately 700 BC to 150 AD very definite advances in physics were made by the Greeks. Thales (546 BC) appears to have recognised certain aspects of static electricity. Pythagoras (580-500 BC) founded the pythagorean school of philosophy, his most important contribution was probably the famous pythagorean theorem of geometry. Democritics (460-370 BC) postulated an atomic nature of matter.

Plato (427-347 BC) was one of the latest Greek philosophers in this period but his contribution to physics was relatively insignificant. However, his famous pupil Aristotle (384-322 BC) was the first man to write a book on physics. He also contributed so much to science that for many centuries his influence was paramount. He is criticised today on the ground that his famous theories were not based upon experimental evidence, but upon abstract arguments. Aristotle tried to explain why things happen and not "how" they happen. Modern science on the other hand tries to explain "how" nature behaves by putting a lot of emphasis upon precise quantitative measurements. In other words, Aristotle's conclusions were more deductive than inductive. Euclid (450-374 BC) made some contribution to geometrical optics. In the Greek period, Archimedes (287-212 BC) of Alexandrian school is perhaps the best known of all the Greek physicists. Unlike Aristotle he used experiment as the base for his theoretical reasoning. He made a notable contribution to optics, mechanics, hydrostatics, hydraulics and other fields of physics. Who is not familiar with "Archimedes's Principle"? He is also sometimes called the founder of mathematical physics because of accurate application of mathematics to problems of physics. To summarize the Greek period as a whole, it is fair to state that this whole period was characterized by deductive rather than inductive reasoning; that observation was for the most part superificial rather than critical; that there was more passive curiosity than active experimentation and more "armchair" speculation than fact finding. One wonders why this was like that! It is easy to answer this question when one remembers that experimentation in physics is a physical effort. The social status of the freeborn Greek citizen was considered to be above the slaves. This prevented them to undertake manual labour with the result that no experimental activity worth the name was carried out by the Greeks. Were they not shy of manual work they would have contributed more to the advancement of science.

The Islamic Era (600 - 1500 AD)

In this era the venue of science shifted to the Islamic world. All Greek manuscripts were translated into Arabic and a lot of scientific activity took place in Muslim countries. The Muslim scientists and their contribution will be discussed later in this chapter. However, it can be said safely that Muslim world was being illuminated with the light of science while the Europe was plunged into darkness as there was practically no advancement in physics till 1550 in Europe. Between 700 to 1100 a beginning was made toward a revival of learning in Europe. Universities were developed under the patronage of the church where no experiment was undertaken and criticism of Aristotle's views was not allowed.

New awakening in Physics (1500 - 1700 AD)

The period of new awakening commences with Copernicus (1473 - 1543), Galilio (1564-1642), Kepler (1571 - 1630) and Gilbert (1540 - 1603) who all paved the way for Isaac Newton (1642-1727). Galilio and Kepler established the fundamental ideas of modern celestial mechanics based upon observations. This period owes a lot to Islamic period. In this era the whole work of Muslim scientists was transferred to Europe. Sir Isaac-Newton who is considered as the greatest genius the world ever produced, was born in this period. His contribution to physics includes his three famous laws of motion and the law of gravitation. He invented calculus and contributed a lot to the field of optics. A part of classical mechanics in which Newton's laws are applied is called Newtonian Mechanics and is applicable to most of the systems even today.

Newton's contemporaries include Huygens (1629 - 1695) who proposed wave theory of light, famous physicist Boyle (1627 - 1691); well known for his Boyle's law of gases, Hooke (1635-1703) who gave us Hooke's law and Pascal (1623 - 1662) who is remembered for his law of fluid pressure.

Classical Physics Period (1700 - 1890 AD)

The achievement of eighteenth and nineteenth centuries are so large that it is difficult to enumerate them. As physics was gathering momentum so it had to be considered in sub-divisions like mechanics, heat, light and electricity. The work of Bernoulli (1700 - 1782) in hydro-dynamics and gas theory; D, Alembert (1717-1783), Euler (1707 - 1783), Lagrange (1736 - 1831) and Laplace (1749 - 1827) in theoretical mechanics is highly commendable.

During this period the Newtonian mechanics was so well-established that it almost seemed that the end of physics was in sight. The physicists thought that they had achieved and explained everything. This period is named as classical period.

In the field of heat, Fahrenheit (1686 - 1736) developed thermometers and temperature scale; Black (1728 - 1799) introduced the concepts of latent and specific heats: Rumford (1753 - 1814), Joule (1818 - 1889) and Rowland (1848 - 1901) established that heat was merely a form of energy which fitted very well into the Newtonian mechanics; Carnot (1796 - 1832), Mayer (1814 - 1878), Helmholtz (1821 - 1894), Kelvin (1824 - 1907) stated laws of thermodynamics which helped the physicists to unify the concepts of heat with those of mechanics. Several physicists determined the velocity of light, but the present accepted value was found by Michelson. Huygen's principle became well established by the experimental work of Young (1773 - 1829) and Fresnel (1768 - 1827). Malus (1775 - 1812) and Brewster (1781 - 1868) discovered "polarization of light". The most remarkable event of this period is the resolution of the controversy about the nature of light. Maxwell (1831 - 1879) proposed his famous electromagnetic wave theory of light which satisfied all parties to the controversy.

In this period more attention was given to electricity than to any other branch of physics. It was found in the eighteenth century that electricity and magnetism were interrelated. Gray (1670 - 1763), Franklin (1706 - 1790), Cavendish (1731 - 1810) and Coulomb (1736 - 1806) did significant work in electrostatics. Galvani (1737 - 1798) and Volta (1745 - 1827) were pioneers in current electricity. In the later part of this period Faraday (1791-1867), Ampere (1775 - 1836); Wheatstone (1824 - 1887) and Hertz (1857 - 1894) contributed a lot to the field of electricity. Faraday was the outstanding experimentalist of his time and Maxwell was the most outstanding theoretical physcist of this period. In this period people thought that the science of physics was fully organized and well integrated especially by the work of Maxwell, so that nothing was left to be discovered by future physicists. They believed that probably future generations of physicists would have to content themselves with extending the accuracy of known information to the next decimal place. The story of modern physics which began about 1890 gave a severe jolt to the sleeping physicists. The noteworthy discoveries of electron, x-rays and radioactivity could not be explained on the basis of classical mechanics. So a new branch of physics known as "modern physics" was developed to explain these new phenomena".

1.4 PERIOD OF MODERN PHYSICS (1890 AD. to the present)

Crookes (1832 - 1919) had discovered cathode rays in 1878 but J.J. Thomson (1856 -1940) explained their nature as negatively charged particles called electrons. Lorentz (1853 - 1928) formulated an electron theory of matter in 1895 in which vibrating electrons accounted for the electromagnetic radiations predicted earlier by Maxwell and discovered in 1888 by Hertz. In 1895 Rontgen (1845 - 1923) discovered x-rays. In 1896 Becquerel discovered radioactivity. The quantum theory was introduced by Planck (1858 - 1947) in 1900 to be followed by the theory of relativity in 1905 by Einstein (1879 - 1955). These two theories brought a revolution in the fundamental philosophy of physics and provided the key to the solution of several problems left over from the preceding period. The quantum theory explained the phenomena of heat radiation and the photo-electric effect immediately. The theory .of relativity explains certain phenomena associated with matter such as electrons, atoms and molecules, moving with extraordinary large velocities.

From about 1911 much interest has been focussed upon atomic physics. Rutherford (1871 - 1937) established the concept of the nuclear atom. Bohr devised a model of the atom in 1913 which was like the solar system and its planets. Quantum and relativistic physics have completely revolutionised our concepts in physics. During the year 1920 more advances were made in electronics, spectroscopy and nuclear physics, Compton removed all doubts of the quantum theory in 1923 by his experiments with x-rays. De-Broglie introduced the concepts of wave mechanics in 1924. Heisenberg, Dirac and Schrodinger contributed significantly to develop quantum mechanics. Davison and G.P.Thomson produced experimental evidence for the wave nature of electrons in 1927 and 1928. In 1932 Chadwick discovered the neutron, a fundamental particle with no electrical charge. Lawrence invented the cyclotron in the same year.

Joliot and his wife Irene Curie discovered artificial radioactivity in 1934. Fermi and others produced artificial radioactivity by neutron capture. Hann in 1938 discovered uranium fission which is the basis of the atomic bomb. Transistor was invented by Bardeen, an invention which has revolutionised the modern electronics. Discovery of high temperature superconductivity in 1986 by Muller and Bendoz has opened doors for high speed trains and transmission lines without energy losses.

We have briefly gone through some of the achievements of physics right upto the present time. List of scientists and their discoveries is far from complete nor it is possible to write more due to limitation on the scope of this book. Research today is not a privilege of individual physicists rather it is the age of group research. Hundred of scientists who may never receive recognition for their work have contributed to the development of science.

1.5 TEACHING OF ISLAM AND SCIENCE

The basic principles of science are observations and reasoning. Islam stresses both. The very first verse of the Holy Quran revealed to the Holy Prophet (SAW) states:

اِقْرَاْ بِاسْمِ رَبِّكَ الَّذِیْ خَلَقَ ۚ خَلَقَ الْاِنْسَانَ مِنْ عَلَقٍ ۚ

اِقْرَاْ وَرَبُّكَ الْاَكْرَمُ الَّذِیْ عَلَّمَ بِالْقَلَمِ عَلَّمَ الْاِنْسَانَ مَا لَمْ يَعْلَمْ ؕ

"Read; In the name of your Lord who is the Creator. He has created man from a clot. Read: and your Lord is the Most Generous who teaches by the pen. Teaches man that he did not know."

The importance given by God Almighty to reading, writing and scientific knowledge is clear from this verse. The Holy Quran in so many verses has again and again ordained the believers to observe and think. Two are quoted here:

إِنَّ فِي خَلْقِ السَّمٰوٰتِ وَالْاَرْضِ وَاخْتِلَافِ الَّيْلِ
وَالنَّهَارِ وَالْفُلْكِ الَّتِي تَجْرِي فِي الْبَحْرِ بِمَا يَنْفَعُ
النَّاسَ وَمَا أَنْزَلَ اللّٰهُ مِنَ السَّمَاءِ مِنْ مَّاءٍ فَأَحْيَا
بِهِ الْاَرْضَ بَعْدَ مَوْتِهَا وَبَثَّ فِيهَا مِنْ كُلِّ دَآبَّةٍ
وَّتَصْرِيفِ الرِّيٰحِ وَالسَّحَابِ الْمُسَخَّرِ بَيْنَ السَّمَآءِ
وَالْاَرْضِ لَاٰيٰتٍ لِّقَوْمٍ يَّعْقِلُوْنَ

"Surely, in the creation of the Heavens and the Earth and in the interchange of night and day and in the ships which sail through the sea and in the rain which God sends down from Heaven giving life to the earth after its death and scattering over it all kinds of cattle and in the change of winds and in the clouds that are made to serve between the Heavens and the Earth - are signs for those who understand."

أَفَلَا يَنْظُرُوْنَ إِلَى الْإِبِلِ كَيْفَ خُلِقَتْ وَإِلَى السَّمَآءِ كَيْفَ رُفِعَتْ
وَإِلَى الْجِبَالِ كَيْفَ نُصِبَتْ وَإِلَى الْاَرْضِ كَيْفَ سُطِحَتْ

"Will they not observe the camels how they are created. And the Heaven how it is raised? And the hills, how they are set up?"

The two verses quoted above are the orders from God to observe, think and these now are recognised as basic principles of modern science. Islam, thus laid down the foundation of modern scientific thought 1400 years before.

Numerous other verses can be quoted in which the Holy Quran invites the readers to observe and think.

افلا ينظرون ○ افلا يتفكرون ○ افلا يتدبرون

("Don't you see? Don't you think? Don't you plan?")

The Holy Prophet (SAW) told his followers that learning of science is the duty of every Muslim man or woman.

طلب العلم فريضة على كل مسلم. (ابن ماجه بیہقی)

It was the result of the Quranic injunctions and the Holy Prophet's(SAW) pronouncement that the spirit of enquiry spread rapidly through out the Muslim world. And the science with experimentation became well established in the Islamic era as we shall describe in the next article.

1.6 CONTRIBUTIONS TO PHYSICAL SCIENCE BY THE ISLAMIC WORLD.

As we learnt in previous pages the Greek were "arm-chair" philosophers, they thought and argued without making observations, Islam changed this attitude. Obeying the injunction of Allah the Muslims observed, thought and reasoned. They not only translated all available Greek science books into Arabic but also established observatories and laboratories at various places. They revised the astronomical tables of

Ptolemy and determined with precision the orbits of the Sun, the Moon and the Planets. Al-Beruni, a great mathematician determined the earth's circumference and divided its surface into latitudes and longitudes which we find marked on the globe even today.

Abu Abdullah Muhammad Ibn Jabir Al-Battani was the greatest astronomer of his time. He improved upon the calculations of the orbits of the Moon and certain planets. He proved the possibility of annual eclipses of the Sun. He popularised the trignometric ratios, sine, cosine, tangent etc., as we use them today.

The Arabs introduced the Arabic system of numbers, enriched Algebra by the solution of the third and fourth degree equations. Al-Khawrizimi wrote a number of books on algebra, astronomy and arithmatic which were used as textbooks in European universities till the sixteenth century. He was also responsible for the introduction of the Arab numerals, called "Algorioms" after him, to the west.

The Muslim contributions in the field of chemistry and physics are also very important. The earlier chemists were alchemists. According to Joseph Hell "The increase in the number of artifically prepared substances, the perfection of methods handed down from the Greeks and application of these methods to most diverse materials are the striking achievements of the Arabs in the domain of chemistry. If, in this direction they advanced considerably beyond the Greeks, it was due to the fact that in the place of hazy mystical speculations they introduced objective experiments in the study of nature."

A great name in alchemy was Jabir Bin Hayyan who worked in Kufa around 776 AD. He described scientifically the two important chemical operations of calcination and reduction. He improved the methods of evaporation, sublimation, melting and crystallisation.

Ibn-al-Haitham (Alhazen) of Basra (965 AD.) developed optics to its highest degree at that time.

His book on "optics" which was translated into Latin is still available. He opposed the theory of Euclid and Ptolemy that the eye sends out visual rays to the object. He discussed, the propagation of light, colours, optical illusions, reflection and refraction of light rays through transparent media. All the medieval western writers on optics based their optical works largely on Alhazen's "Optticae-Thesaurus". Omar Khayyam who was both a poet and mathematician improved upon the calender in 1079.

Robert Briffault in his book the 'Making of Humanity' has honestly expressed the indebtedness of Europe to the Arabs in the following words, "it was under the influence of the Arabs and the Moorish revival of culture and not in the 15th century that a real renaissance took place. Spain not Italy was the cradle of the rebirth of Europe. After steadily sinking lower and lower into barbarism it has reached the darkest depths of ignorance and degradation when cities of the Sarcenic world Baghdad, Cairo, Cordova and Tolido were growing centres of civilization and intellectual activity. It was there that the new life arose which was to grow into a new phase of human evolution. From the time when the influence of their culture made itself felt, began the stirring of a new life. Science is the most momentous contribution of Arab civilisation to the modern world but its fruit were slow in ripening. Not until after Moorish culture had sunk back into darkness did the giant which it had given birth to rise in his might".

With decline of Islamic empire the scientific activity ceased in Islamic world and all the work of Muslim scientists were translated into European languages. Roger Bacons learnt Arabic and Arabic sciences and he declared without reservation that the knowledge of Arabic and Arabic sciences was for his contemporaries, the only way to true knowledge. The experimental method of Arabs was by Bacon's time widespread and well established throughout Europe. Europe advanced with the help of Muslim science while the Islamic countries sank into darkness politically, scientifically and economically.

Pakistan came into being in 1947. At that time in the regions which constitute Pakistan today there was not a single Ph.D in physics and no laboratory of physics worth mentioning. After August 1947 some physicists migrated from India. The names of the pioneers of physics in Pakistan may be counted on fingers. They are Prof. Muhammad Rafi Chaudhri, Prof. Basir Pal, Prof. Majeed Mian and Prof. Mujteba Karim. Prof. Abdus Salam the only Nobel laureate from Pakistan started his career after the establishment of Pakistan and soon migrated to England to hold a chair of physics at Imperial college, London. Later on he founded the International Centre for Theoritical physics at Trieste (Italy) which is directed by him. Previously it was thought that there were four different forces of repulsion or attraction in the universe, he reduced this number to three in about 1969. He shared Nobel Prize with Weinberg and Glashov in 1979 for this work. Adequate facilities for physics research are now available at the Centre of Excellence in Solid State Physics, Centre for High Energy Physics, both located at the University of the Punjab, New Campus, Lahore; Department of Physics, Quaid-i-Azam University, Islamabad; Abdul Qadeer Khan Nuclear Research Laboratories, Kahuta; Pakistan Institute of Nuclear Science and Technology, Nilore; Institute of Silicon Technology Islamabad; The National Institute of Electronics, Islamabad;

Institute of Standards, Islamabad; Defence Science and Technology Organisation, Rawalpindi and Physics Departmant, University of Karachi.

1.7 PHYSICS AND THE MODERN MAN.

As we look around we find physics being used everywhere. The new discoveries of physics have always affected human life. At the time of discovery the discoverers do not realize the full significane of their work. When Micheal Faraday discovered the principles of electromagnetic induction in 1829, few people at that time could have realized just how important electricity would become a century or more later. People asked him "What is the use of electricity? Mr. Faraday"! His reply was classic. "What use is a new born baby." Who knows where a new discovery would lead to? Look at the computer revolution which has been brought about with the introduction of microchip. Just over 30 years Bardeen invented the transister. Now thousands of transisters can be made on a single chip called integrated circuit. The use of integrated circuits has changed the whole concept of computing telecommunication, entertainment, flying and even day to-day activities as shopping or visiting a bank.

We are all users of physics now. Faraday discovered electricity; we use it now in fans, in lights and many other house-hold appliances. Refrigerators and air-conditioners are there for our service because of Carnot. Telephone is a gift from Grahm Bell. We have television in every house because J.J. Thomson made cathode-ray tube for us. Aeroplanes make use of physical laws of aerodynamics. We enjoy the luxury of riding automobiles due to invention of internal combustion engine. Technology, the whole collection of means and equipment, which man has made to improve his lot on this planet or even in space is based on physics. Physicist tried to learn about radio waves which resulted in the formation of a group of specialists who tried to make this new gain useful;. In this way radio engineering and electronics arose as a special off shoot of physics. Skyscrapers, bridges and roads are designed by civil engineers whose work is based on physical principles, so well tested that the physicists no longer doubt their applicability. In the same way, the technique of release of nuclear energy is becoming a specialized, important and fascinating field of its own called nuclear engineering. In medicine, even before a baby is born, physics is at work with ultrasonic scanning machines to picture the unborn child in the womb. This is the way with physics; it fathers other sciences and their useful applications which we call technology. Such issues make a background knowledge of physics important for everyone. The individual without knowledge of this subject is at a disadvantage. In this scientific age, a background in physics is as important as was the ability to read and write in medieval times.

1.8 MEASUREMENT AND THE SYSTEM OF UNITS

Physics is the science of measurement and observation coupled with logic and reason. For the record of our observations and for the comparison of our measurements with those of others we must agree on a common language, definition of the terms and a set of standards. If the length of one metre is different for different persons, a customer buying 3 metres of cloth will either get more or less cloth from different shopkeepers. Similarly, if a grocer's kilogram weighs less than the standard kilogram he is considered dishonest. These two examples illustrate the necessity of having standards of various kinds for different sort of measurements. These standards are expressed in different measuring units. The fundamental units of measure in science are those for length, time and mass.

Length

In scientific laboratories all over the world, the length is measured in metres (meters). *Until 1961 the metre was defined as the distances between two finely drawn lines on a metal bar housed in the International Bureau of Weights and Measures, near Paris. Copies* of this bar were distributed to various National Standards Laboratories throughout the world. However, in 1961 in accordance with an international agreement the metre was redefined in terms of the wave length of the orange light emitted by Krypton atoms. Since all isotopes of krypton 86 emit the same wave length of the orange light, a length standard can be established in any laboratory when required. This standard is called an Atomic Standard of Length. The advantage of this atomic standard of length is that it is possible to report length in terms of atomic standard to a precision of 1 part in 100000000, (10^8), almost a hundred times better than was previously possible. Metre is convertable into smaller or greater units of length by merely dividing or multiplying by factor of 10. For example:

10 milimetre (mm) = 1 centimetre (cm)
10 cm = 1 decimetre (dm)
1000 m = 1 kilometre (km)
or 10^3 m = 1 km
or 1 m = 10^{-3} km
Similarly, 1 m = 100 cm
or 1 m = 10^2 cm
or 1 cm = 10^{-2} m

Powers of 10 are very useful in expressing very small or very large numbers. For examples:

1 light year, LY (the distance light will travel in a year)

$= 9460000000000000$ m

$= 9.46 \times 10^{15}$ m

Radius of proton $= .0000000000000012$ m $= 1.2 \times 10^{-15}$ m

Time

Second is the basic unit of time in all systems of units. It used to be defined as 1/86, 400 of the mean solar day. Slight irregularities in the rotation of earth affect the duration of the mean solar day. Since 1967 the international standard of time has been based on the vibrations of Cesium atoms. The second is now defined by atomic vibrations to be the duration of 9,192,631,770 periods of the radiations corresponding to the transition between the two hyperfine levels of the ground state of the Cesium 133 atom. The various time units are:

1 second = 9192631770 vibrations of Cesium atom

1 minute = 60 sec,1 hour = 3600 sec, 1 day = 86400 sec

1 year $= 3.156 \times 10^{7}$ sec

Mass

The third fundamental physical quantity mass is a measure of the amount of matter in an object. Since the international standard of mass has been a cylinder of platinum irridium housed in the International Bureau of Weights and Measures and designated as 1 kilogram (kg). The smaller units are:

1 kg $=$ 1000 gm $= 10^{3}$ gm

The Fundamental of SI. Units

We have defined above a system of units in which the fundamental physical quantities length, time and mass are measured in the following units:

Length	: Metre (m)
Mass	: kilogram (kg)
Time	: Seconds (sec)

This system of units used to be called as M.K.S. system of units. Here M stands for metre, K for kilogram and S for seconds.

Recent trend is to adopt this system world wide. Therefore, now this system of unit is called SI. units. SI. units stands for System International 'd' unites. Three more units have been added to make a set of six basic SI. units; the metre (m) for length; the kilogram (kg) for mass; the second (sec) for time; the ampere (A) for electric current; the Kelvin (K) for absolute temperature; the Candela (cd) for luminous intensity of light sources. The last three units will be defined when needed.

Derived Units

Other units in the SI. system for speed, force, energy, electrical resistance etc., are combinations of basic or fundamental units and, therefore, are called derived units. For example, speed or velocity is expressed as metres per second and acceleration as metre per sec per sec.

Dimensions of Physical Quantities

In the last paragraph we said that velocity is expressed as metres per second. We say that velocity has the dimensions of length divided by time, often written as lt^{-1}. Similarly, the dimension of area are l^2 and that of volume as l^3.

An equation containing magnitudes of physical quantities should have the same dimensions on both sides of the equation.

Use of Prefix

Sometime we use prefix to a unit to express appropriate units. For example, centi means 1/100, therefore, centimetre means 1/100 of a metre. The commonly used prefixes are listed below:

Prefix	*Symbol*	*Multiple*	*Prefix*	*Symbol*	*Multiple*
tera	T	10^{12}	centi	c	10^{-2}
giga	G	10^{9}	milli	m	10^{-3}
mega	M	10^{6}	micro	μ	10^{-6}
kilo	K	10^{3}	nano	n	10^{-9}
hecto	h	10^{2}	pico	p	10^{-12}
deca	da	10^{1}	femto	f	10^{-15}
deci	d	10^{-1}	atto	a	10^{-18}

1.9 SIGNIFICANT FIGURES AND THE DEGREE OF ACCURACY

On the first day of your physics laboratory you are given a scale on which rulings are spaced every mm and you are asked to measure the length and width of your physics book in metres and compute its surface area with the help of a calculator. A care free student will record his observations in his note book like this.

Length = 0.23 m, width = 0.17 m

On the other hand an over cautious student will report his measurement as

Length = 0.2335 m

Width = 0.1780 m

When checked by the instructor both were found to be incorrect, why? The first was wrong because the scale was graduated to mm or one thousandth of a metre. One could measure lengths accurately with this scale upto 1 mm. The first student only used the scale upto 1 cm reducing the accuracy of measurment by 1/10 of a metre. The second was wrong because he reported the result upto 1/10000 of metre which was not possible with the use of the supplied scale as it was graduated upto 1mm only.

The correct observations are:

Length = 0.233m,

Width = 0.178m,

Because we are sure about these measurements upto third digit beyond decimal and have no idea of the value of the fourth. The number of digits about which we do feel reasonably sure is called the significant figures. In the above example the significant figures are 3 in both the measurements. The greater the accuracy of our measuement, the larger the number of significant figures we can use.

When we write four significant figures we imply that a fifth digit will have no meaning. If our accuracy were ten times greater, we would use another significant figure. To write additional figures beyond the limit of accuracy of the instrument used is to mislead people into believing which is not correct. For finding the area the students used electronic calculators and got the following results:

The area obtained by the care free student = 0.0391 m^2

The area obtained by the overcautious student = 0.041563 m^2

The area obtained by multiplying the correct length and breadth = 0.041474 m^2

This time all the three results are wrong, the correct answer is 0.041 for the area. Because when two numbers are multiplied together, their product connot have more accuracy or more significant figures than the less accuracy of the two factors. The results are not improved by merely carrying out simple arithmatical operations to many figures. The same thing applies to division. Never carry out a division beyond the number of significant figures in the least accurate measurement you are using.

Examples:

$$3.3 \times 5.345 = 17$$

$$2.200 \times 4.5530 = 10.02$$

$$0.0022 \div 1,555556 = 0.0014$$

When the figures are added and subtracted the decimal location of the least significant figures determines the number of figures that should apear in the result:

Example:

$$\begin{array}{r} 2.3129 \\ +\,\underline{1.5348} \\ \underline{3.8477} \end{array}$$

However, if one of these number is poorly known we have,

$$\begin{array}{rrl} 2.3129 & 2.3129 & \\ +\;\underline{1.5} & -\;\underline{1.5} & \\ \underline{3.8} & \underline{0.8} & \text{and not } 0.8129 \end{array}$$

SUMMARY

Physics is the study of the properties of matter and energy and of their mutual relationship. Various branches of physics are, classical mechanics, heat and thermodynamics, statistical mechanics,optics, acoustics,electricity and magnetism, quantum mechanics, atomic physics, nuclear physics, solid state physics and biophysics. In the period 3000 BC to 600 AD people were familiar with some principles of physics, but their approach was that of an arm-chair philosopher, speculations with out experimentation.

Archimedes is the best known Greek physicist of this period. Muslims were the first to introduce experimentations in physics. They translated scientific workof Greeks into Arabic and tried to support the contents of these books with experimental evidences. They did it because Allah in Holy Quran and the Holy prophet (SAW) in hadith have ordained the muslims to acquire knowledge of science. Islamic era produced a large number of muslim scientists who enriched the scientific knowledge by their contribution. The period 1500- 1700 is called the period of New Awakening in physics due to a lot of discoveries and inventions were made by physicsists in this period. The period 1700 is termed as that of classical physics. At the end of this period thought that understanding of physical phenomena on the basis of physics was complete and nothing was left to be discovered. The period from 1890 to the present is the period of Modern physics.In this period electron, X-rays, radioactivity, superconductivity, fission and fusion were discovered. Quantum theory and relativity brought a revolution in theoretical physics.

QUESTIONS

1.1 What is physics? How is it different from chemistry?

1.2 Name the household appliances in your house which make use of physics principles.

1.3 Quote some more Quranic verses and hadiths which encourage the acquisition of scientific knowledge.

1.4 Why Islamic countries are backward in science?

1.5 Why do you want to study physics?

1.6 Name some of the current famous scientists of Pakistan and describe their contribution toward science.

1.7 Make a list of scientists whose laws you have studied in your school.

1.8 Why MKS system of units is called as SI units?

1.9 Which one is bigger:

a kilometre or a mile, A kilogram or a seer, A yard or a metre.

1.10 A student was asked to measure the length of a straight line with a scale on which minimum division is 1 mm. He recorded his three observations as follows:

(1) 0.8325m (2) 0.83m (3) 0.833m

Is his record correct? If not why?

PROBLEMS

1.1 Electronic rest mass is 9.11×10^{-31} kg

(a) Write it without the use of powers of ten

(b) Convert it into gm

1.2 The electronic charge is .00000000000000000016 C. Find its charge to mass ratio and express the results in C/kg using powers of ten.

1.3 Proton rest mass is 1.67×10^{-27} kg. Compute the ratio of proton mass to the electronic mass.

1.4 Speed of light is 3×10^{8}m sec^{-1}. Compute mc^2 for the proton and electron. In what units you will express the result?

1.5 Density of air is 1.2 kg/m^3. Change it into gm/cm^3.

1.6 Standard atmospheric pressure is expressed as 1.013×10^5 N/m^2. What are its dimensions?

1.7 Compute (i) 3.856×3.19 (ii) $10.23 \div 8.5489$

Express your result in significant figures.

2

SCALARS AND VECTORS

2.1 SCALARS AND VECTORS

Physical quantities used in various fields of science are either scalars or vectors.

A scalar is a physical quantity which is completely described by a number with proper units. The number representing the physical quantity is called its magnitude. Some of the well known examples of scalars are mass, distance, temperature, energy and electrical charge as they require only magnitude with proper units for their complete description. The scalars are added, subtracted and multiplied by the usual arithmetical rules.

A physical quantity which requires magnitude in proper units as well as direction is called a vector. In order to explain the definition of a vector, consider Fig. 2.1. If we are told that a person moves a distance of 10 m from the point of O, he may be anywhere on the circumference of a circle of radius 10 m with its centre at O. This means that the distance 10 m is not sufficient to locate the position of the person. Therefore, an additional information of direction of motion must be supplied with the distance. Now if we say that the person has moved 10 m west of O, then his position is precisely located and the displacement vector OA is completely determined. Other

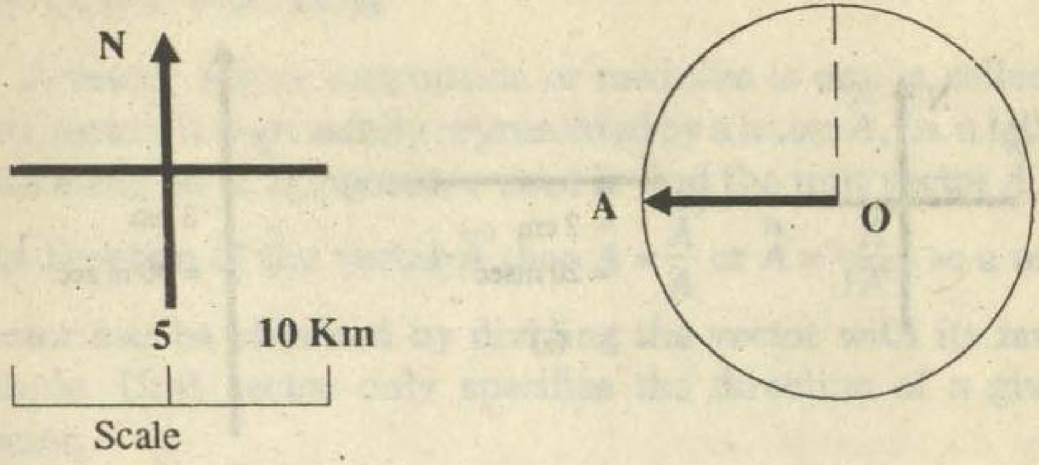


Fig. 2.1 *Displacement from O to A*

familiar examples of vectors are velocity, acceleration, force, momentum, electric field etc. These quantities require a number in proper units alongwith the direction for a complete description. The combined effect of two vectors cannot be found by a simple arithmetical procedure but by a process called Vector addition, vector subtrcation and Vector multiplication. Simple arithmetical addition and subtraction of vectors can be carried out in the case of parallel and antiparallel vectors.

2.2 VECTOR REPRESENTATION

a) Symbolic Representation

A vector is usually represented by a bold face letter **A** or a letter with an arrow drawn above or below it i.e., $\vec{A}$ or $\underrightarrow{A}$. The magnitude of a vector, alternatively known as modulus, is denoted by the roman letter A for $|\vec{A}|$

b) Graphical Representation

A vector is graphically represented by a straight line with an arrow-head in the direction of the vector. The length of the line according to the scale chosen represents the magnitude of the vector. To illustrate it further, consider two bodies moving with velocities 20 msec^{-1} toward east and 30 msec^{-1} toward north, respectively. These vectors are graphically rep-

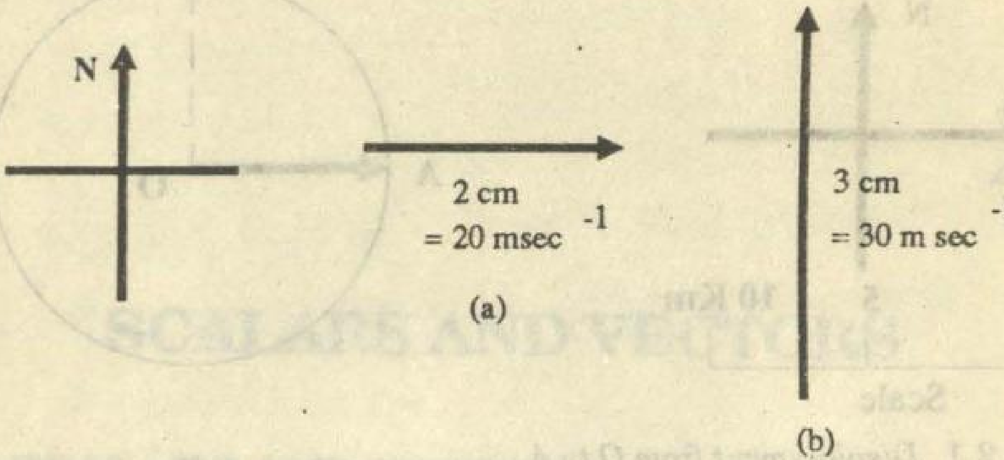


Fig. 2.2 *Representation of a vector.*

resented as in Fig. 2.2. We choose a scale such that 1 cm represents 10 msec^{-1}. Thus the velocity of the first body is represented by a line of length 2 cm, drawn from west to east with the arrow-head at the eastern end (Fig. 2.2a). Similarly, the velocity of the other body is represented by a 3 cm line, drawn from, south to north with the arrow-head at the northern end (Fig 2.2b). The arrow head will be referred to as head and the other end as tail in subsequent discussion.

In the above example, the directions of velocities were described with respect to the geographical axes. In general, the direction of a vector in a plane is measured with respect to two lines, known as axis, drawn pėrpendicular to each other, which form the coordinate system as shown in Fig. 2.3.

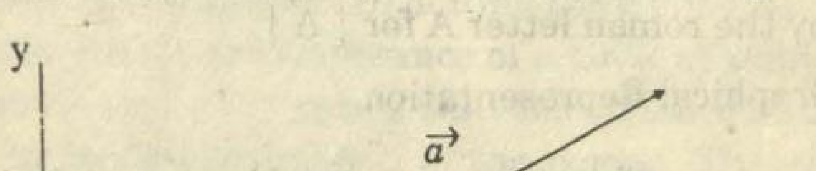


Fig. 2.3 *Specification of a vector* $\vec{a}$*.*

2.3 UNIT VECTOR

A vector whose magnitude or modulus is one is called a unit vector. It is generally represented by a letter $\hat{A}$, i.e. a letter with a cap on it. Suppose we need to find the unit vector $\hat{A}$, in the direction of the vector $\vec{A}$ then $\hat{A} = \frac{\vec{A}}{A}$ or $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ so a unit vector can be obtained by dividing the vector with its magnitude. Unit vector only specifies the direction of a given vector.

In three dimensional space, three mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ along x, y, z axes (rectangular or cartesian coordinates) respectively, are shown in Fig. 2.4.

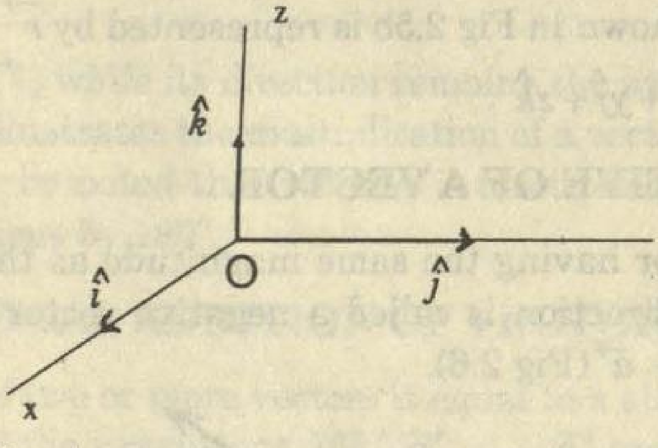


Fig. 2.4 *Unit vectors*

In this case $\vec{A}$ will have three components (A_x, A_y, A_z) along x, y and z axes. Hence $\vec{A}$ is expressed as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

2.4 POSITION VECTOR

In two dimensional cartesian coordinate system, the distance and direction of the point P (x, y) from the origin may be described by an arrow from the origin (0,0) to the point P (x,y) as shown in Fig. 2.5a. The usual notation for this position

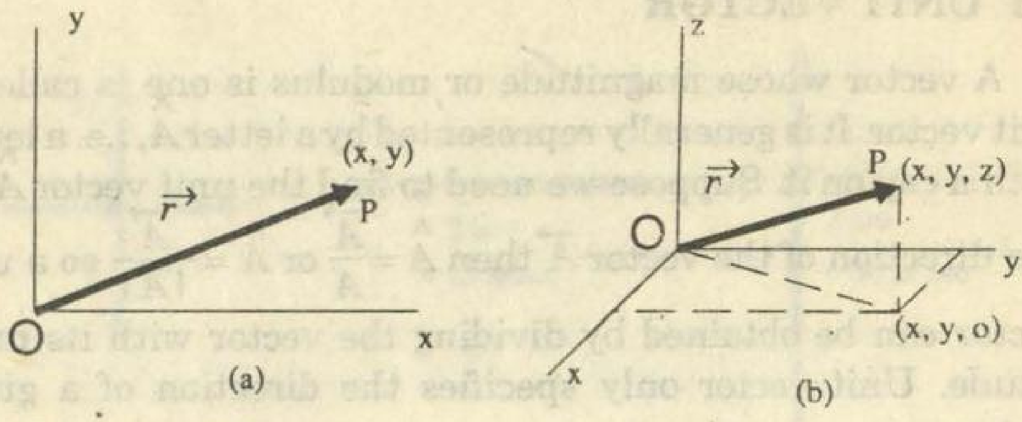


Fig. 2.5 *Position Vector*

vector is $\vec{r}$ which can be written as

$$\vec{r} = x\hat{i} + y\hat{j}$$

In three dimensional spaces the position vector $\vec{r}$ of a point P (x,y,z) as shown in Fig 2.5b is represented by $\vec{r}$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

2.5 NEGATIVE OF A VECTOR

The vector having the same magnitude as that of $\vec{a}$, but opposite in direction is called a negative vector of $\vec{a}$ and is denoted by $-\vec{a}$ (Fig 2.6).

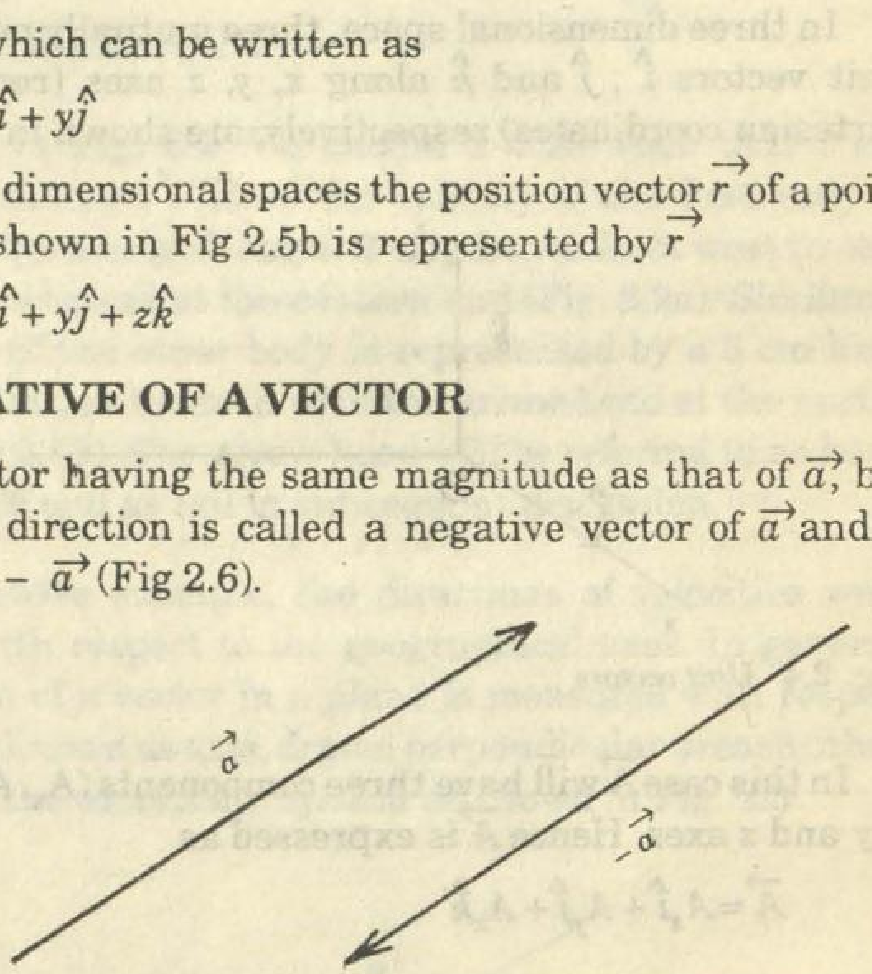


Fig. 2.6 ·*Specification of a negative vector.*

2.6 NULL VECTOR OR ZERO VECTOR

It is a vector having zero magnitude and arbitrary direction. It is denoted by $\vec{0}$.

2.7 MULTIPLICATION OF A VECTOR BY A NUMBER

When a certain vector $\vec{a}$ is multiplied by a number n, then the magnitude of the resultant vector becomes

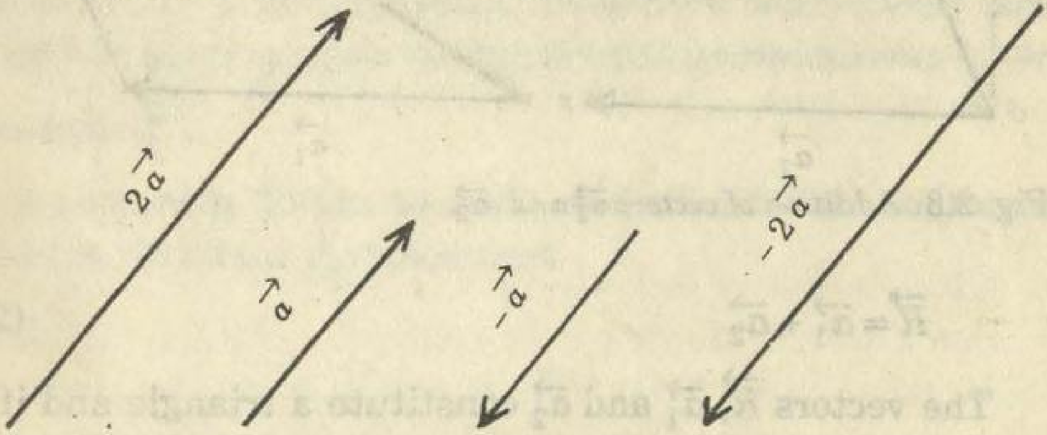


Fig. 2.7 *Multiplication of a Vector by a Scalar.*

$|n\vec{a}| = n\,|\vec{a}|$, while its direction remains the same as that of $\vec{a}$. Fig. 2.7 illustrates the multiplication of a vector by 2, –1, and–2. It may be noted that when $\vec{a}$ is multiplied by –1, its direction changes by 180°.

2.8 GRAPHICAL ADDITION OF VECTORS

The sum of two or more vectors is equal to a single vector which is called their resultant. If $\vec{a}_1, \vec{a}_2, \ldots\ldots \vec{a}_n$ are n vectors, their resultant $\vec{a}$ is represented by,

$$\vec{a} = \vec{a}_1 + \vec{a}_2 + \ldots\ldots \vec{a}_n$$

$$\text{or } \vec{a} = \sum_{r=1}^{n} \vec{a}_r$$

Where Σ (sigma) stands for summation. Then the vector $\vec{R}$, which is drawn from the tail of $\vec{a}_1$ to the head of $\vec{a}_2$, is the sum of $\vec{a}_1$ and $\vec{a}_2$ (Fig. 2.8). The vector $\vec{R}$, called the "resultant of $\vec{a}_1$ and $\vec{a}_2$ is represented by the equation.

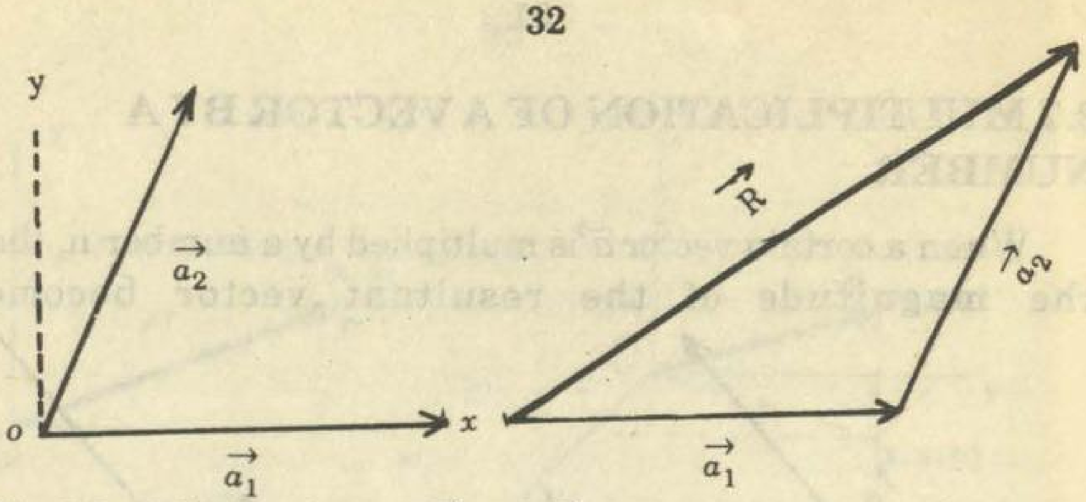


Fig. 2.8 *Addition of vectors* $\vec{a}_1$ *and* $\vec{a}_2$.

$$\vec{R} = \vec{a}_1 + \vec{a}_2 \tag{2.1}$$

The vectors $\vec{R}$, $\vec{a}_1$ and $\vec{a}_2$ constitute a triangle and if the directions (orientations) and magnitudes of $\vec{a}_1$ and $\vec{a}_2$ are known, the ordinary trigonometric laws can be applied to find the direction and magnitude of $\vec{R}$. Graphically, $\vec{a}_1$ and $\vec{a}_2$ can be drawn according to the selected scale. The length and orientation of $\vec{R}$ can be measured with a ruler and a protractor. It may be noted that order of additions of vectors is immaterial i.e.

$$\vec{a}_1 + \vec{a}_2 = \vec{a}_2 + \vec{a}_1$$

It may also be seen from the geometry that the tails of $\vec{a}_1$ and $\vec{a}_2$ can be placed at the common origin and $\vec{R}$ is then the diagonal (starting from the tails of $\vec{a}_1$ and $\vec{a}_2$) of a paral-

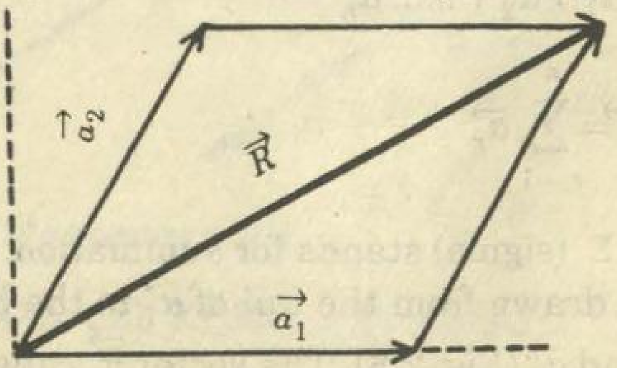


Fig. 2.9 *Parallelogram method of vector addition.*

lelogram whose sides are $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ (Fig. 2.9)

This is commonly known as the parallelogram method of vector addition. In all these manipulations, the absolute position of vector is un-important. They have been moved about as wished keeping their magnitudes and orientations intact.

Example 2.1

A ship sails 50 km to north and then 95 km southwest. Find the resultant displacement.

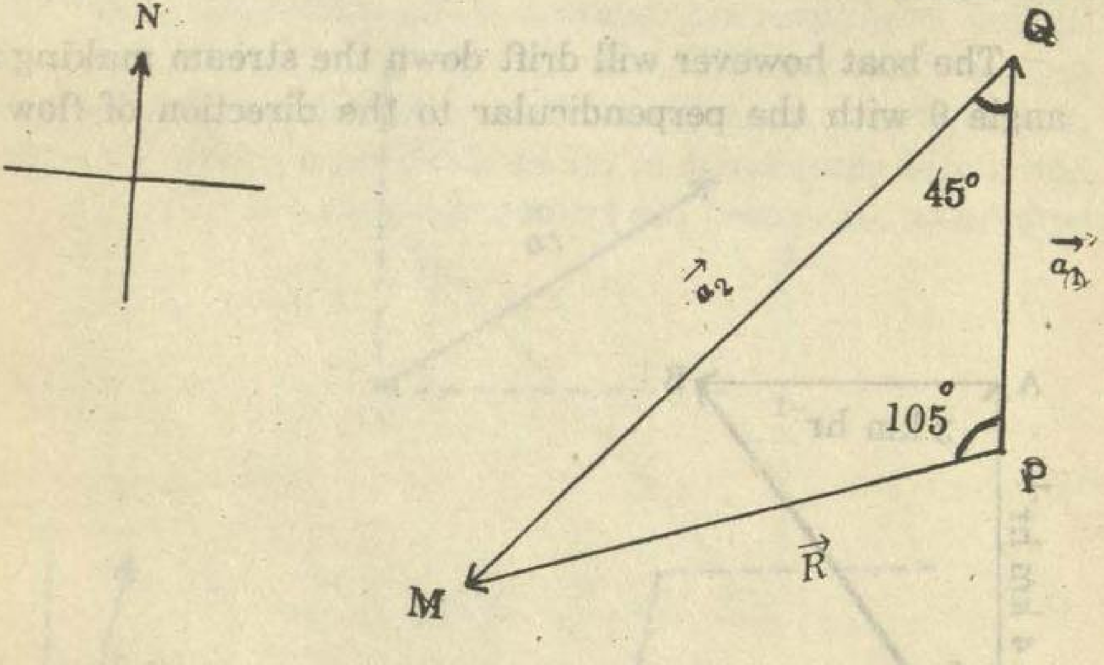


Fig. 2.10 *Shows the vector representation of displacement of a ship and its resultant vector by graphical method.*

In Fig. 2.10 PQ represents the displacement 50 km due to north and QM represents the displacement 95 km south west according to the scale represented above. The resultant displacemnet R is represented by PM. On measuring PM and converting it according to the sacle, it is found to be 70 km approximately and the angle QPM is 105° approximately i.e., 105° approximately South-west.

Example 2.2

A boat whose velocity in still water is 4 km hr^{-1} is required to cross a stream flowing with a velocity of 3 km hr^{-1}. What will be speed and velocity of the boat while crossing the stream if it points directly to the opposite bank perpendicular to the direction of flow of stream.

Solution

The boat however will drift down the stream making an angle θ with the perpendicular to the direction of flow of

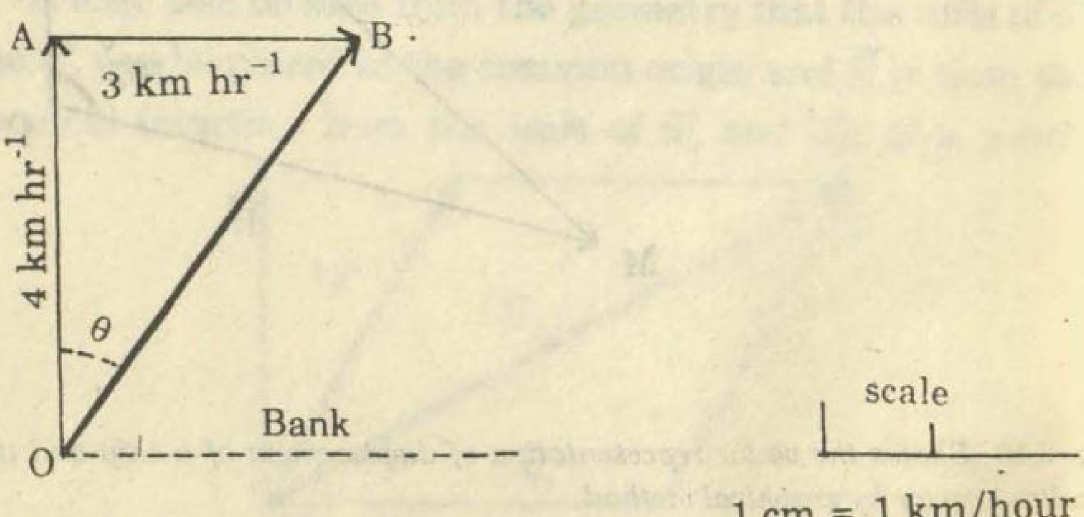


Fig. 2.11 *Boat crossing a river.*

stream. On drawing a vector diagram of the two velocities according to a scale 1 cm = 1 km hr^{-1}, the resultant velocity is $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$, where the length of OB = 5cm. Hence the resultant velocity of the boat is 5 km hr^{-1} making an angle $\theta = 36^o\ 44'$ with $\overrightarrow{OA}$, so the boat moves making an angle of $53^o\ 16'$ with the bank.

2.9 SUBTRACTION OF VECTORS

The difference between $\vec{a_1}$ and $\vec{a_2}$ is $\vec{R}$ which by definition, is the sum of $\vec{a_1}$ and $-\vec{a_2}$ e.g.

$$\vec{R} = \vec{a_1} - \vec{a_2} = \vec{a_1} + (-\vec{a_2})$$

Fig. 2.12 (a,b) illustrates the subtraction of one vector from the other.

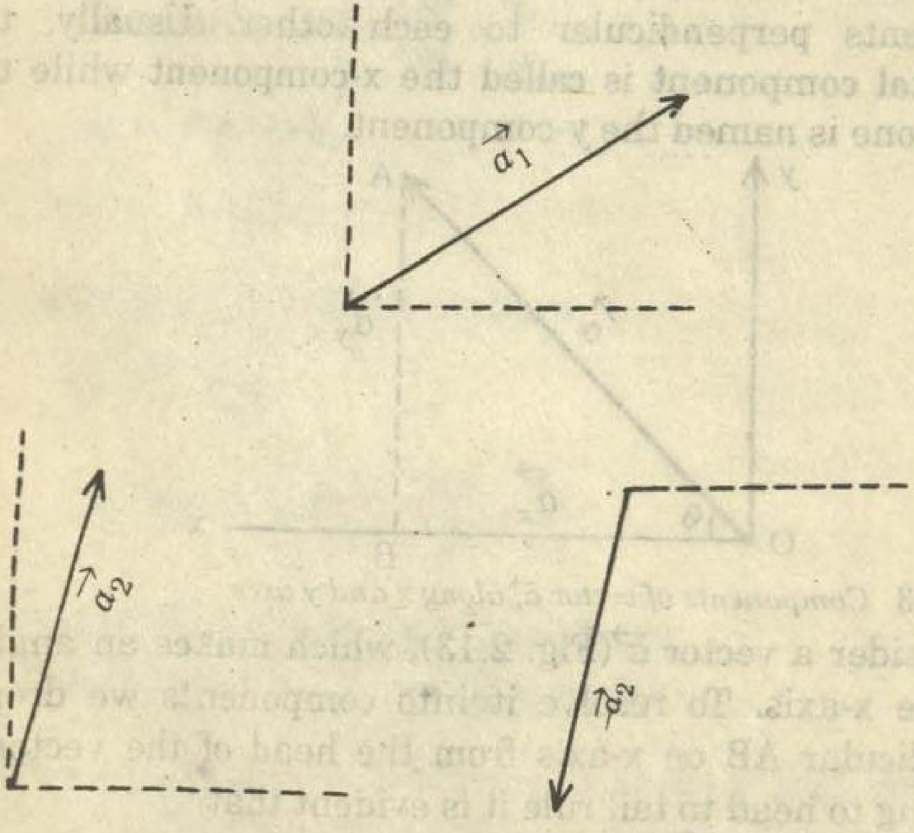


Fig. 2.12 a *Specification of vectors $\vec{a_1}$, $\vec{a_2}$ and $-\vec{a_2}$*

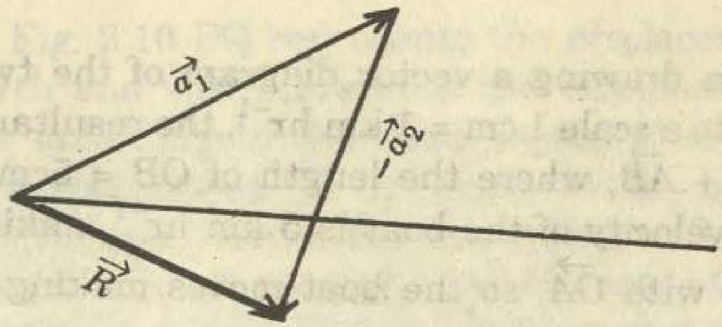


Fig. 2.12 b *Subtraction of $\vec{a_2}$ from $\vec{a_1}$.*

2.10 RECTANGULAR COMPONENTS OF A VECTOR

The process of vector addition can be reversed. The reverse process is known as the resolution of a vector into .its components. Addition of vectors gives a unique result while resolution of a vector leads to a number of results unless further conditions are imposed on its resolution. Generally, one of the conditions is to resolve a vector into its components perpendicular to each other. Usually, the horizontal component is called the x-component while the vertical one is named the y-component.

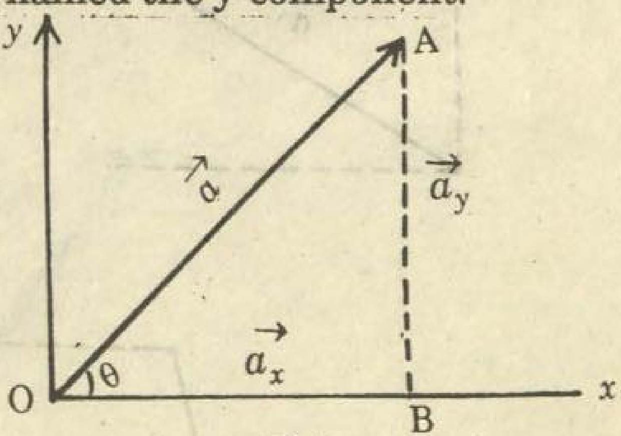


Fig. 2.13 *Components of vector $\vec{a}$, along x and y axes.*

Consider a vector $\vec{a}$ (Fig. 2.13), which makes an angle θ with the x-axis. To resolve it into components we drop a perpendicular AB on x-axis from the head of the vector $\vec{a}$. According to head to tail rule it is evident that

$$\vec{OA} = \vec{OB} + \vec{BA}$$

$$\text{or } \vec{a} = \vec{a_x} + \vec{a_y} \tag{2.3}$$

In other words, the vector $\vec{a}$ has been resolved into mutually perpendicular components. These components are also referred as rectangular components. If a_x and a_y are the magnitudes of the components and $\hat{i}$ and $\hat{j}$ are the unit vectors along x - and y - axis, then Eq. 2.3 can be written as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \tag{2.4}$$

Now consider the triangle OAB (Fig. 2.13).

$$\text{Here } \sin\theta = \frac{AB}{OA} = \frac{a_y}{a}$$

$$\text{or } a_y = a \sin\theta \tag{2.5}$$

$$\text{and } \vec{a_y} = a \sin\theta\, \hat{j}$$

$$\text{Similarly, } \cos\theta = \frac{OB}{OA} = \frac{a_x}{a}$$

$$\text{or } a_x = a \cos\theta \tag{2.6}$$

$$\text{and } \vec{a_x} = a \cos\theta\, \hat{i}$$

From Eqs. 2.5 and 2.6 we have

$$a^2 = a_x^2 + a_y^2 \tag{2.7}$$

$$\text{and } \tan\theta = \frac{AB/OA}{OB/OA} = \frac{AB}{OB} = \frac{a_y}{a_x} \tag{2.8}$$

2.11 ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

The advantages of resolution of a vector into its rectangular components become obvious when one has to deal with several vectors making different angles with x - axis.

In the first instance, addition of two vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ will be considered and then it will be generalized to n vectors, $\overrightarrow{a_1}$,

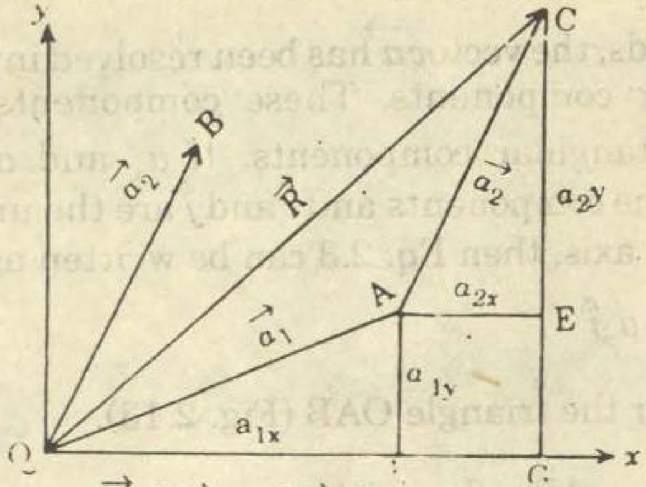


Fig. 2.14 *The sum $\overrightarrow{R}$ of $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ shown in cartesian components.*

The two vector $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ are represented by OA and OB.

The vector $\overrightarrow{a_2}$ is added to $\overrightarrow{a_1}$ by the head-to-tail rule of vector addition. Thus the resultant $\overrightarrow{R} = \overrightarrow{a_1} + \overrightarrow{a_2}$ is represented by the vector $\overrightarrow{OC}$ (Fig. 2.14). Since,

$$\overrightarrow{a_1} = a_{1x}\,\hat{i} + a_{1y}\,\hat{j} \tag{2.9}$$

$$\overrightarrow{a_2} = a_{2x}\,\hat{i} + a_{2y}\,\hat{j} \tag{2.10}$$

$$\overrightarrow{R} = R_x\,\hat{i} + R_y\,\hat{j} \tag{2.11}$$

Where R_x and R_y are the x and y components of $\overrightarrow{R}$ respectively. In the figure above

$$a_{1x} = OF \text{ and } a_{2x} = AE = FG$$

$$OG = OF + FG$$

$$= OF + AE$$

$$or\ R_x = a_{1x} + a_{2x} \tag{2.12}$$

Eq. 2.12 states that the sum of the magnitude of the x-components of the two vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$, is equal to the magnitude of the x-component of their resultant $\overrightarrow{R}$. Similarly, it can be proved that the sum of the y-components of vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ is equal to the y-component of the resultant $\overrightarrow{R}$, and is given by

$$R_y = a_{1y} + a_{2y} \tag{2.13}$$

Using the results of section 2.10 , the x and y components of the given vectors are found. These components are added separately to obtain x and y components of the resultant and thus the resultant vector is determined.

Consider n-vectors $\overrightarrow{a_1}$, $\overrightarrow{a_2}$, ... $\overrightarrow{a_n}$ located at the point O (Fig. 2.15).

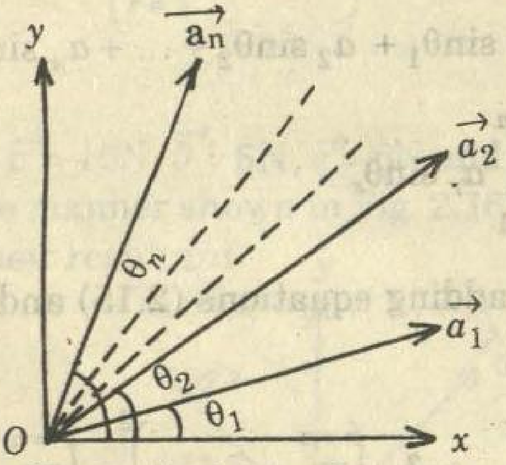


Fig. 2.15 *Addition of vectors by their resolution along x and y axis.*

If $a_{1x}, a_{2x}, \dots, a_{nx}$ are the cartesian components along x-axis of the n-vectors, then R_x, the x-component of the resultant $\overrightarrow{R}$, is given by

$$R_x = a_{1x} + a_{2x} + \dots\dots + a_{nx} \tag{2.14}$$

As discussed in Eq. 2.6.

$$R_x = R \cos\theta$$

where $R = |\overrightarrow{R}|$

$$a_{1x} = a_1 \cos\theta_1$$

$$a_{2x} = a_2 \cos\theta_2$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$a_{nx} = a_n \cos\theta_n$$

$$\therefore R\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2 + \ldots + a_n\cos\theta_n$$

$$R\cos\theta = \sum_{r=1}^{r=n} a_r \cos\theta_r \tag{2.15}$$

Similarly,

$$R\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2 + \ldots + a_n\sin\theta_n$$

$$\text{or } R\sin\theta = \sum_{r=1}^{r=n} a_r \sin\theta_r \tag{2.16}$$

Squaring and adding equations (2.15) and (2.16). We obtain,

$$(R\cos\theta)^2 + (R\sin\theta)^2 = \left(\sum_{r=1}^{r=n} a_r\cos\theta_r\right)^2 + \left(\sum_{r=1}^{r=n} a_r\sin\theta_r\right)^2$$

$$R^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\sum_{r=1}^{r=n} a_r\cos\theta_r\right)^2 + \left(\sum_{r=1}^{r=n} a_r\sin\theta_r\right)^2$$

$$\text{or, } R^2 = \left(\sum_{r=1}^{r=n} a_r\cos\theta_r\right)^2 + \left(\sum_{r=1}^{r=n} a_r\sin\theta_r\right)^2$$

$$\text{i.e., } R = \sqrt{\left(\sum_{r=1}^{r=n} a_r\cos\theta_r\right)^2 + \left(\sum_{r=1}^{r=n} a_r\sin\theta_r\right)^2} \tag{2.17}$$

Dividing equation (2.16) by (2.15) we get

$$\tan\theta = \frac{\sum_{r=1}^{r=n} a_r \sin\theta_r}{\sum_{r=1}^{r=n} a_r \cos\theta_r}\cdot$$

$$or\ \theta = \tan^{-1}\left(\frac{\sum_{r=1}^{r=n} a_r \sin\theta_r}{\sum_{r=1}^{r=n} a_r \cos\theta_r}\right) \qquad (2.18)$$

Example 2.3

Four forces $\overrightarrow{a} = 10N$, $\overrightarrow{b} = 5N$, $\overrightarrow{c} = 6N$ and $\overrightarrow{d} = 6N$ are acting at a point in the manner shown in Fig. 2.16. Using resolution method, find their resultant.

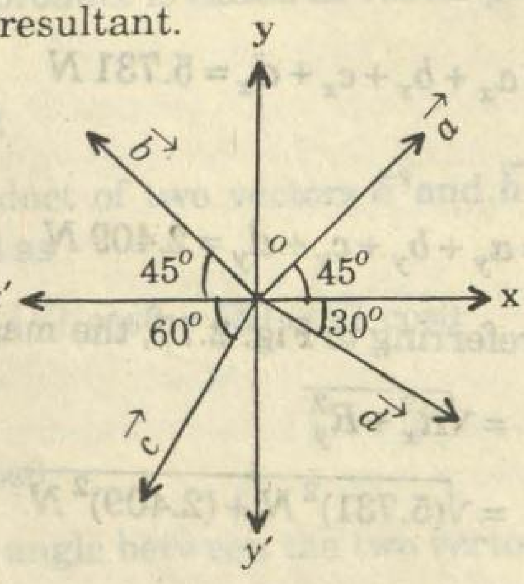


Fig. 2.16

Solution

As the vector $\overrightarrow{a} = a_x\,\hat{i} + a_y\,\hat{j}$,

$$\text{then } a_x = a\cos45^o N = 7.07\ N$$

$$a_y = a\sin45^o N = 7.07\ N$$

Similarly, for $\overrightarrow{b}$.

$$b_x = -\,b\cos 45^\circ N = -3.535\ N$$

$$b_y = b\sin 45^\circ N = 3.535\ N$$

For $\vec{c}$

$$c_x = -\,c\cos 60^\circ N = -3.00\ N$$

$$c_y = -\,c\sin 60^\circ N = -5.196\ N$$

For $\vec{d}$

$$d_x = d\cos 30^\circ N = 5.196\ N$$

$$d_y = -\,d\sin 30^\circ N = -3.00\ N$$

Then,

$$R_x = a_x + b_x + c_x + d_x = 5.731\ N$$

and

$$R_y = a_y + b_y + c_y + d_y = 2.409\ N$$

Hence referring to Fig. 2.17, the magnitude

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(5.731)^2 N + (2.409)^2 N}$$

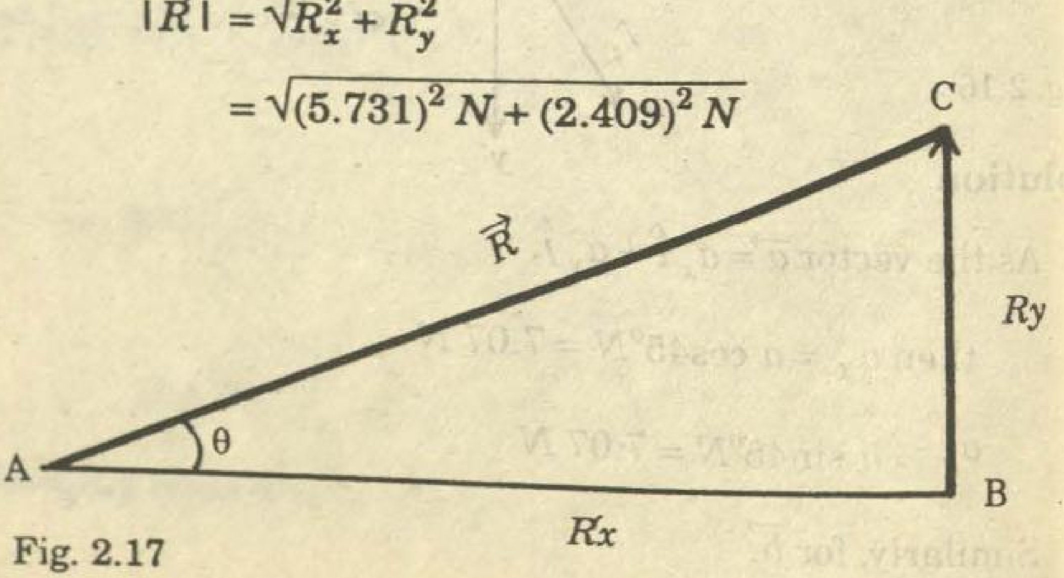


Fig. 2.17

$$= 6.217\,N$$

$$\text{and } \tan\theta = \frac{BC}{AB} = 0.420$$

$$\therefore \theta = \tan^{-1} 0.420$$

$$= 22^{\circ}\,48'$$

that is the resultant makes an angle $22^{\circ}\,48'$ with the x-axis.

2.12 MULTIPLICATION OF TWO VECTORS

There are two types of multiplication of vectors.

i) When the product of two vectors results into a scalar quantity, the product is termed as scalar product or dot product.

ii) when the multiplication of two vectors results into a vector, the product is called as vector product or cross product.

i) Scalar Product

The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is written as $\vec{a}\cdot\vec{b}$ and is defined as

$$\vec{a}.\vec{b} = |\vec{a}|\;|\vec{b}|\cos\theta = |\vec{b}|\;|\vec{a}|\cos\theta$$

$$\vec{a}.\vec{b} = \vec{b}.\vec{a}$$

$$\vec{a}.\vec{b} = ab\cos\theta \tag{2.19}$$

where θ is the angle between the two vectors.

Consider two vectors $\vec{a}$ and $\vec{b}$ as shown in Fig. 2.18. Draw a perpendicular from A to OB and from B to OA.

Thus $\vec{a}.\vec{b} = |\vec{a}|\;(|\vec{b}|\cos\theta)$

or, $\vec{a}\cdot\vec{b}$ = (magnitude of a) times (scalar projection of $\vec{b}$ on $\vec{a}$)

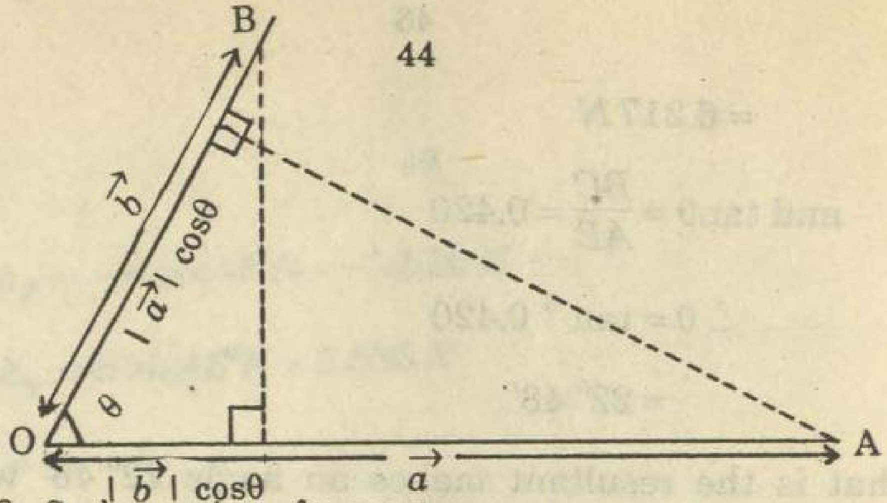


Fig. 2.18 *Scalar product of two vectors*

Also $\vec{b} \cdot \vec{a} = |\vec{b}| \, (|\vec{a}| \cos\theta)$

= (magnitude of b) times (scalar projection of $\vec{a}$ on $\vec{b}$)

Hence, the scalar or dot product of two vectors is the product of the modulus of either vector and the scalar projcetion of the other in its direction. Thus the dot product of two vectors is a scalar quantity.

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

A simple example of a dot product is the work which is a scalar quantity obtained by the scalar product of two vectors force $\vec{F}$ and displacement $\vec{S}$.

i.e., Work = $\vec{F} \cdot \vec{S} = FS \cos\theta$

It may be noted that the scalar product of two vectors is not similar to the product of twc numbers. The product of two non-zero numbers is never equal to zero, whereas the scalar product of two non-zero vector is equal to zero if these are at right angle to each other. For example the unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ are perpendicular to each other, therefore,

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \qquad (2.20)$$

The scalar product of a vector $\vec{a}$ with itself i.e. $\vec{a} \cdot \vec{a}$ is called the square of the vector. In this case $\theta = 0$, and $\cos 0 = 1$

Hence

$$\vec{a}.\vec{a} = |a|^2 = a^2 \qquad (2.21)$$

and so for the unit vectors,

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1 \qquad (2.22)$$

From our previous knowledge, $\vec{a}$ and $\vec{b}$ can be written as,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

Hence, the scalar product of $\vec{a}$ and $\vec{b}$ will be

$$\vec{a}.\vec{b} = \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right).\left(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}\right)$$

$$= a_xb_x + a_yb_y + a_zb_z \qquad (2.23)$$

Characteristics of Dot Product

i) since $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ hence scalar product is commutative.

ii) If a.b = 0, then either of the two vectors $\vec{a}$ or $\vec{b}$ is a null vector or the vectors are mutually persendicular.

iii) If the vectors $\vec{a}$ and $\vec{b}$ are parallel to each other, then $\vec{a}.\vec{b} = \pm ab$ because when

$\theta = 0^\circ, \cos\theta = 1$

$\theta = 180^\circ, \cos\theta = -1$

iv) $(m\,\vec{a}).(n\,\vec{b}) = mn\,(\vec{a}.\vec{b})$
$= mn\,\vec{a}.\vec{b} = \vec{a}.\,mn\,\vec{b} = n\vec{a}.\,m\vec{b}$

where m and n are numbers. Thus dot product obeys associative law.

ii) Vector Product

Consider two vectors $\vec{a}$ and $\vec{b}$ in a certain plane making an

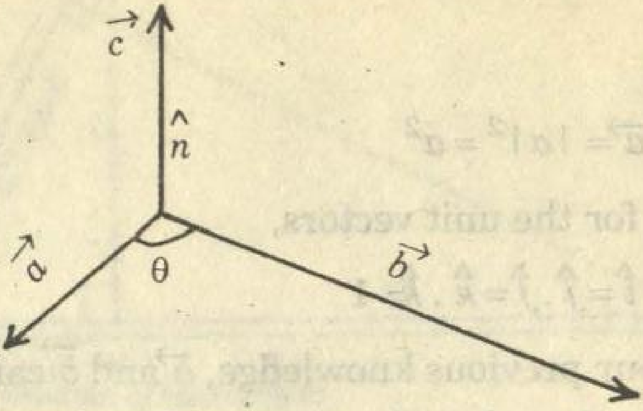


Fig. 2.19 *Vector product of two vectors*

angle θ with each other (Fig. 2.19). The vector product $\vec{c}$ is defined as

$$\vec{c} = \vec{a} \times \vec{b} = |a|\ |b|\ \sin\theta\ \hat{n} \tag{2.24}$$

Where $\hat{n}$ is a unit vector normal to the plane, containing $\vec{a}$ and $\vec{b}$. The direction of this vector is obtained with the help of a right hand screw (Fig. 2.20), whose axis is perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. $\vec{b}$ is rotated from $\vec{a}$ to $\vec{b}$ through the angle θ (which lies between 0 and 180). The direction along which the screw advances indicates the direction of the resultant vector. The direction of the resultant vector can also be found by a simple rule, called right hand rule. Hold your right hand in the manner shown in Fig. 2.20. The direction of the thumb should be perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. If the fingers are folded in the direction in which $\vec{a}$ rotates

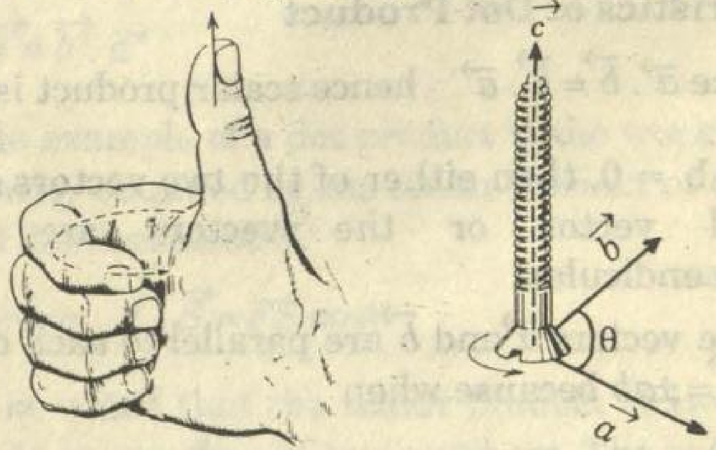


Fig. 2.20 *The Right hand Rule.*

toward $\vec{b}$ through the angle θ ($0 < \theta < \pi$), then the thumb points towards the direction of the resultant vector $\vec{c}$. Thus the vector product of two vectors is a vector quantity.

$$\text{Magnitude of } \vec{a} \times \vec{b} = |\vec{a} \times \vec{b}|$$

$$= |\vec{a}|\ |\vec{b}| \sin\theta$$

$$= (OA)\ (OB)\ \sin\theta$$

= area of the parallelogram in which the two adjacent sides are a and b.

Characteristics of a Vector product

i) On reversing the order of the vectors, the sign of the vector product is reversed. In order to find the direction of the vector product $\vec{a} \times \vec{b}$, the sense of rotation of the right hand screw is from $\vec{a}$ to $\vec{b}$. On the other hand to find the direction of the vector $\vec{b} \times \vec{a}$, the direction of rotation of the right hand screw will be from $\vec{b}$ to $\vec{a}$. Thus the direction of vector product in this case i.e. $\vec{b} \times \vec{a}$, is opposite to the direction of the vector product $\vec{a} \times \vec{b}$ (i.e.,)

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} \qquad (2.25)$$

Therefore, the vector multiplication is not commutative.

ii) The vector product is associative.

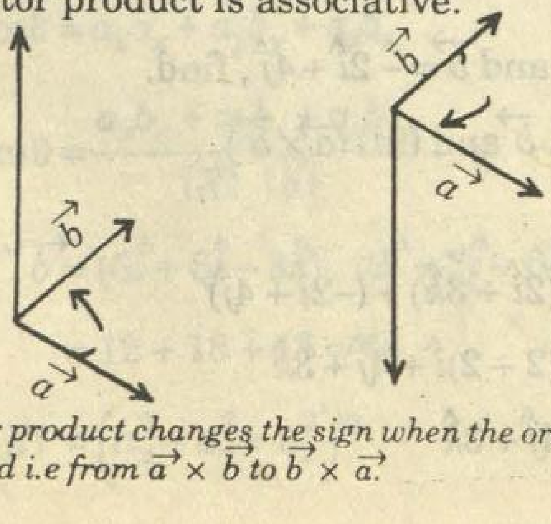


Fig. 2.21 *Vector product changes the sign when the order of vectors in the product is reversed i.e from $\vec{a} \times \vec{b}$ to $\vec{b} \times \vec{a}$.*

If m is a scalar, then

$$(m\vec{a} \times \vec{b}) = \vec{a} \times (m\vec{b}) = m\,(\vec{a} \times \vec{b})$$

iii) If $(\vec{a} \times \vec{b}) = 0$, then either of the two vector is a null vector or vectors $\vec{a}$ and $\vec{b}$ are parallel to each other. For the unit vector $\hat{i}, \hat{j}$ and $\hat{k}$, we have the following relations:

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \tag{2.26}$$

$$\left.\begin{aligned} &\hat{i} \times \hat{j} = \hat{k};\ \hat{j} \times \hat{i} = -\hat{k} \\ &\hat{j} \times \hat{k} = \hat{i};\ \hat{k} \times \hat{j} = -\hat{i} \\ &\hat{k} \times \hat{i} = \hat{j};\ \hat{i} \times \hat{k} = -\hat{j} \end{aligned}\right\} \tag{2.27}$$

The Vector Product of two Vectors

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ is given by

$\vec{a} \times \vec{b} = \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right) \times \left(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}\right)$

$= \hat{i}\left(a_y b_z - a_z b_y\right) + \hat{j}\left(a_z b_x - a_x b_z\right) + \hat{k}\left(a_x b_y - a_y b_x\right)$. An example of a vector product is the force $\vec{F}$ acting on a charged particle moving with a velocity $\vec{V}$ in a magnetic field $\vec{B}$ and is given by,

$$\vec{F} = q\vec{V} \times \vec{B}$$

where q is the charge on the particle

$\vec{F}$ is perpendicular to the plane containing $\vec{V}$ and $\vec{B}$

Example 2.4

If $\vec{a} = 2\hat{i} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + 4\hat{j}$, find.

i) $\vec{a} + \vec{b}$ (ii) $\vec{a} \cdot \vec{b}$ and (iii) $(\vec{a} \times \vec{b})$

Solution

$$\begin{aligned} i)\ \vec{a} + \vec{b} &= (2\hat{i} + 3\hat{k}) + (-2\hat{i} + 4\hat{j}) \\ &= (2 - 2)\hat{i} + 4\hat{j} + 3\hat{k} \\ &= 4\hat{j} + 3\hat{k} \end{aligned}$$

$$ii)\ \vec{a}\,.\,\vec{b} = (2\hat{i} + 3\hat{k})\,.\,(-2\hat{i} + 4\hat{j})$$

$$= (-4\hat{i} \times \hat{i}) + (8\hat{i} \times \hat{j}) - (6\hat{k} \times \hat{i}) + (12\hat{k} \times \hat{j})$$

$$= -4$$

$$iii)\ \vec{a} \times \vec{b} = (2\hat{i} + 3\hat{k}) \times (-2\hat{i} + 4\hat{j})$$

$$= (-4\hat{i} \times \hat{i}) + (8\hat{i} \times \hat{j}) - (6\hat{k} \times \hat{i}) + (12\hat{k} \times \hat{j})$$

$$= 0 + 0 + 8\hat{k} - 6\hat{j} - 12\hat{i}$$

$$= -12\hat{i} - 6\hat{j} + 8\hat{k}$$

Example 2.5

Find the angle between the vectors $\vec{a}$ and $\vec{b}$,

$$\vec{a} = 6\hat{i} + 6\hat{j} - 3\hat{k}$$

and $$\vec{b} = 2\hat{i} + 3\hat{j} - 6\hat{k}$$

Solution

Since, $\vec{a}\,.\,\vec{b} = |\vec{a}|\ |\vec{b}| \cos\theta$

We know that

$$\vec{a}\,.\,\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

Hence

$$|\vec{a}|\ |\vec{b}| \cos\theta = a_x b_x + a_y b_y + a_z b_z$$

$$\text{or } \cos\theta = \frac{a_x b_x + a_y b_y + a_z b_z}{|\vec{a}|\ |\vec{b}|}$$

$$\vec{a}\,.\,\vec{b} = (6\hat{i} + 6\hat{j} - 3\hat{k})\,.\,(2\hat{i} + 3\hat{j} - 6\hat{k})$$

$$= 12 + 18 + 18 = 48$$

$$|\vec{a}| = \left(a_x^2 + a_y^2 + a_z^2\right)^{1/2}$$

$$= \left(6^2 + 6^2 + (-3)^2\right)^{1/2}$$

$$= \sqrt{81} = 9$$

$$|\vec{b}| = \left(b_x^2 + b_y^2 + b_z^2\right)^{1/2}$$

$$= \left(2^2 + 3^2 + (-6)^2\right)^{1/2}$$

$$= \sqrt{49} = 7$$

$$\therefore \cos\theta = \frac{48}{9 \times 7} = 0.7619$$

or $\theta \approx 40^o$

Example 2.6

Show that

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} . \vec{b}|^2 = |\vec{a}|^2 \ |\vec{b}|^2$$

Solution

Since $\vec{a} \times \vec{b} = |\vec{a}| \ |\vec{b}| \sin\theta \ \hat{n}$

Hence $|\vec{a} \times \vec{b}| = |\vec{a}| \ |\vec{b}| \sin\theta \ |\hat{n}|$

$$= |\vec{a}| \ |\vec{b}| \sin\theta$$

Also $\vec{a} . \vec{b} = |\vec{a}| \ |\vec{b}| \cos\theta$

Therefore

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} . \vec{b}|^2$$

$$= [\,|\vec{a}| \ |\vec{b}| \sin\theta]^2 + [\,|\vec{a}| \ |\vec{b}| \cos\theta]^2$$

$$= |\vec{a}|^2 \ |\vec{b}|^2 \left(\sin^2\theta + \cos^2\theta\right)$$

$$= |a|^2 \ |b|^2$$

SUMMARY

A scalar is a physical quantity which is completely specified by a magnitude with appropriate unit.

A vector is a physical quantity which is completely specified by a magnitude in appropriate unit and a direction.

A vector is represented by $\vec{a}$, its magnitude by a and unit vector along the direction of $\vec{a}$ by $\hat{a}$. Thus $\vec{a} = a\ \hat{a}$

Addition of $\vec{a}$ and $\vec{b}$ gives a new vector $\vec{c}$ such that

$$\vec{c} = \vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Subtraction of $\vec{b}$ from $\vec{a}$ results in a new vector $\vec{d}$ such that

$$\vec{d} = \vec{a} - \vec{b}$$

Multiplication of $\vec{a}$ by a scalar n simply changes the magnitude of the new vector by n i.e., $\vec{a} = n\ \vec{a}$.

Scalar product of $\vec{a}$ *and* $\vec{b}$.is a scalar quantity given by,

$$\vec{a} \cdot \vec{b} = ab \cos\theta = \vec{b} \cdot \vec{a}$$

Where θ is the angle between $\vec{a}$ and $\vec{b}$.

Vector product of $\vec{a}$ and $\vec{b}$ is *a* vector such that

$$\vec{a} \times \vec{b} = ab \sin\theta\hat{n} = -\vec{b} \times \vec{a}$$

Where n is a unit vector normal to the plane containing $\vec{a}$ and $\vec{b}$, Direction of $\hat{n}$ is obtained by the right hand rule.

Addition of n-vectors $\vec{a_1}\ \vec{a_2}\ \vec{a_3} \dots \vec{a_n}$ is carried out by using their components. If $\vec{R}$ is their resultant, then

$$R^2 = \left(\sum_{r=1}^{n} a_r \cos\theta_r\right)^2 + \left(\sum_{r=1}^{n} a_r \sin\theta_r\right)^2$$

Where θ_r is the angle of $\overrightarrow{a_r}$ with x-axis.

and

$$\tan\theta = \frac{\sum_{r=1}^{n} a_r \sin\theta_r}{\sum_{r=1}^{n} a_r \cos\theta_r}$$

QUESTIONS

2.1 Give five example each of Scalars and Vectors not mentioned in this chapter.

2.2. Can the magnitude of the resultant of two vectors be greater than the sum of magnitudes of individual vectors?

2.3. Is it possible to add three vectors of equal magnitude but different directions to get zero? If you think so, illustrate with a diagram.

2.4. If $\vec{c}$ is the vector sum of $\vec{a}$ and $\vec{b}$, does $\vec{c}$ have to lie in the plane determined by a and b?

2.5. Three vectors can be added to obtain a resultant. Can the process be reversed, i.e., can a vector be resolved into three components lying along specified directions in a plane?

2.6. Can a scalar product of two vectors be negative?If your answer is yes, give an example, if no, provide a proof.

2.7. If $\overrightarrow{a}.\overrightarrow{b} = o$, can it be concluded that a and b are perpendicular to each other.

2.8. If $\vec{a} \times \vec{b} = 0$, can it be concluded that $\vec{a}$ and $\vec{b}$ are parallel (or antiparallel)? What other information, if any, is needed to prove that $\vec{a}$ and $\vec{b}$ are parallel (or antiparallel).

2.9. Suppose $\vec{a}$ is a non-zero vector. If $\vec{a} \cdot \vec{b} = 0$ and also $\vec{a} \times \vec{b} = 0$, where $\vec{b}$ is an unknown vector. What can you conclude about $\vec{b}$?

PROBLEMS

2.1. Represent the following vector quantities with the help of straight lines:

Vector	*Magnitude*	*Direction*
i) Velocity	100 km sec^{-1}	30^{o} north east.
ii) Force	50 N	120^{o} with the x-axis.
iii) Displacement	150 km	south - west.

2.2. A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30^{o} east of north for 25 km. Find the total displacement of the car from the starting point.

2.3. A mule is pulling a barge along a canal exerting a force of 650 N at an angle of 30^{o} to one side of the direction of motion of the barge. Calculate the work done by the mule $\vec{F} \cdot \Delta \vec{S}$ as it moved 100 m.

2.4. At a certain instant the position of a stone in a sling is given by $r = 1.70$ m along the positive x-axis. The linear momentum $\vec{p}$ of the stone is 12 kg m sec^{-1} along the positive y-axis. Calculate the angular momentum

$\vec{L} = \vec{r} \times \vec{p}$ of the stone about the origin.

2.5. Vector $\vec{T}$ lies along the positive y-axis and $|T| = 3$. $\vec{S}$ is in the first quadrant of the x-y plane, making an angle of 60° with the x-axis as shown in the Fig. 2.22, S = 2. Calculate $(\vec{S} \times \vec{T})$

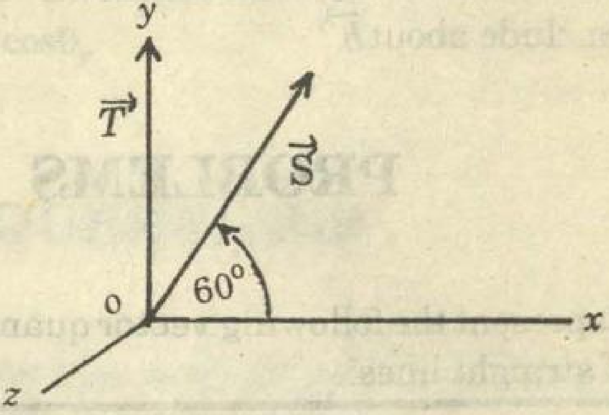


Fig. 2.22

2.6. What is the displacement of the point of a wheel intially in contact with the ground when the wheel rolls forward ½ rev? Take the radius of the wheel as R and the x-axis in the forward direction.

2.7. Vector $\vec{a}$ is 8 in the upward direction, $\vec{b}$ has a magnitude 6 and the scalar product $\vec{a} \cdot \vec{b} = 0$. What are the possible directions and magnitude of $\vec{a} \times \vec{b}$.

2.8. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{a} . \vec{b} = 12$ and $\vec{a} \times \vec{b} = 0$, find $\vec{b}$.

2.9. Find the value of q, for which the following two vectors will become perpendicular to each other

$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{b} = 13\hat{i} + q\hat{j} + 2\hat{k}$

2.10. Supose $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

(a) Find a unit vector in the same direction as $\vec{a}$.

(b) Find a vector in the same direction as $\vec{a}$ but of magnitude 12.

(c) Find a vector perpendicular to $\vec{a}$?

2.11 A force of 20 N acts on a body and moves it 2m. The line of motion of the body and the direction of the force are shown in Fig. 2.23. Calculate the work done, $W = \vec{F} \cdot \Delta\vec{S}$, by using the two forms of the scalar product.

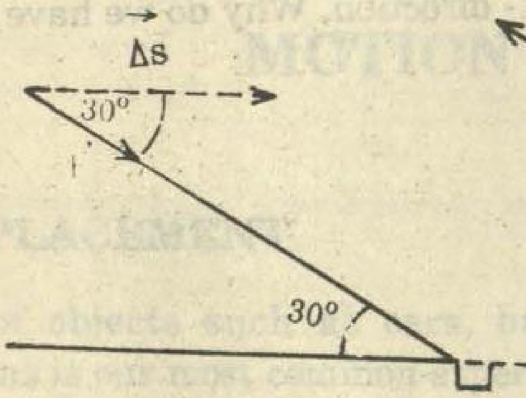


Fig. 2.23

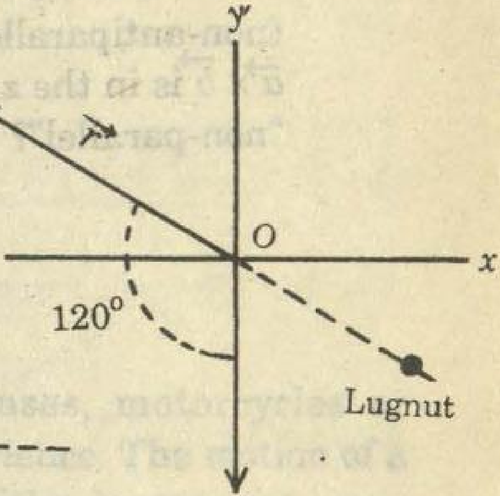


Fig. 2.24

2.12. While changing a tyre a mechanic exerts a force of 500 N on the end of a 0.6 m long tyre wrench, as shown in Fig. 2.24. Calculate the magnitude of the torque $\vec{\tau} = \vec{r} \times \vec{F}$ at the lugnut using two forms of vector product.

2.13. The equation $\vec{F} = q(\vec{v} \times \vec{B})$ gives the force $\vec{F}$ on an electric charge q moving with a velocity $\vec{v}$ through a magnetic field B. Calculate the force $\vec{F}$ on a proton having a charge 1.60×10^{-19} C) moving with a velocity $(2.1\hat{i} + 0.2\hat{j} + 1.3\hat{k}) \times 10^{5}$ $msec^{-1}$ in a magnetic field 0.62 Tesla

2.14. At a certain instant the position of a stone in a ring is given by $\vec{r} = 1.7\,\hat{i}\,m$. The linear momentum $\vec{p}$ of the stone is $12\hat{j}$ kg msec^{-1}. Calculate the angular momentum $\vec{L} = \vec{r} \times \vec{p}$ of the stone about the origin.

2.15. The electro-magnetic pointing vector $\vec{S}$ is defined by $\vec{S} = \vec{E} \times \vec{H}$ where E and H are electric and magnetic fields for $\vec{E} = 10.1\hat{i} + 0.2\hat{j} + 0.6\hat{k}$ and $\vec{H} = -0.4\hat{i} + 9.8\hat{j} + 0.1\hat{k}$, Calculate S. (disregard units for the time being).

2.16. If $\vec{a} = a_1\,\hat{i} + a_2\,\hat{j}$ and $\vec{b} = b_1\,\hat{i} + b_2\,\hat{j}$ are any non-parallel (non-antiparallel) vectors in the x-y plane. show that $\vec{a} \times \vec{b}$ is in the z - direction. Why do we have to specify "non-parallel"?

3

MOTION

3.1 DISPLACEMENT

Motion of objects such as cars, buses, motorcycles or pedestrains is our most common experience. The motion of a body is described by ascribing it a position in space at every instant of time. The change in position with time then determines the quantity of motion. The physical quantity which represents the motion of an object is called displacement and is a measure of the difference in position between two points or places. The shortest distance between these two points is the magnitude of this quantity, whereas its direction is that of the vector drawn from the initial point to the final point. Therefore, displacement is a vector quantity.

In Fig. 3.1, let us consider an aeroplane which starts from Karachi and flies north west to Quetta. After disembarking some of its passengers it flies east towards Lahore. From Lahore it flies in the north westren direction and lands at Peshawar. The net displacement representing the journey from Karachi to Peshawar will be represented by a straight line originating from Karachi and terminating at Peshawar in the north east direction.

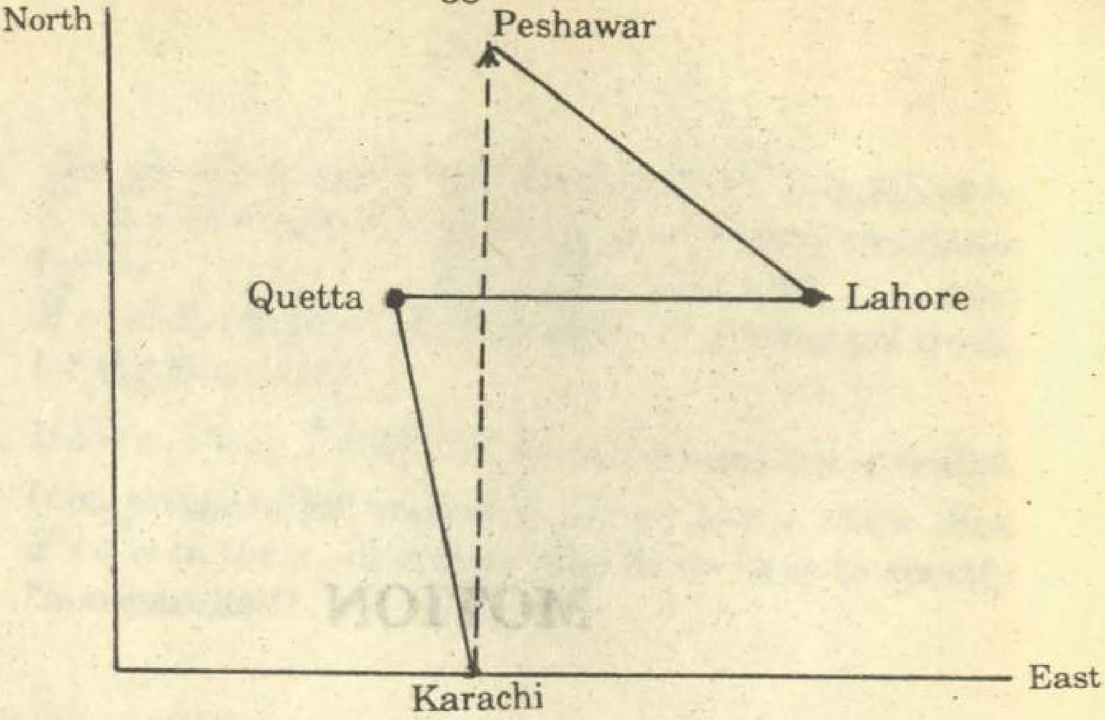


Fig. 3.1 *Flight of an aeroplane from Karachi - Quetta, Lahore, Peshawar.*

3.2 VELOCITY

In order to define motion of objects we will need to define many other quantities like speed, velocity, acceleration etc; which are not fundamental but derived. *Velocity is defined as the rate of change of displacement.* Thus if a body undergoes a displacement $\vec{d}$ during the time interval t, then its velocity is written as:

$$\vec{V} = \frac{\vec{d}}{t} \qquad (3.1)$$

Since displacement is a vector quantity velocity is also a vector with dimension lt^{-1} and is measured in metre per second in SI units. When a body undergoes equal displacements in equal intervals of time, however, small, the body is said to move with uniform velocity and its motion is called uniform motion. In case the body covers different distances in equal intervals of time, it is then said to move with a variable velocity and the motion of the body is said to be non-uniform.

In such situations the concept of average velocity becomes useful.

Let a body be at position x_1 at time t_1 and at a position x_2 at time t_2, then the average velocity V_{av} is defined as

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \therefore\ V_{av} = \frac{\Delta x}{\Delta t} \tag{3.2}$$

It may be noted that the average velocity is independent of the path chosen by the body between the position x_1 and x_2

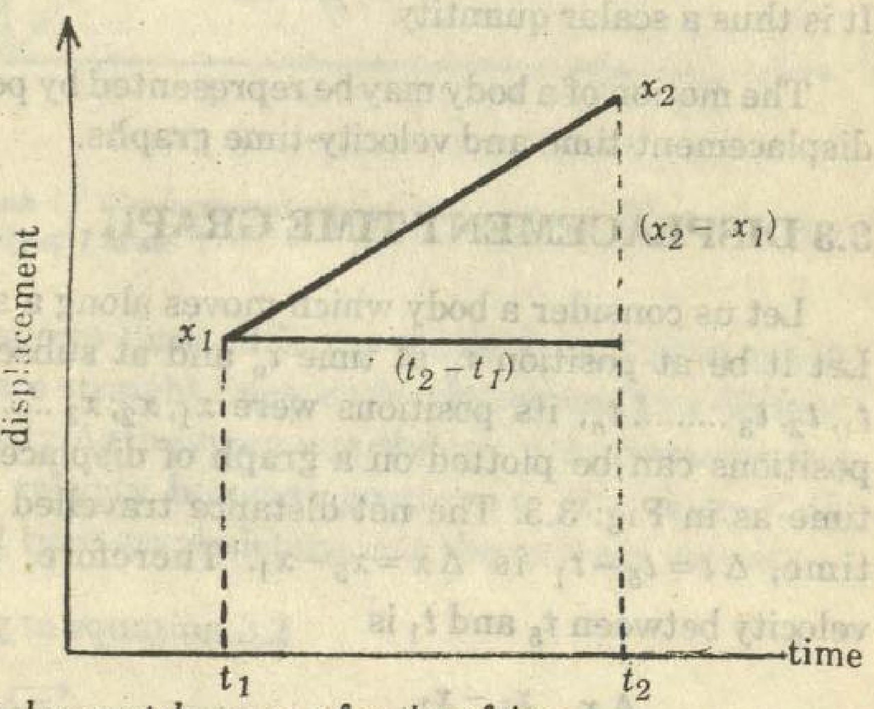


Fig. 3.2 *Displacement drawn as a function of time.*

Example 3.1

A particle moving along the x-axis is located at $x_1 = 15$ m at $t_1 = 5$ sec and at $x_2 = 33$m at $t_2 = 13$ sec. Find its displacement and average velocity during this interval.

Solution

The displacement is given by

$$\Delta x = x_2 - x_1 = 33\text{m} - 15\text{m} = 18\text{m}$$

The average velocity

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{33m - 15m}{13sec - 5sec} = \frac{18m}{8sec} = 2.25\ m\ sec^{-1}$$

Speed

The speed of a body is determined by the amount of distance it travelled in a given time interval in any direction. It is thus a scalar quantity.

The motion of a body may be represented by position-time, displacement-time and velocity-time graphs.

3.3 DISPLACEMENT TIME GRAPH

Let us consider a body which moves along a straight line. Let it be at position x_o at time t_o and at subsequent times $t_1, t_2, t_3 \ldots\ldots\ldots t_n$, its positions were $x_1, x_2, x_3 \ldots\ldots\ldots x_n$. These positions can be plotted on a graph of displacement versus time as in Fig: 3.3. The net distance travelled between the time, $\Delta t = t_5 - t_1$ is $\Delta x = x_5 - x_1$. Therefore, the average velocity between t_5 and t_1 is

$$V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_5 - x_1}{t_5 - t_1}$$

If the velocity with which the body is moving is constant, then clearly, the average velocity is always the same regardless of the particular time interval, chosen for computation. We will see this clearly if we plot the motion of a body moving with constant velocity of 1 m sec^{-1} on a displacement-time graph. If at the beginning the position of the body was marked zero meter, its positions at the end of subsequent seconds will be marked 2 m, 3 m, 4 m,. The displacement time-graph will look like the one given below:-

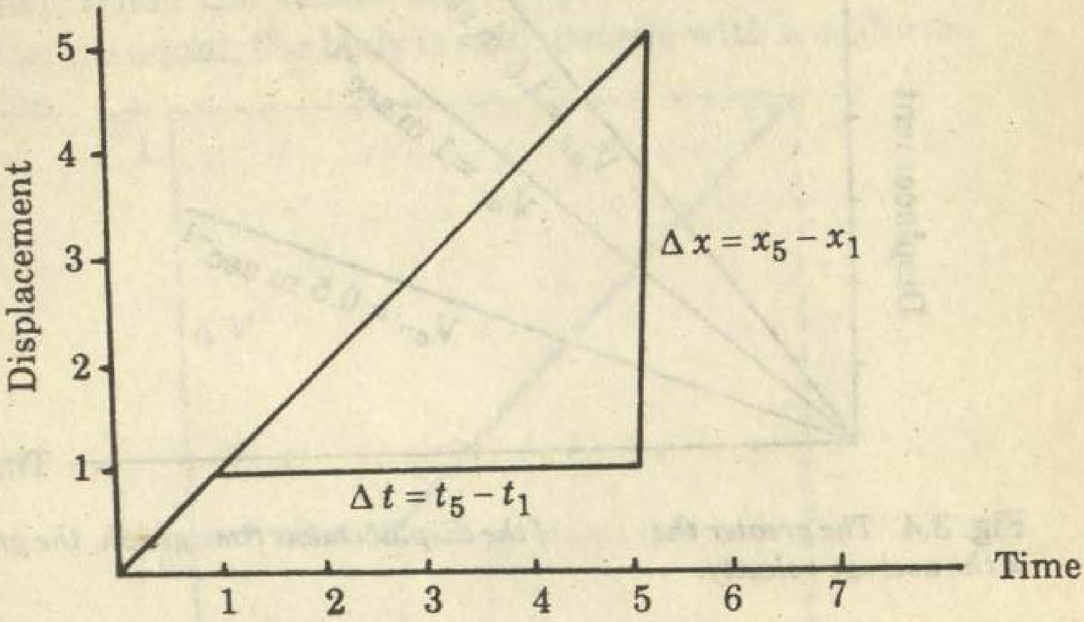


Fig. 3.3 *Graph of displacement versus time for an object moving at constant velocity of 1 m* sec^{-1}.

Fig. 3.4 shows three different displacement-time graphs all of which are straight lines, each corresponds to a different average velocity. As the steepness or slope of the line increases, the average velocity becomes greater i.e. the slope of the displacement time graph determines the average velocity.

According to equation 3.2

$$\vec{V}_{av} = \frac{\overrightarrow{\Delta x}}{\Delta t}$$

If we go on making the time interval Δt smaller and smaller, Δx will also become smaller and smaller. If Δt approaches to zero then $\frac{\Delta x}{\Delta t}$ will approach to a value of $\vec{V}$ which is called instantaneous velocity.

$$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t} \qquad (3.3)$$

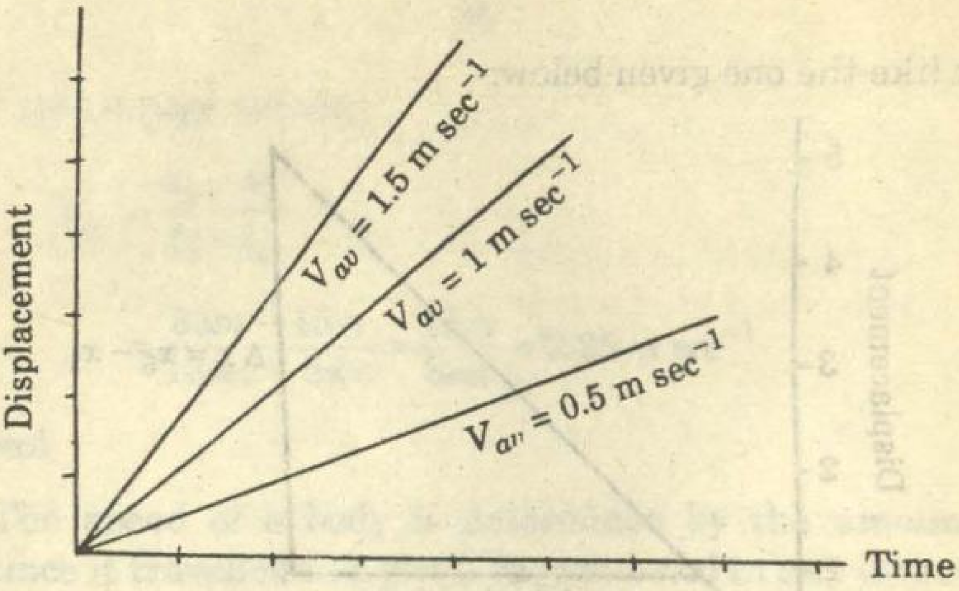


Fig. 3.4 *The greater the [illegible] of the displacement time graph, the greater is the average velocity.*

3.4 ACCELERATION

A body may change its speed or direction or both, The quantity which describes such changes is called acceleration. *It is defined as change of velocity (not speed) per unit time (second). Its unit is m sec^{-2} with dimension lt^{-2}.*

If the velocity of a body changes by an amount $\Delta \overrightarrow{V}$ during the time interval Δt, then its average acceleration is defined as

$$\overrightarrow{a}_{av} = \frac{\Delta \overrightarrow{V}}{\Delta t} \qquad (3.3')$$

as $\overrightarrow{V}$ is a vector quantity so the acceleration is also a vector quantity its direction is along $\Delta \overrightarrow{V}$, the change in velocity. As Δt approaches to zero, the average acceleration will approach the value of instantaneous acceleration at the instant t. i.e;

$$\overrightarrow{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{V}}{\Delta t} \qquad (3.4)$$

If the velocity of the body is decreasing the acceleration is negative. Negative acceleration is known as retardation or deceleration. When the values of average and instantaneous acceleration are equal, the body is said to move with a uniform acceleration.

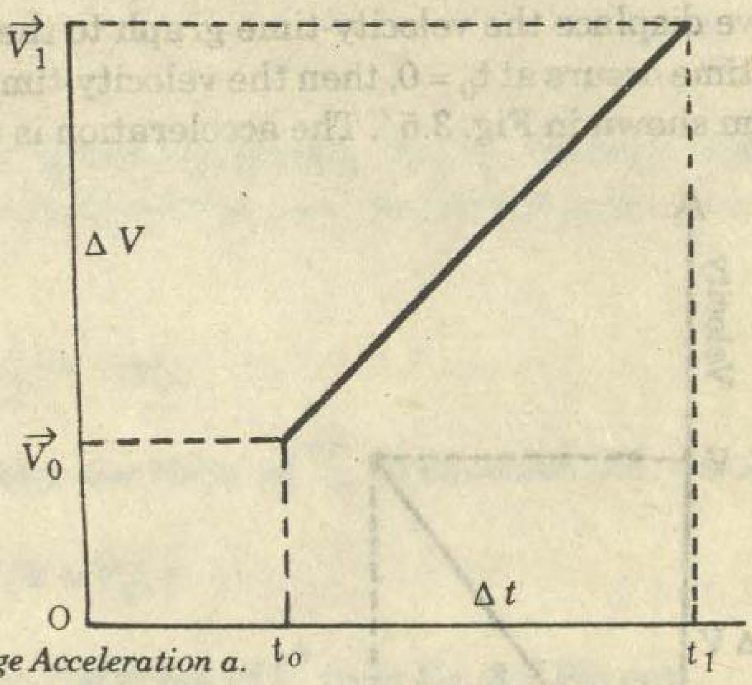


Fig. 3.5 *Average Acceleration a.*

3.5 VELOCITY-TIME GRAPH

Velocity-time graphs are useful for the representation of the motion of an object. Fig. 3.5 shows the velocity-time graph for a body moving with uniform acceleration. Average acceleration from the graph is obtained as

$$\vec{a}_{av} = \frac{\vec{V}_1 - \vec{V}_0}{t_1 - t_0} = \frac{\Delta \vec{V}}{\Delta t} \tag{3.5}$$

By definition the instantaneous acceleration $= \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$ therefore, the slope of the velocity-time graph at any instant gives the value of the instantaneous acceleration.

3.6 UNIFORM ACCELERATION

Since the velocity-time graph is a straight line in Fig. 3.5

therefore, $\overrightarrow{a_{av}} = \overrightarrow{a}$ = constant i.e. the body moves with constant uniform acceleration.

If we displace the velocity-time graph to the left until the initial time occurs at $t_0 = 0$, then the velocity-time graph takes the form shown in Fig. 3.5′. The acceleration is then given by

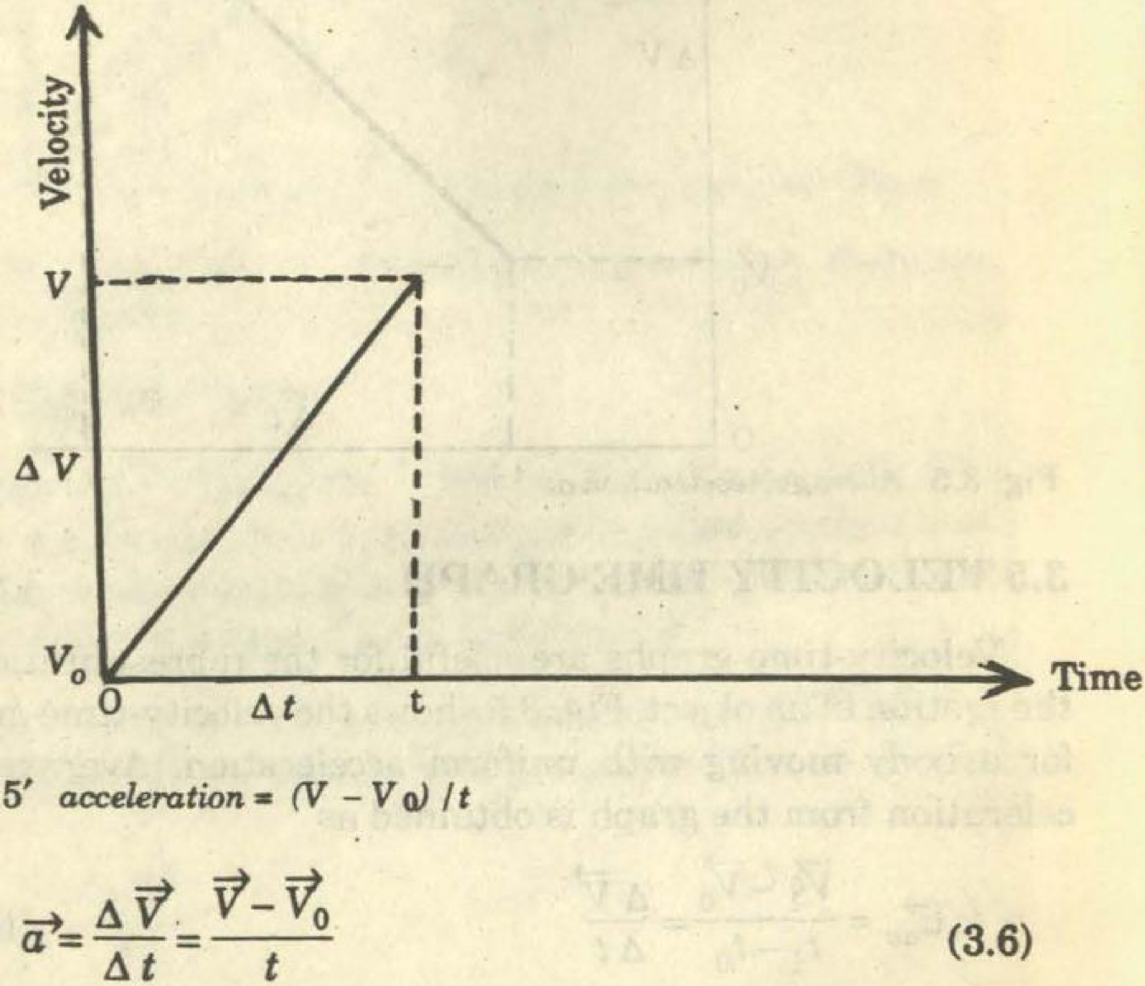


Fig. 3.5′ *acceleration* = $(V - V_0)/t$

$$\overrightarrow{a} = \frac{\Delta \overrightarrow{V}}{\Delta t} = \frac{\overrightarrow{V} - \overrightarrow{V_0}}{t} \tag{3.6}$$

from which we get

$$\overrightarrow{V} = \overrightarrow{V_0} + \overrightarrow{a}\, t \tag{3.7}$$

That is the velocity at the time t is equal to the initial velocity $\overrightarrow{V_0}$ plus the additional velocity acquired as a result of constant acceleration that acts during the time t.

Next we seek an expression for the distance travelled by the body moving with constant acceleration. If we consider the initial position to be at the origin so that x = 0, distance travelled is given by the product of the average velocity and the time.

$$x = V_{av}\, t \tag{3.8}$$

From the Fig. 3.5 ′, it is clear that the average velocity in the case of uniform acceleration is just the average of $\vec{V}$ and $\vec{V}_0$

$$V_{av} = \frac{1}{2}(V + V_0) \tag{3.9}$$

Substituting the value of $\vec{V}_{av}$ in equation 3.8, we obtain

$$x = \frac{1}{2}(V + V_0)\, t \tag{3.9 a}$$

By putting the value of $\vec{V}$ from Eq. 3.7 We get

$$x = \frac{1}{2} V_0\, t + \frac{1}{2}(V_0 + at)\, t$$

$$x = \frac{1}{2} V_0\, t + \frac{1}{2} V_0\, t + \frac{1}{2} a\, t^2$$

$$x = V_0\, t + \frac{1}{2} a\, t^2 \tag{3.10}$$

Eq. 3.9a shows that on the velocity-time graphs the area under the velocity time curve represents the total distance travelled.

We can get another expression for $\vec{V}$ in terms of $\vec{a}$ and x, by eliminating t from Eq.3.7 and substituting in Eq.3.10,

$$x = V_0\left(\frac{V - V_0}{a}\right) + \frac{1}{2} a \left(\frac{V - V_0}{a}\right)^2$$

$$ax = V_0\, V - V_0^2 + \frac{1}{2}\left(V^2 + V_0^2 - 2V\, V_0\right)$$

$$2ax = V^2 + V_0^2 - 2V_0^2$$

$$= V^2 - V_0^2$$

$$\text{or } V^2 = V_0^2 + 2ax \qquad (3.11)$$

In case of uniform acceleration and for the initial conditions x = 0 and $\vec{V} = \vec{V_0}$ at t = t_0, we can summarize the results of this article as, for a = constant

$$\left.\begin{aligned} .V &= V_0 + at \\ x &= V_0 t + ½ at^2 \\ V^2 &= V_0^2 + 2ax \end{aligned}\right\} \qquad (3.1.$$

Example 3.2

In a limited distance race, a contestant in a car reaches a velocity of 300 km hr^{-1}, after covering a distance of 0.45 km at constant acceleration. Find the acceleration.

Solution

Here V_0 = 0 and V = 300 km hr^{-1} = 83.33 m sec^{-1}

By using Eq.3.11, we get the acceleration,

$$a = \frac{V^2}{2x} = \frac{(83.33 \text{ m sec}^{-1})^2}{2 \times 0.45 \text{ Km}} = 0.092 \text{ m sec}^{-2}$$

Thus the acceleration achieved is 0.092 m sec^{-2}

Example 3.3.

A person is driving at a uniform velocity of 70 km hr^{-1}, when he suddenly meets an obstacle on the road 50 metre ahead. (a) Find out the deceleration, if he has to stop just before hitting the obstacle. (b) Find out the deceleration if he hits the obstacle at velocity of 5 km hr^{-1}.

Solution

(a) $V_0 = 70 \text{ km hr}^{-1} = 19.44 \text{ m sec}^{-1}$, $V = 0$, $x = 50 \text{ m}$

Applying the formula

$V^2 = V^2_0 + 2\,ax$, We have

$0 = (19.44 \text{ m sec}^{-1})^2 + 2a \times 50\text{m}$

$$a = -\frac{(19.44\, m\, sec^{-1})^2}{100\, m} = -\,3.77\, m\, sec^{-2}$$

(b) In the second part of the example. $V = 5 \text{ km hr}^{-1} = 1.38 \text{ m sec}^{-1}$ $V_0 = 70 \text{ km hr}^{-1} = 19.44 \text{ m sec}^{-1}$, $x = 50$ m, hence

$$a = \frac{(1.38 \text{ m sec}^{-1})^2 - (19.44 \text{ m sec}^{-1})^2}{2 \times 50\, m}$$

$$a = -\,3.76\text{msec}^{-2}$$

Example 3.4

In the example 3.3, if the driver is able to achieve a constant deceleration of 10 m sec^{-2} (a) what distance the car travels if the brakes are applied at 80 km hr^{-1}? (b) at 120 km hr^{-1}?

Solution

(a) At 80 km hr^{-1}, $V = 0$,

$$V_0 = 80 \text{ km hr}^{-1} = \frac{80}{3.6} \text{ m sec}^{-1} = 22.22 \text{ m sec}^{-1},$$

and $a = -\,10 \text{ m sec}^{-2}$

By using Eq. 4.11,

$$x = \frac{-V_0^2}{-2a} = \frac{(22.22 msec^{-1})^2}{2 \times 10\, msec^{-2}} = 24.69\, m$$

(b) $V_0 = 120 \text{ km hr}^{-1} = \frac{120}{3.6} \text{ m sec}^{-1} = 33.33 \text{ m sec}^{-1}$,

$a = -10 \text{ m sec}^{-2}$

$$x = \frac{-1110.88\ m^2\ sec^{-2}}{-2 \times 10\ m\ sec^{-2}} = 55.54\ m$$

Thus a fifty percent increase in initial velocity increases the stopping distance by almost 125 percent. That is why it is always advisible to drive at moderate speed and to maintain sufficiently large distance from the vehicles in front if cruising at high speed. As the stopping distance increases quadertically with cruising velocity.

Example 3.5

A driver slams on the brakes when the velocity of his car is 120 km hr^{-1}. The car skids along the road undergoing a constant deceleration of 16 km hr^{-1}sec^{-1} for 50 meters before colliding with a parked car. (a) What is the collision velocity (b) How many seconds will elapse before collision.

Solution

We know that $V^2 = V_0^2 + 2ax$

$a = -16 \text{ km hr}^{-1} \text{ sec}^{-1}$

$V_0 = 120 \text{ km hr}^{-1}$

Therefore,

$$V^2 = (120 \text{ km hr}^{-1})^2 - 2 \times 16 \frac{\text{km}}{\text{hr} \cdot \text{sec}} \times 50 \text{ m} .$$

$$= 14400 \left(\text{km hr}^{-1}\right)^2 - 32 \text{ km hr}^{-1} \cdot \frac{50}{1000} \cdot 3600 \text{ km hr}^{-1}$$

$$= (14400 - 5760)\left(\text{km hr}^{-1}\right)^2$$

$$= 8640\left(\text{km hr}^{-1}\right)^2$$

$$V = 92.95\ \text{km hr}^{-1}$$

The collision will be at a velocity of almost 93 km hr^{-1}.

(b) To find the elapsed time, consider Eq. 3.7 which gives:

$$t = \frac{V - V_0}{a}, \text{ where}$$

$$V = 92.95\ \text{km hr}^{-1}$$

$$V_0 = 120\ \text{km hr}^{-1}$$

$$a = -\ 16\ \text{km hr}^{-1}\ \text{sec}^{-1}$$

$$\text{therefore, } t = \frac{(92.95 - 120.00)\ \text{km hr}^{-1}}{-16\ \text{km hr}^{-1}\text{sec}^{-1}} = 1.68\ \text{sec.}$$

The collision will take place within nearly two seconds after applying the brakes.

Variable Acceleration

In the preceding paragraph, we have discussed the case of motion with constant acceleration. The best example of such a motion is the free fall of bodies under constant acceleration due to gravity. The examples in which a body moves with non-uniform or variable acceleration, can be illustrated by the following velocity-time graph.

In this case the gradient or slope increases with time so the acceleration increases with time. To determine the acceleration at any point on the curve a tangent to the curve at that point must be drawn. The tangent is then used to form a

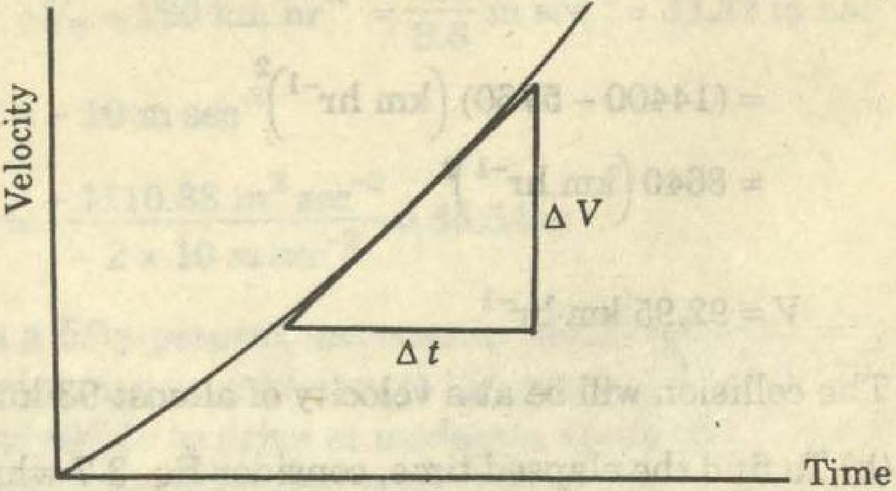


Fig. 3.6 *Velocity time graph for variable acceleration.*

gradient triangle as shown in the Fig. 3.6. The acceleration is determined from the ratio of the perpendicular to the base of the triangle.

3.7 LAWS OF MOTION

So far we have studied motion of a body without considering about the cause of motion. Galileo studied the causes of motion of objects in an objective consistent, systematic and experimental manner and arrived at the equation (Eq. 3.12) describing motion under uniform acceleration. He came very close to the understanding what we now called the first Law of motion.

Issac Newton, born in the year Galileo died in (1642) restated and generalized Galileo's finding in the form of two laws and added a third and presented these in his now world famous book "Mathematical principles of Natural Philosophy" in the year 1687.

Newton's three laws express the relationship among force, mass and motion of a body in a mathematical form together with his law of Universal Gravitation.

They constituted the first modern physical theory, a structure so remarkable that it dominated the landscape of human thought for two centuries. It still remains the governing theory for celestial objects and objects on the earth moving with velocities small in comparison with the velocity of light. Newton stated the first law of motion as follows:

"A body continues its state of rest or uniform motion in a straight line unless it is compelled to change that state of motion by a force impressed upon it.

This property of body that opposes any change in its state of motion is called 'INERTIA'. Newton's first law introduces the idea of force as the agent causing a body to change its state of motion or that of rest. 'Mass' the quantity of matter in any body is the quantitative measure of inertia.

Illustration of the first law of motion

Some common examples of inertia will be useful in clarifying the concept of Newton's first Law. A car standing on the road without petrol will require a lot of effort to start it moving but once it starts moving, only a modest push will keep it rolling. When brakes are applied to a fast moving bus, the bus comes to a halt but the passengers and other loose objects will tend to continue their motion at the same speed and in the same direction. As a result, all of them will be thrown forward. An object dropped vertically downward from a moving train does not hit the ground along a vertical line but describes a curve as beside having a velocity in the downward direction due to gravitational acceleration, it also has a horizontal component equal to the velocity of the train at the time it was dropped. The resultant of two velocities leads to the motion of the dropped objects in a curved path.

In real life situations it is difficult to test the law of inertia. A car needs a force (provided by the engine when it burns the fuel) to start from rest thus satisfying the requirement of the first law. When it attains a certain velocity the engine is switched off. The speed of the car is gradually reduced finally becoming zero. This is an apparent contradiction to the first law. The reality, however, is that there is another force due to friction between the ground and the tyres of the car which opposes the motion of the car and finally brings it to rest. It is thus not the absence of force provided by the engine but the force of friction which brings the car to stop. Thus satisfying the first law. If it were possible to remove this frictional force, it would be reasonable to assume that the car will continue to move with the same constant speed it had at the time the engine was switched off and stopped supplying further force. This is in fact what happens in space. Once a space ship is beyond the pull of earth's gravity, it continues to move with constant velocity even without the thrust of its engines.

Law of Acceleration

Newton's first law tells us how a body behaves when there is no net force acting on it. Let us now consider a situation when there is either a single applied force or two or more applied forces, whose vector sum is not zero. This situation can be dealt with the help of Newtons second Law which states.

'The effect of an applied force on a body is to cause it to accelerate in the direction of the force. The acceleration is in direct proportion to the force and is inversely proportional to the mass of the body'.

The second Law of motion tells us that when a force is applied to a body at rest it will begin to move in the direction of the force and will move faster and faster as long as the force remains in operation.

If a body is moving in a straight line and a force is applied in the direction of its motion it will increase its velocity and will continue to do so as long as the force continues in operation. If, however, the force is applied in the direction opposite to the direction of motion, the acceleration will again be in the direction of the force, causing the body to show down. If such a force continues long enough, the body will slow down to a stop and then will move with an increasing velocity in the direction in which the force is being applied, that is, opposite to its initial direction of motion.

The law also implies that if 'the force applied to a moving object is not along the line of its motion the acceleration will still be in the direction of the force. The effect in general will be to change both the direction and the speed of the motion. We will discuss such cases in much greater detail in the next chapter.

Newton's second Law can be written in an equation form as follows:

$$\vec{F} = m\,\vec{a} \qquad (3.13)$$

Where $\vec{F}$ is the net applied force on the object, $\vec{a}$ is the resulting acceleration of the object and m is the inertial mass of the object.

As can be seen from Eq. 3.13, the force $\vec{F}$ is a vector and the equation is a vector equation, while m is the scalar proportionality constant.

We can use the second Law of motion to express the unit of force. The SI unit of force is called Newton. It is obtained by putting m = 1 kg and $\vec{a}$ = 1 m sec^{-2}, in equation 3.13,

i.e., Force = Mass × acceleration

$$\text{Newton} = (1\text{kg}) \times (1\text{ m sec}^{-2})$$

Hence one Newton is the force which produces an acceleration of 1 m sec^{-2} in a body of mass 1 kg.

Therefore, Newton = $\frac{\text{Kilogram metre}}{\text{sec}^2}$

The dimensional units of force, are given by

$$\{F\} = m\,lt^{-2} \qquad (3.14)$$

Force is a derived quantity since it has been derived from the fundamental quantities as mass, length and time by using the second Law of motion.

Mass is commonly defined as the amount of matter contained in a body. The idea that mass measures an object's response to acceleration is made quantitatively by Newton's second Law. According to Eq. 3.13 for a fixed force, the larger mass of a body the smaller its acceleration. Take a rubber ball lying on the floor. It has relatively little mass and therefore little inertia. We can produce acceleration easily by kicking it with our foot. A ball of similar size but made of lead has much more mass hence more inertia, therefore acceleration can't be produced easily in it.

We can check this quantitatively through Eq. 3.13. Since F is constant but masses of rubber and lead balls are different So

$$F = m_1 a_1 \text{ for rubber ball} \quad \text{and} \qquad (3.15)$$
$$F = m_2 a_2 \text{ for lead ball} \qquad (3.16)$$

From the above equations we get.

$$m_1 a_1 = m_2 a_2 \quad \text{or,}$$

$$\frac{a_1}{a_2} = \frac{m_2}{m_1} \qquad (3.17)$$

The larger mass thus experiences the smaller acceleration and vice versa. Hence inertial mass is the intrinsic measure of a body's resistance to acceleration. The Eq. 3.13 is also consistent with the law of inertia. In a zero force environment $\vec{F} = 0$. Substituting $\vec{F} = 0$. in Eq. 3.13 gives us $\vec{a} = 0$ and the zero acceleration will not change the state of body i.e., the body is at rest or moving with uniform velocity. Observers in such a zero force environment are called inertial observers and such a frame of reference is called inertial frame of reference, i.e.; a frame of reference which is not being accelerated, it may be at rest or moving with uniform velocity.

Example 3.6

Two Askimos names A and B are able to exert a force of 400 N on the sled plane to race their sleds across a frozen lake. A's sled with two people on it has a combined mass of 120 kg, B's sled has one person on it with a combined mass of 40 kg. Assuming that there is no friction between the sleds and the ice, find the acceleration of the two sleds and their separation after ten seconds.

Solution

Using the equation $\vec{F} = m\,a$, we get in A's case F = 400 N and m = 120 kg. Hence

$$a = \frac{400\,N}{120\text{ kg}} = \frac{10}{3}\,m\,sec^{-2}$$

In B's case F = 400 N, m ′ = 40 kg. Hence

$$a' = \frac{400\,N}{40\text{ kg}} = 10\,m\,sec^{-2}$$

The 120 kg sled has a smaller acceleration than the 40 kg sled. Now let us find the distance travelled by each in ten seconds from the start of the race.

For A, $$x = \frac{1}{2}\,a\,t^2 = \frac{1}{2}\cdot\frac{10}{3}\times(10)^2 = \frac{500}{3}\,m$$

For B, $x' = \frac{1}{2} a' t^2 = \frac{1}{2} \cdot 10 \times (10)^2 = 500\, m$

Separation between B and A will be $x' - x = \frac{1000}{3}\ m$

Hence sled B will gain $\frac{1000}{3}\ m$ or $\frac{1}{3}\ km$ over sled A in ten minutes.

Newton's Third Law of Motion

Newton's first two laws concentrate only on the motion of body, but it is a common experience that whenever one body disturbs the motion of another body it is itself disturbed. The 'action' of one body on another is always accompanied by a reaction of the second on the first. Newton's third Law tells us exactly how the forces of 'action' and 'reaction' are related. The body reacts to its environment as described by Newton's third Law which states *"To every action (force) there is always an equal and opposite reaction (force).*

This law tells us that a body thus cannot experience a force from its environments without exerting an equal and opposite force on its environments. Forces in nature always occur in pairs. An action-reaction pair is composed of forces equal in magnitude, but opposite in direction acting on different bodies. We can generalize from the above statement and say that the force exerted on a body by its environment is equal in magnitude but opposite in direction to the force exerted on the environment by the body.

We can formulate Newton's third Law as follows:

Consider the interaction between bodies A and B as in the Fig. 3.7. The force exerted by A on B is F_{AB} and the force exerted by B on A is $-F_{BA}$. Then according to Newton's third Law.

$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad (3.18)$$

This relation holds whether A (or B) experience other forces or not.

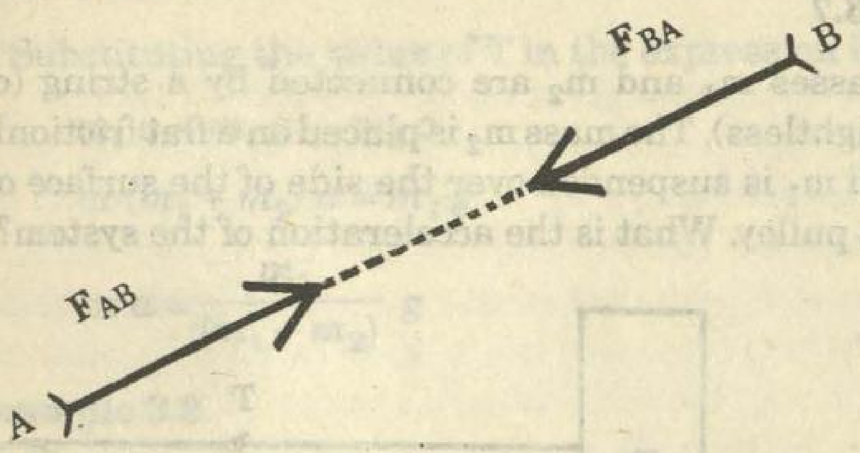


Fig. 3.7

When the two bodies A and B interact, the force they exert on each other in Fig. 3.7 say F_{AB}, the force exerted on the body A by the body B and F_{BA} the force exert on body B by the body A are precisely the same in magnitude but exactly opposite in direction.

We must be careful not to confuse Newton's second and third Law. For example, let us explain the following dilemma.

The force we exert on the ground is equal in magnitude but oppositely directed from the force exerted on us by the ground. The vector sum of these two forces is zero. If the net force is always zero, then how could you ever jump off the ground?

The dilemma is resolved by the observation that although the two forces form action reaction pair, they are acting on different bodies! Whether we jump off the ground, depends on the net force acting on us. The force we exert on the ground is not a part of the net force exerted on us It is the reaction of the ground acting vertically upward as a result of our exertion on the ground vertically downward minus our weight. If the net force is positive we will be able to jump. The net force acting on the ground is the push we give to the ground by our feet. This force also includes our weight.

Example 3.7

Two masses m_1 and m_2 are connected by a string (considered weightless). The mass m_2 is placed on a flat frictionless surface and m_1 is suspended over the side of the surface on a frictionless pulley. What is the acceleration of the system?

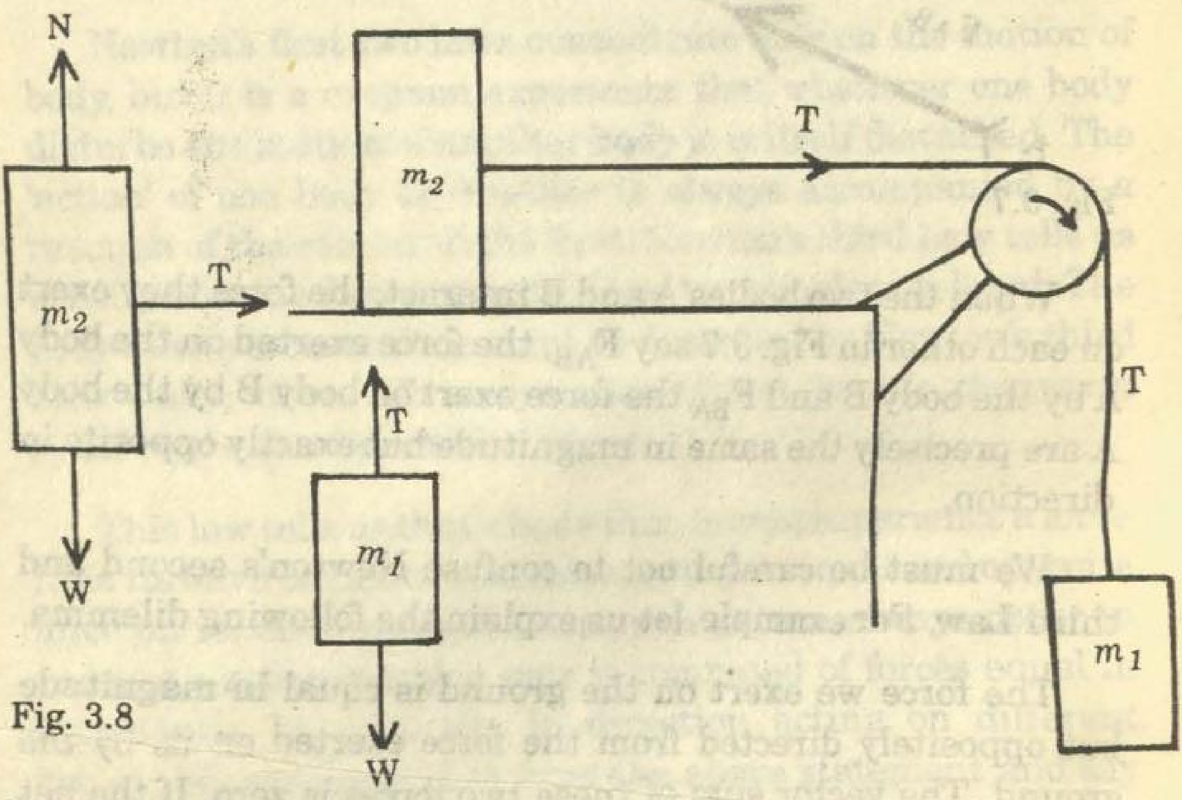


Fig. 3.8

Solution

The Fig 3.8 illustrates the situation and also shows the forces acting on each of the mass

Net force on m_1:

$$F_{net\,1} = m_1 g - T = m_1 a$$

Net force on m_2

$$F_{net\,2} = T = m_2 a$$

The net force on m_2 is just T, the tension in the string since N (the reaction force of the surface on m_2) and W_2 cancel each other. In each case the acceleration is the same as the string remains tight.

Substituting the value of T in the expression for $F_{net\ 1}$:

$$m_1 a = m_1 g - m_2 a$$

$$\text{or } (m_1 + m_2)\, a = m_1 g$$

$$\text{or } a = \frac{m_1}{(m_1 + m_2)}\, g$$

Example 3.8

Consider two blocks of mass m_1 and m_2 resting on a frictionless surface, in contact with each other. If a force $\vec{F}$ is applied on m_1, does it gets transmitted to m_2 ?

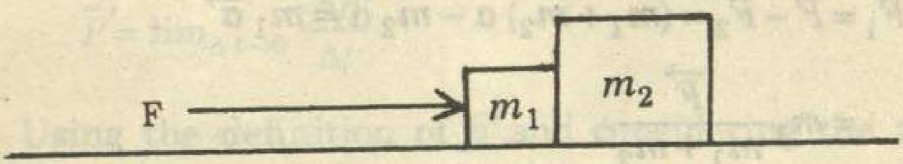


Fig. 3.9 *A force F is applied to m_1 does it get transmitted to m_2 ?*

Solution

Third law tells us that whatever force m_1 exerts on m_2, must be balanced by the force exerted by m_2 on m_1. But third law does not tell us about the magnitude of this force, the second law does. The total mass of the two blocks is $m_1 + m_2$, the only external force is F_1 therefore the acceleration of the two blocks together is

$$\vec{a}=\frac{\vec{F}}{m_1+m_2}$$

The acceleration of m_2 is of course also $\vec{a}$, so that the applied force on m_2 that gives rise to this acceleration is

$$\vec{F}_2=m_2\,\vec{a}=m_2\frac{\vec{F}}{m_1+m_2}$$

Which is smaller than the external force $\vec{F}$ by the fraction $\frac{m_2}{m_1+m_2}$.

Similarly the net force acting on m_1 is

$$\vec{F}_1=m_1\,\vec{a}=m_1\frac{\vec{F}}{(m_1+m_2)}$$

The net force acting on m_1 is $\vec{F}-\vec{F}_2$

$$\vec{F}_1=\vec{F}-\vec{F}_2=(m_1+m_2)\,a-m_2\,a=m_1\,\vec{a}$$

$$=m_1\frac{\vec{F}}{m_1+m_2}$$

Which is just the sum of the external force $\vec{F}$ and $-\vec{F}_2$ the reaction force applied by m_2 on m_1. So the force applied on m_1 does get transmitted to m_2.

3.8. LINEAR MOMENTUM

In addition to the concept of force, mass and acceleration, we define a new derived Vector quantity called momentum. The momentum of an object is defined to be the product of the object's mass and its velocity. Because mass is a scalar and velocity is a vector the product $m\vec{v}$ is a vector. Commonly denoted by the symbol p. As we have considered the case of a body of mass m_1 moving along a straight line with velocity $\vec{V}$, the momentum of the body is known as the Linear Momentum, which is given as

$$\text{Momentum } \vec{p} = m\vec{V} \tag{3.19}$$

So far we have considered mass of the body to be constant but there are important cases, in which mass changes with time. For example a rocket is propelled by the ejection of mass (usually in the form of gases at high velocity) and therefore the mass of the rocket-plus fuel system decreases with time. We can therefore restate Newton's Second Law in terms of the time rate of change of momentum. That is the average force $\vec{F}$ is given by the expression

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

and the instantaneous force as

$$\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta\vec{p}}{\Delta t} \tag{3.20}$$

Using the definition of $\vec{p}$ and considering the mass to remain constant in time we have

$$\vec{F} = \lim_{\Delta t \to 0} \frac{\Delta(m\vec{V})}{\Delta t}$$

$$= m \lim_{\Delta t \to 0} \frac{\Delta\vec{V}}{\Delta t}$$

or using the definition of acceleration,

$$\vec{F} = m\vec{a}$$

We have thus recovered the form of the second law of Newton that we used in Eq. (3.13). Eq. (3.20) is the most general statement regarding force. In the special case that the mass remains constant Eq. 3.20 is equivalent to Eq. 3.13.

Example 3.9

A 60 kg man jumps into a swimming pool from a diving board 7 metres above the water. It takes 0.5 sec. for the water to reduce his velocity to zero. What an average force does the man exert on the water?

Solution

The man falls freely under gravity. Hence his velocity after falling through a height h is given by the relation

$$V^2 = 2\,ah$$

In the present case, $a = g = 9.8\ \text{m sec}^{-2}$, $h = 7\text{m}$, hence

$$V^2 \doteq 2 \times 9.8 \times 7 = 137.2\ m^2\ sec^{-2}$$
$$\text{or } V = 11.7\ m\ sec^{-1}$$

Therefore the man's momentum on striking the water was

$$p_1 = m\,V = 60\ \text{kg} \times\ 11.7\ \text{m sec}^{-1}.$$
$$= 702\ \text{kg. m sec}^{-1}.$$

$$\therefore F = p_1 - p/t = 702/0.5 = 1404\ \text{N}$$

Conservation of linear momentum

We now consider the importance of the concept of linear momentum. Supposes we have a system that is isolated i.e. the constituents of the system interact with one another but there is no external agency that acts on them in any way. Truly isolated objects are not possible in the real physical world but a group of objects whose mutual interaction is much greater than their interaction with other objects can be treated as if they were isolated. For example, consider two ice hockey pucks (flat objects used in the game of ice hockey) that slide almost without friction on ice and collide with one another. The individual motions are influenced in a small way by the gravitational force due to surrounding objects and to an even greater extent by friction, but the collisional interaction between the two pucks so overwhelms these other interactions that for most purposes the latter can be neglected.

If a group of bodies constitute an isolated system there is no net force acting on this system. Since F = 0, Eq. 3.20 requires that there be no change in the linear momentum with time in other words $\vec{p}$ = constant. This is an extremely important result:

If there is no external force applied to a system the total linear momentum of that system remains constant in time. This is the statement of the principle (or law) of the conservation of linear momentum. This principle is valid universally not only for a classical system but also for atoms and molecules.

If the system consists of two objects of masses m_1 and m_2 moving with velocities V_1 and V_2 respectively, then the total linear momentum is

$$\vec{p} = \vec{p_1} + \vec{p_2}$$

$$= m_1 \vec{V}_1 + m_2 \vec{V}_2 = \text{constant} \tag{3.21}$$

Suppose we have two masses at rest connected by a compressed spring as shown in Fig. 3.10. In this condition the total

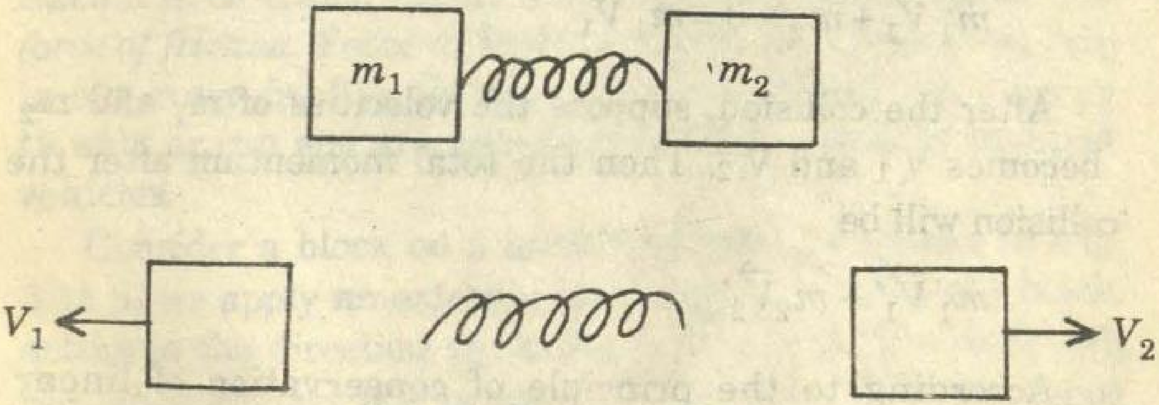


Fig. 3.10 *The motion of masses after release is determined by the conservation of linear momentum.*

linear momentum of the system is zero and the conservation law tells us that it must always be zero. After the release of the spring, the objects move away from each other with velocities $\vec{V}_1$ and $\vec{V}_2$ thus

$$0 = \vec{p}_1 + \vec{p}_2$$

$$\text{so that } m_1 \vec{V}_1 + m_2 \vec{V}_2 = 0 \qquad (3.22)$$

Considering only the magnitudes of the velocities

$$m_1 V_1 = - m_2 V_2 \qquad (3.23)$$

Where the negative sign indicates that the velocities are oppositely directed.

Example 3.10

Let us consider the case in which a ball of mass m_1 moving along a straight line with velocity V_1 hits a ball of mass m_2 which is at rest before the collision. What will be the velocities of m_1 and m_2 after the collision?

Solution

Before the collision, the total momentum of the two balls of mass m_1 and m_2 is given by

$$m_1 \vec{V}_1 + m_2 \times 0 = m_1 \vec{V}_1$$

After the collision, suppose the velocities of m_1 and m_2 becomes V'_1 and V'_2. Then the total momentum after the collision will be

$$m_1 \vec{V}_1' + m_2 \vec{V}_2'$$

According to the principle of conservation of linear momentum, the total linear momentum before and after the collision must be the same. Hence

$$m_1\vec{V}_1 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

$$\vec{V}_2' = \frac{m_1}{m_2}(\vec{V}_1 - \vec{V}_1') \tag{3.23}$$

Thus after the collision the ball m_2 will move along the same direction as the ball m_1 was moving before the collision with a velocity given by Eq. (3.23). From this equation one also obtains the relation

$$m_2 V_2' = m_1\left(V_1 - V_1'\right)$$

Where $m_2\ V_2\ '$ is the momentum gained by m_2 and $m_1(V_1 - V_1')$ is the momentum lost by m_1. Thus the principle of the conservation of linear momentum also implies that momentum lost by one part of the system (m_1) will be gained by the other part (m_2).

3.9. RESISTIVE FORCES

According to Newton's first law, a body in motion will continue to move unless acted upon by an external net force. A body moving through a viscous medium such as air or water eventually comes to rest, similarly a body in motion on a solid surface eventually comes to rest. There are always forces which resist the motion of a body. These forces arise as a result of the interaction of the moving body with its environment. *Such a force, which resists the motion of a body is called the force of friction*. Force of friction plays a very important role in our every day life, for example, forces of friction allow us to walk or run and are necessary for the motion of wheeled vehicles.

Consider a block on a horizontal table as shown in Fig. 3.11. If we apply an external horizontal force $\vec{F}$ to the block, acting in the direction as shown in Fig. 3.11, the block will remain stationary if F is not too large. The force that keeps the block away from moving acts to the left is the force of friction. As long as the block is in equilibrium $\vec{f} = \vec{F}$. Since the block is stationary we call this force of friction as the force of static friction f_s

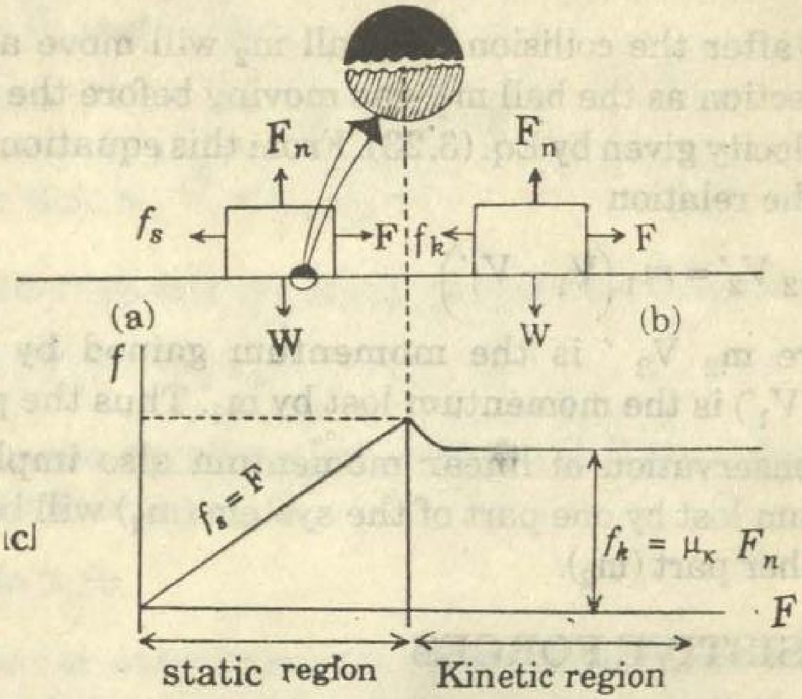


Fig. 3.11 *The force of friction $\vec{f}$ between a block and a rough surface is opposite to the applied force $\vec{F}$ (a) The force of static friction equals the applied force. (b) When the applied force exceeds the force of kinetic friction f_k the block accelerates to the right (c) A graph of the applied force versus the magnitude of the frictional force. Note that $f_{s\ maximum} > f_k$.*

Experiments show that this force arises from the roughness of the two surfaces so that the contact is made only at few points as shown in the "magnified view" of the surfaces in the Fig. 3.11a. The frictional force has its origin in the electrostatic forces at the atomic and moleculer level at the points where the surfaces meet. We will therefore not look at frictional force from the microscopic point of view but will look only at its effects at the macroscopic level.

If we increase the magnitude of $\vec{F}$ as in Fig. 3.11b the block will eventually slip. When block is on the verge of slipping, f_s is maximum. When F exceeds $f_{s\ max}$ the block moves and accelerates to the right. When the block is in motion the retarding frictional force becomes less than $f_{s\,max}$ (Fig. 3.11 c). When the block is in motion, we call the retarding force, the force of kinetic friction f_k. The unbalanced force in the x-direction, $F - f_k$, produces an acceleration to the right. If $F = f_k$ the block moves to the right with constant speed. If the applied force is removed, then the frictional force acting to the left would decelerate the block and eventually bring it to rest.

In a simplified model we can imagine that the force of kinetic friction is less than $f_{s\,max}$ because of the reduction of the roughness of the two surfaces when the object is in motion. When the object is stationary the contact points are said to be "cold-welded". When the object is in motion these small welds become ruptured and the frictional force decreases.

Experimentally you will find that both f_k and f_s are proportional to the normal force acting on the block and depends on the roughness of the two surfaces in contact. The experimental observations can be summarized as follows:

1. The force of static friction f_s between any two surfaces in contact is opposite to the applied force and can have values given by

   $$f_s \leq \mu_s F_n \qquad (3.24)$$

   Where the dimensionless constant μ_s is called the coefficient of static friction and F_n is the normal force. The equality in Eq. 3.24 holds when the block is on the verge of slipping that is, when $f_{s\,max} = \mu_s F_n$. The inequality holds when the applied force is less than that value.

2. The force of kinetic friction is opposite to the direction of motion and is given by

$$f_k = \mu_k F_n \tag{3.25}$$

Where μ_k is the coefficient of kinetic friction.

3. The value of μ_k and μ_s depends on the nature of surfaces, but μ_k is generally less than μ_s. Typical values of μ_s range from 0.01 for smooth surfaces to 1.5 for rough surfaces.

   Furthermore, the co-efficients of friction are nearly independent of the area of contact between the surfaces. Though the co-efficient of kinetic friction μ_k varies with speed, we shall neglect any such variations.

Example 3.11

A Skier moving horizontally at 20 m sec^{-1} coasts to a stop as a result of the frictional force exerted by the snow on her skis. If the total mass of the skier and skis is 80 kg and normal force 784 N, what will be the stopping distance, if we neglect the air resistance?

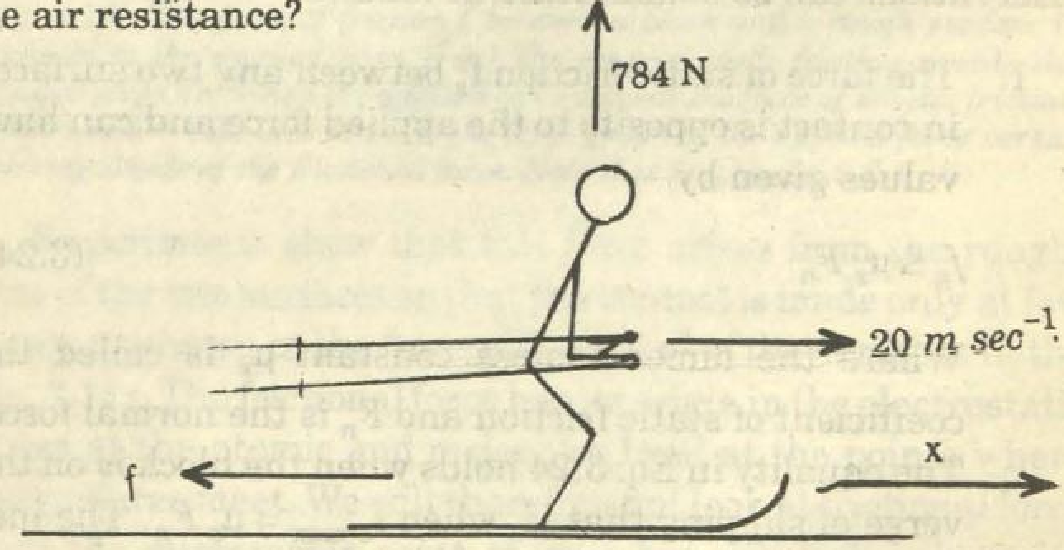


Fig. 3.12 *Skier Sking at 20 m sec^{-1} comes to a stop due to frictional resistance of the snow, directed in the opposite direction.*

Solution

First we determine the acceleration of the skier. The only horizontal force acting is f. So using Eq. 3.25 we get.

$$f = -\mu_k F_n$$

For the snow $\mu_k = 0.18$, $F_n = 784$ N, therefore

$$f = -\ 0.18 \times\ 784$$
$$= -\ 141.12\ \text{N}$$

Using Newton's second Law we find the acceleration

$$a = \frac{f}{m} = -\frac{141.12\ N}{80\ Kg} = -1.76\ m\ sec^{-2}$$

Because the force is constant the acceleration is also constant. The stopping distance will be given by the Eq. 3.11

$$V^2 = V_0^2 + 2ax$$

since $V = 0$, $V_0 = 20\ m\ sec^{-1}$, $a = -\ 1.76\ m\ sec^{-2}$

hence, $x = \dfrac{20 \times 20\ m^2\ sec^{-2}}{2 \times 1.76\ m\ sec^{-2}} = 113.6$ m

Hence, the skier will come to a stop after covering 113.6 m.

Fluid Friction

As stated earlier, a body experiences a resistive force when it moves through a fluid, which can be either air, gas or liquid. Just as in the case of a body moving along a solid surface experiences a frictional force opposing its motion, similarly the fluid also exerts a frictional force on the body.

The size, shape and orientation of an object determines the fluid force on the object. Swimmers and sky divers change their shape and orientation by bending, twisting and moving their arms and legs. This allows to manipulate the fluid forces and consequently to control their speed and direction of motion. As fluid force has the simplest expression for objects with perfectly spherical shape we will limit our considerations to the study of fluid forces acting on moving spheres.

Fog and mist are collections of tiny water droplets which are spherical in shape. Close examination reveals that these droplets fall very slowly. The effect of drag on the droplet's speed is large in comparison with the drag's effect on a freely falling cricket ball. At the low speeds of water droplets the drage force is well represented by Stoke's Law. The Law expresses the drag F_D on a sphere of radius r moving with velocity V as

$$F_D = 6\pi\eta r V \tag{3.26}$$

Where η (eta) is an empirical quantity called the viscosity. This quantity is analogous to the coefficient of kinetic friction μ_k. The SI unit of viscosity is kilogram per meter per second.

Consider the vertical motion of a fog droplet subject to the force of gravity and drag. The droplet begins to move from rest. After a time they acquire a speed V downward. Two forces act on the droplet, its weight mg downward and the dragging force $6\pi\eta r V$ upward. The droplet's downward acceleration is determined from Newton's Second Law:

$$F = mg - 6\pi\eta r V = ma$$

or dividing by m we get

$$a = g - \frac{6\pi\eta r V}{m} \tag{3.27}$$

The most striking feature of the above result is the velocity dependence of the acceleration. Freely falling objects experience a common acceleration g.

The droplet's initial acceleration is g for V = 0 but then falls to zero, as the velocity increases till the two forces become equal in magnitude. At this point the velocity is at the maximum value called the terminal velocity V_T. Putting a = O in Eq. 3.27 we get

$$V_T = \frac{mg}{6\pi\eta r} \tag{3.28}$$

We see from the above expression that the terminal velocity is directly related to the object's mass. The more massive an object, the faster it falls through a fluid.

To determine the size dependence of the terminal velocity, we introduce the mass density ρ

$$m = \left(\frac{4}{3}\pi r^3\right)\rho$$

Where $\frac{4}{3}\pi r^3$ is the volume of the droplet. Substituting this expression for m in Eq. 3.28 we get

$$V_T = \frac{4}{3}\pi r^3 \rho \frac{g}{6\pi\eta r}$$

$$\text{or } V_T = \frac{2\rho g r^2}{9\eta} \tag{3.29}$$

The terminal velocity of a sphere of given material (fixed ρ) varies directly with the square of the radius. For example, doubling the radius produces a fourfold increase in terminal velocity.

Example 3.12

The radius of a small fog droplet is found to be 5.1×10^{-6} m. Find out the settling speed of the droplet in air, for which $\eta = 1.9 \times 10^{-5}$ kg m^{-1} sec^{-1}?

Solution

For water $\rho = 10^3 \text{ kg m}^{-3}, g = 9.8 \text{ m sec}^{-2}$. Using Eq. 3.29 we get,

$$V_T = \frac{2 \times 10^3 \text{ kg m}^{-3} \times 9.8 \text{ m sec}^{-2} \times (5.1 \times 10^{-6} \text{ m})^2}{9 \times 1.9 \times 10^{-5} \text{ kg m}^{-1} \text{ sec}^{-1}}$$

$$= 2.981 \times 10^{-3} \, m \, sec^{-1}$$

Fluid friction tends to be much less pronounced than friction between solids. Pulling a raft through, or on water requires less force than sliding the same raft over even a very smooth surface. As one might expect, frictional forces caused by air or gases are much less than those caused by liquids. A fact which has been usefully employed in the invention of hover crafts we will describe them in more detail later. Objects moving through air encounter very little friction at low speeds as given in Eq. 3.27, but as speed increases marked by air friction becomes powerful retarding force. Friction with atmosphere air generates such heat that meteorites burn up before they hit the earth causing shooting stars often seen in the northern hemisphere during the month of August. Space crafts which are to be brought back to Earth are provided with high temperature shields to prevent them from being consumed by the heat of friction as they enter the earth's atmosphere on the return journey at speeds in excess of 50,000 km hr^{-1}.

Hover Crafts

Hover Crafts are surface effect machines invented for transportation over water, marshes and ground. They make use of the comparatively low frictional forces associated with air when moderate speeds are concerned. Hover crafts have the ability to hover just above the surface of the ground or water and move with great rapidity on cushion of air continuously being forced out between the bottom of the vehicle and the ground or water surface. These vehicles are now in regular use as ferries for short distance travel across channels and islands. They can attain speed over 150 km hr^{-1} as against the speed of the surface boats of around 60 km hr^{-1}.

Friction - Good or bad

Friction is advantageous or disadvantageous depends entirely on the results desired. Friction between rubber and concrete is certainly desirable. It will be impossible to walk if the friction between the soles of the shoes and the ground were absent. All automotive transportation is based upon friction between the tyres and the road. On the other hand, in machines having moving parts like, the piston in the cylinder, wall friction is indeed undesirable and every effort is made to minimize the effect. A lot of research is being done all over the world to develop newer and newer lubricants to minimise friction in machine. This theory of lubrication is very complicated, but it is sufficient to say here that the lubricants provides a fluid film between the two solid surfaces thus substituting much less fluid friction in place of greater friction which exist between solid surfaces. In the table below approximate coefficients of kinetic friction μ_k are given in Table 3.1.

Table. 3.1 Coefficient of kinetic friction μ_k

Material	*(T^o C)*	μ_k
Waxed Ski on dry snow	0	0.04
Waxed Ski on dry snow	– 10	0.18
Waxed Ski on dry snow	– 40	0.40
Ice on ice	– 10	0.035
Ebonite on ice	– 10	0.05

Material	*$(T^o C)$*	μ_k
Brass on ice	– 10	0.075
Rubber on dry land		1.02

3.10. MOTION OF A BODY ON AN INCLINED PLANE

Galileo was the first to formulate the law of inertia later incorporated as the first law of motion by Newton in his mechanics. Contrary to every day experience, Galileo asserted from his "thought experiment" that constant motion requires no force. He was studying the motion of various object on an inclined plane.

He noted that in the case of planes that slope downward there is already present a cause of acceleration, while on planes sloping upward there is retardation. See Fig. 3.13.

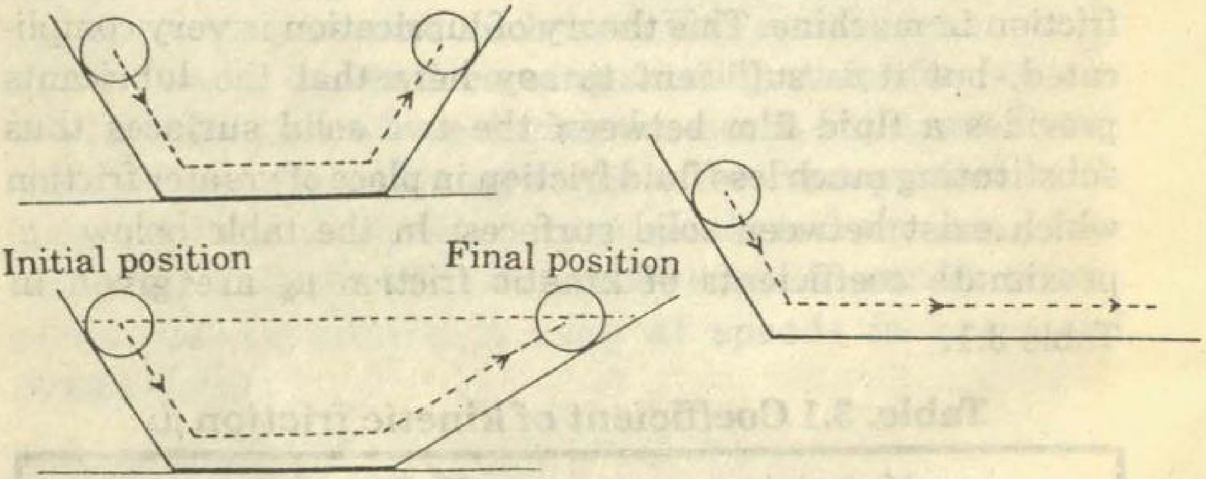


Fig. 3.13 *Galileo observed that a ball tends to rise to its original height regardless of the slope of the incline. With zero slope, the original slope could never be attained. Therefore, he argued that motion along a horizontal line should be perpetual.*

From this experience he reasoned that when the plane slopes neither upward, nor downward there should be neither retardation nor acceleration. Motion along a horizontal plane should be constant.

Galileo's experiment dealing with the motion along the inclined plane of a body is a clear example of dilution of forces. By decreasing the slope continuously, the strength of the force of gravity is gradually decreased till for a horizontal plane gravity has been totally eliminated so far as its effect on horizontal motion is concerned.

Consider a block of mass m sliding along an inclined plane having an angle of inclination θ at A (Fig 3.14). The force along the plane is mg sin θ when θ = 90°, sin 90° = 1.

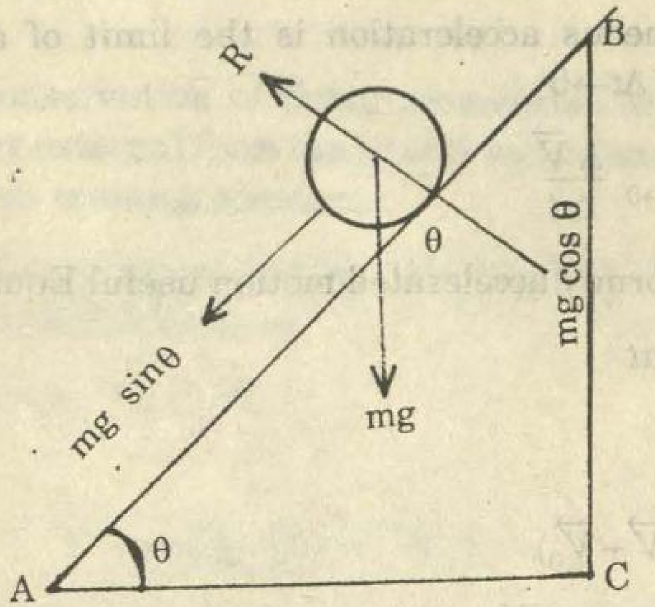


Fig. 3.14

Therefore the force has the maximum value = mg. when θ = 0°, sin 0° = 0 the force also reduces to zero i.e. as θ is varied from 90° to 0°, the force varies from a maximum value of mg to zero. In other words, the force can be diluted from its maximum value mg to any desired value by choosing the angle *θ*.

SUMMARY

Average velocity is the rate of change of displacement.

$$\vec{V}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

Instantaneous velocity is the limit of the average velocity as $\Delta t \to 0$

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

Average acceleration is the rate of change of velocity.

$$\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t}$$

Instantaneous acceleration is the limit of average acceleration as $\Delta t \to 0$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

For Uniformly accelerated motion useful Equations are:

$$\vec{V} = \vec{V}_0 + at$$

$$\vec{x} = \vec{V}_{av}\, t$$

$$\vec{V}_{av} = \frac{1}{2}(\vec{V} + \vec{V}_0)$$

$$\vec{x} = \vec{V}_0\, t + \frac{1}{2}\vec{a}t^2$$

$$\vec{V}^2 = V_0^2 + 2\vec{a}x$$

Newton's first law states that every body continues in its state of rest, or of constant velocity unless it is caused to change that state by force exerted on it.

The property of a body which opposes any change in its state of motion in a straight line or of rest is called 'Inertia'.

Newton's second law states that the acceleration of a body is directly proportional to the net applied force on it, and is equal to the ratio of the net applied force to the intertial mass. In mathematical form, it can be written as.

$\vec{F} = m\,\vec{a}$

Newton's third law states that to every action there is always an equal and opposite reaction.

Law of force provide for calculation of the force on a body interms of the properties of the body and of its environment.

Linear Momentum of a particle of mass m moving with uniform linear velocity V is defined as

$\vec{p} = m\,\vec{V}$

Law of conservation of linear momentum states in the absence of any external force the total linear momentum of an isolated system remains constant.

Resistive forces oppose motion. Empirical formula for friction in solids and fluids are as

Sliding friction $F = \mu_k F_n$

QUESTIONS

3.1 Under what conditions the Instantaneous and Average velocities will be equal.

3.2. Is it possible for a body to have acceleration when moving with (a) constant velocity (b) constant speed?

3.3 A body is moving along a circle with a constant speed.

a) Is it moving with uniform velocity?

b) If it has acceleration what is the direction of acceleration.

3.4. Discuss the action and reaction forces in the following cases.

a) A boat being rowed in the river.

b) Taking off a rocket.

c) Firing of bullet from the gun.

3.5. When a car takes a turn, the passengers experience a force acting on them away from the centre of the curve, why?

3.6. A particle of mass m is falling freely. What is its weight?

3.7. Can there be acceleration when a body is moving with constant speed.

3.8. A horse exerts a force on the cart in order to pull it. According to Newton's third Law the cart exerts the same force on the horse in the opposite direction. How does the cart move then?

PROBLEMS

3.1. A runner completes a round along a rectangular circuit in 45 sec. The circuit is 400 m long. Find out the average speed and the average velocity of the runner.

3.2. If the velocity time curve is a straight line, the average velocity is half way between the initial and the final velocities in the interval. Sketch a curve of velocity versus time where the average velocity is closer on the curve to the maximum than to the minimum. Repeat for an average velocity that is closer to the minimum.

3.3 By using the data given in the Table 3.2, obtain the average velocity.

Table 3.2. Position and times of a runner for the initial position of a race

Position x (m)	*Time t (sec)*	*Position x (m)*	*Time t (sec)*
0.00	0.00	3.66	0.64
0.31	0.11	4.27	0.74
0.61	0.18	4.88	0.84
0.91	0.25	5.49	0.93
1.22	0.31	6.10	1.03
1.52	0.37	6.71	1.12
1.83	0.43	7.32	1.20
2.44	0.48	7.93	1.29
2.74	0.54	8.53	1.37
3.05	0.59	9.14	1.45

3.4. Plot the displacement-time graph with the help of the data in Table 3.2.

a) Find the average velocity at t = 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 1.0, 1.2, 1.3, 1.5 seconds.

b) Plot the velocity-time graph at the values of t given in part a.

c) Is it an example of constant acceleration / variable acceleration.

3.5. A car moving at 30 km hr^{-1} towards north turns a corner and travels 30 km hr^{-1} to East (Fig. 3.15). The turn requires 4 seconds, determine the average acceleration of the car during the turn. Give its magnitude and direction.

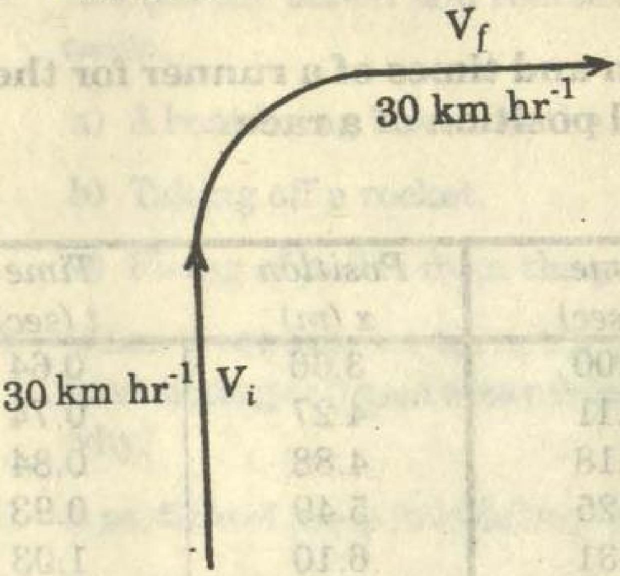


Fig. 3.15

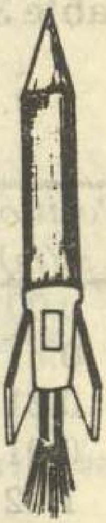


Fig. 3.16

3.6. The velocity of a rocket during the initial launch stage (Fig. 3.16) is given by, $V = 30\,t - 0.5\,t^2$, where V is in metre per second and t is in seconds. Determine the average acceleration of the rocket from (a) $t_i = 0.5$ sec to $t_f = 1$ sec, (b) t = 5 sec, $t_f = 6$ sec.

3.7. A car moving at 16 km hr^{-1} accelerates for 3 seconds the average acceleration over the 3 seconds interval is 32 km hr^{-1}sec^{-1}. What is its speed at the end of the 3 seconds interval.

3.8. A cricket ball is hit so that it travels straight up in the air (Fig. 3.17). A man in the stand observes that it requires 3 sec to reach the maximum height.

a) What was its initial vertical velocity?

b) How high did it rise?

c) What time it will take to reach the ground.

d) What will be its velocity just before hitting the

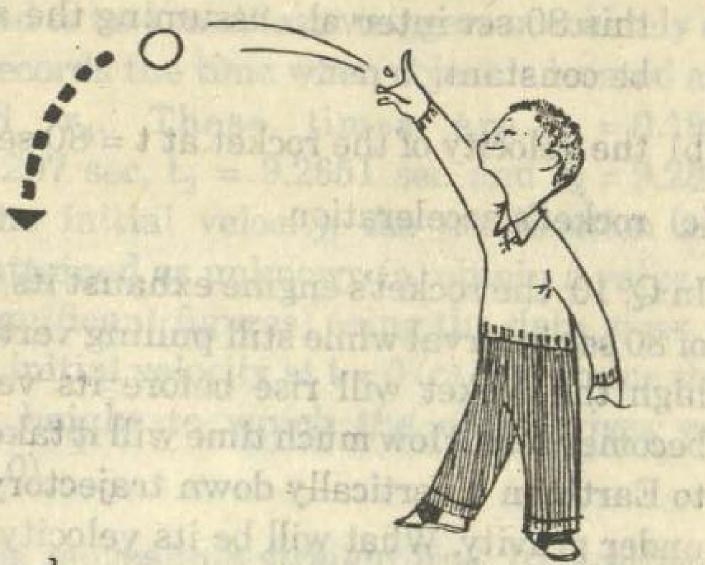


Fig. 3.17

ground.

(Neglect air resistance and assume that the ball was hit one metre above the ground.)

3.9. A student jumps straight up in the air (Fig. 3.18) and is air borne (up plus down) for 0.6 sec. (a) What was his initial velocity? (b) How high did he rise? (Assume a sysmmetric trajectory).

Fig. 3.18

3.10. A rocket starting from rest at t = 0 climbs to an altitude of 9.600 m in 80 sec.

(a) Calculate the rocket's average vertical velocity for this 80 sec interval, assuming the acceleration to be constant.

(b) the velocity of the rocket at t = 80 sec.

(c) rocket's acceleration.

3.11. In Q. 10, the rocket's engine exhaust its fuel at the end of 80 sec interval while still pulling vertically up. How high the rocket will rise before its vertical velocity becomes zero. How much time will it take to come down to Earth on a vertically down trajectory falling freely under gravity. What will be its velocity before hitting the ground?

3.12. In a relay race the second runner does not start from rest. He covers 100 m in 10 sec, finishing with a maximum velocity of 12 m sec^{-1}. Assuming constant acceleration, determine his initial velocity.

3.13. An object travels in vacuum upward past elevations A and B (labelled A′ and B′ downward journey). The object rises to an unknown height and then returns to its starting point, passing A′ and B′ on the way. The distance between two elevations AB or A′B′ is

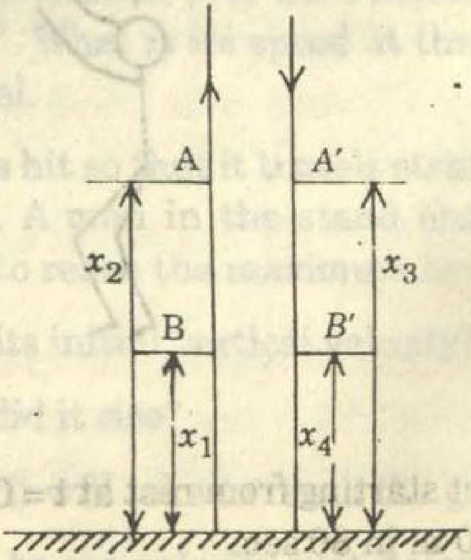


Fig. 3.19

measured to be 1 m. A laser triggers accurately a timer which records the time when object is located at x_1 x_2, x_3 and x_4. These times are $t_1 = 0.1982$ sec. $t_2 = 0.2207$ sec, $t_3 = 9.2651$ sec and $t_4 = 9.2876$ sec. Treat the initial velocity, the acceleration and the height attained as unknown (a) obtain a value of g (to three significant figures) using the data given (b) obtain the initial velocity at t = 0 (c) Determine the total vertical height to which the object rises relative to x (t = 0).

3.14. A particle moves in a straight line. Its position (x) at various moments of time are displayed in the Table 3.3. Determine whether or not the particle remains free of a net force over the entire interval.

Table 3.3

x (*m*)	*t* (*s*)	*x* (*m*)	*t* (*s*)
1.1	0	4.8	3
2.2	1	6.0	4
3.4	2	7.0	5

3.15. The discus used by women athletes has a mass of 1 kg. What is its weight in Newton?

3.16. By giving a sudden pull on a spring attached to a standard kilogram, a student gives it an acceleration of 2.3 m sec^{-2}. Determine the force exerted on the spring by the standard kilogram.

3.17. A 100 kg automobile accelerates at 4 m sec^{-2}. What is the net force on it.

3.18. A constant force F changes the velocity of a 80 kg sprinter from 3 m sec^{-1} to 4 m sec^{-1} in 0.5 sec. Calculate the acceleration of the sprinter.

4

MOTION IN TWO DIMENSIONS

4.1 MOTION IN TWO DIMENSIONS

In this chapter we shall discusss the motion of an object in a plane under uniform acceleration.The motion of this type is described either by cartesian coordinates (x, y) or by plane polar coordinates (r, θ). The velocity and acceleration in polar coordinates are defined in a similar way as are done in cartesian coordinates.

Motion of a projectile and that of a particle moving with uniform speed along a circle are good examples of objects having uniform acceleration in a plane. The path of projectile is confined to a vertical plane. These cases will be discussed in detail.

To find the solution of the problems in two dimensions we resolve the displacements, velocities and acceleration into the components along the x and y axis and use the equations of motions.

$$V_{fx} = V_{ix} + a_x t, V_{fy} = V_{iy} + a_y t \tag{4.1}$$

$$x = V_{ix} t + \frac{1}{2} a_x t^2, y = V_{iy} t + \frac{1}{2} a_y t^2 \tag{4.2}$$

Here V_f and V_i stand for final and initial velocities respectively and are given by,

$\vec{V}_f = V_{fx}\hat{i} + V_{fy}\hat{j}$, $\vec{V}_i = V_{ix}\hat{i} + V_{iy}\hat{j}$ and acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j}$ in terms of their components along x and y-axes.

4.2 PROJECTILE

An object launched in an arbitrary direction in space with the initial velocity having no mechanism of propulsion is called a projectile. Assuming that the projectile does not experience any air resistance, its path is determined by the acceleration due to gravity and is called the trajectory. A golf ball hit into the air, a bullet shot from a gun and an object released from an aeroplane are some examples of projectile motion.

Let us now apply Eq. 4.1 and Eq. 4.2 to a projectile motion, assuming that $\vec{a_x} = 0$, $\vec{a_y} = -g$, is a constant. As the projectile does not experience any air resistance, it has no acceleration along the x-axis and so, $a_x = 0$, therefore the above equations reduce to,

$$V_{fx} = V_{ix}, \quad V_{fy} = V_{iy} - gt \tag{4.3}$$

$$x = V_{ix}\,t \qquad y = V_{iy}\,t - \frac{1}{2}gt^2 \tag{4.4}$$

Suppose that an object is hrown upward from a point O with an initial velocity V_o making an angle θ with the horizontal axis *ox* and *oy* is the other axis perpendicular to *ox* (Fig. 4.1). Suppose further that the magnitude of $\vec{V_o}$ is V_o Hence

$$\vec{V_o} = V_o\cos\theta\,\hat{i} + V_o\sin\theta\,\hat{j}$$

At any time *t*, the velocity of the projectile is $\vec{V} = V_x\hat{i} + V_y\hat{j}$ having the following two components,

$$V_x = V_{ix} = V_o\cos\theta \tag{4.5}$$

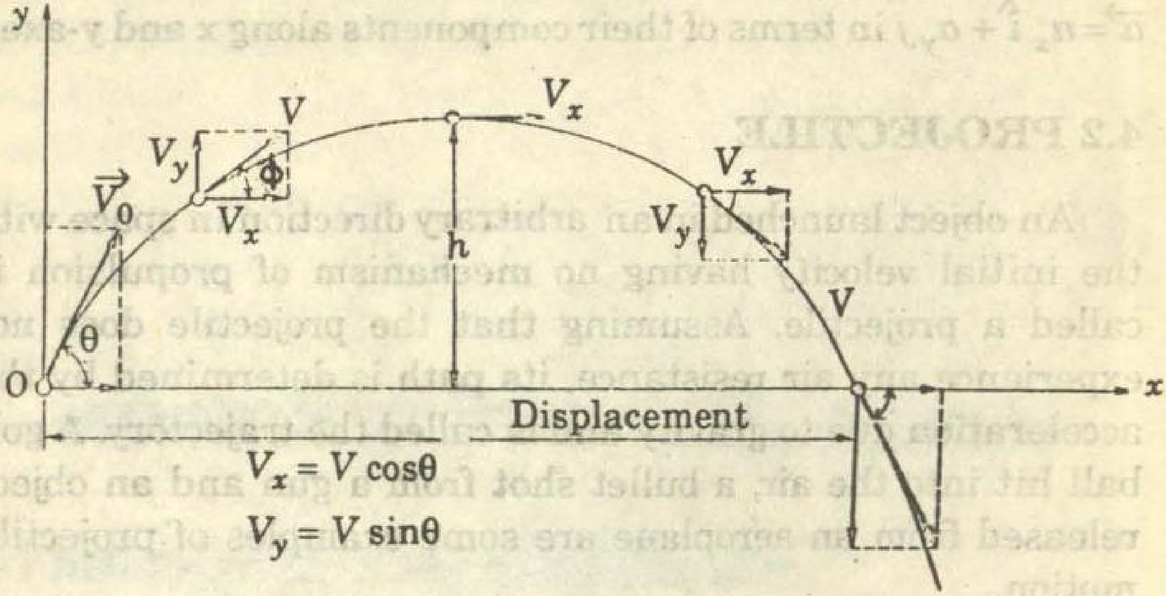


Fig. 4.1 *Trajectory of a projectile*

$$V_y = V_{iy} = V_o\sin\theta - gt \tag{4.6}$$

The upward direction is taken as positive and downward as negative. From these equations it is clear that projectile motion is composed of two independent motions, the horizontal motion with constant velocity and the vertical motion with constant acceleration.

Since all along the trajectories of the projectile the horizontal velocity $V_x = V_o\cos\theta$ remains constant, while the vertical component of velocity has the value $V_y = V_0\sin\theta - gt$ which reduces to zero when the projectile reaches the highest point of the trajectory and starts increasing again (although in the opposite direction) as the projectile continues to fall. The magnitude and the direction of the velocity of the projectile is given by:

$$\vec{V} = V_o\cos\theta\,\hat{i} + (V_o\sin\theta - gt)\,\hat{j} \tag{4.7}$$

$$|\vec{V}| = \sqrt{(V_o\cos\theta)^2 + (V_o\sin\theta - gt)^2}$$

$$= \sqrt{(\cos^2\theta + \sin^2\theta)V_o^2 - 2gtV_o\sin\theta + g^2t^2}$$

$$|\vec{V}| = \sqrt{V_o^2 - 2gtV_o\sin\theta + g^2t^2} \qquad (4.8)$$

and if $\vec{V}$ makes an angle φ with the horizontal then.

$$\varphi = \tan^{-1}\left(\frac{V_o\sin\theta - gt}{V_o\cos\theta}\right) \qquad (4.9)$$

The coordinates (x,y) of the projectile at any time t are given by the following relations.

From Eq. 4.4 we get,

$$x = \left(V_o\cos\theta\right)t \qquad (4.10)$$

$$y = \left(V_o\sin\theta\right)t - \frac{1}{2}gt^2 \qquad (4.11)$$

As from Eq 4.10 t $\frac{x}{V_o\cos\theta}$, substituting the value of t in Eq. 4.11, we have,

$$y = \frac{x}{V_o\cos\theta}V_o\sin\theta - \frac{1}{2}g\left(\frac{x}{V_o\cos\theta}\right)^2$$

$$\text{or} \quad y = x\tan\theta - \frac{1}{2}\frac{gx^2}{V_o^2}\sec^2\theta \qquad (4.12)$$

$$\text{or}, y = ax - bx^2 \qquad (4.13)$$

where a and b are constants having the values $\tan\theta$ and $\frac{g}{2V_o^2}\sec^2\theta$ respectively. Eq. 4.13 represents a parabola. Thus the trajectory of projectile is a parabola as shown in Fig. 4.1. A stroboscopic photograph of a projectile is shown in Fig. 4.2

Fig. 4.3 shows an experimental arrangment in which two

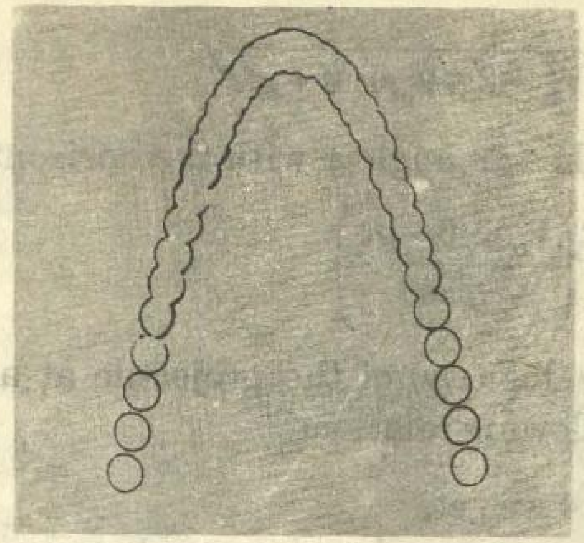


Fig. 4.2 *Stroboscopic photograph of projectile.*

metallic balls are thrown from a given height, ball *A* is given a horizontal velocity V_{ox} while the ball *B*, which is attached to an electromagnet is released simultaneously and allowed to fall vertically. It is seen that the motion of the ball *A* is unaffected while vertical motion of both the balls have the same vertical acceleration.

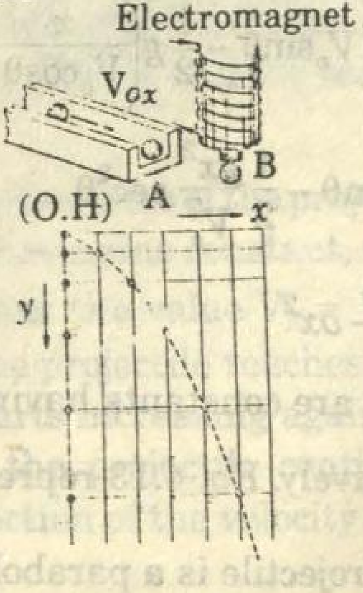


Fig. 4.3 *Experimental arrangement of balls A and B which are released simultaneously.*

From Fig. 4.4, it is clear that both the balls cover equal

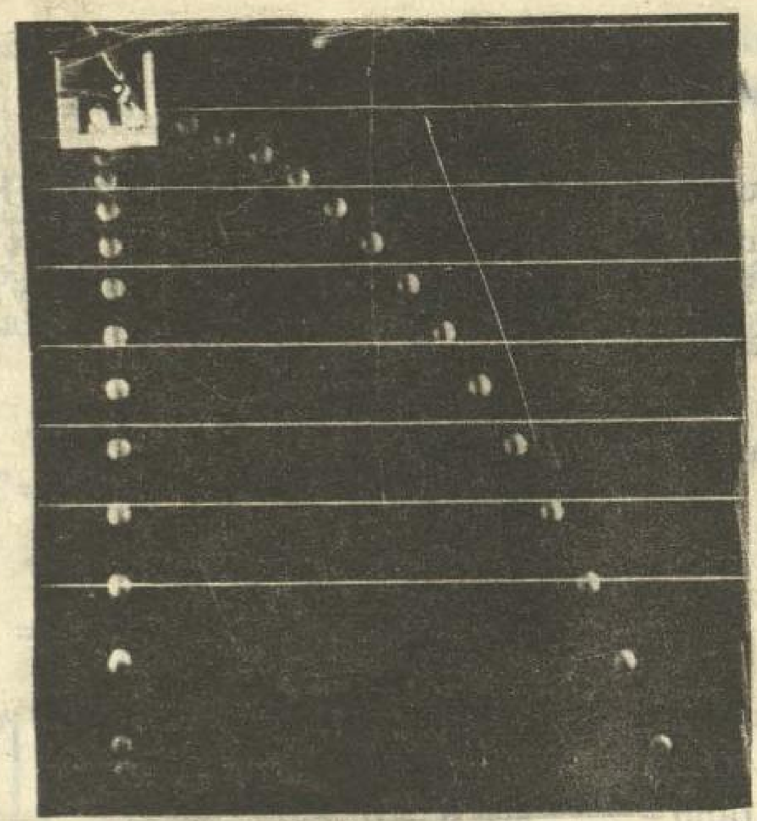


Fig. 4.4 *Stroboscopic photograph of two balls released simultaneously.*

vertical distance during the same interval of time. The ball *A* has constant horizontal velocity and both the balls hit the ground simultaneously.

At the maximum height *H* (Fig 4.1) the vertical component of velocity of projectile becomes zero. According to the Eq. 4.6, the time *T′* taken to achieve it, is

$$0 = V_o \sin\theta - gT'$$

$$\text{or}\,,\ T' = \frac{V_o \sin\theta}{g} \tag{4.14}$$

Substituting the value of *T′* in Eq. 4.11 and making $y_{\text{maximum}} = H$, we get,

$$H = \left(V_o \sin\theta\right)\frac{V_o \sin\theta}{g} - \frac{1}{2}g\left(\frac{V_o \sin\theta}{g}\right)^2$$

$$\text{or, } H = \frac{V_o^2 \sin^2\theta}{2g} \tag{4.15}$$

The range R of the projectile, which is the horizontal distance from its point of projection to the point where it returns to its level of projection is found by setting $y = 0$ in Eq. 4.12, we get,

$$x \tan\theta - \frac{1}{2}\frac{gx^2}{V_o^2}\sec^2\theta = 0$$

$$\text{or, } x\left(\tan\theta - \frac{1}{2}\frac{gx}{V_o^2}\sec^2\theta\right) = 0$$

which requires that either $x = 0$

$$\text{or, } \left(\tan\theta - \frac{1}{2}\frac{gx}{V_o^2}\sec^2\theta\right) = 0$$

When $x = 0$, then it is the point of projection (0, 0) and when,

$$\tan\theta - \frac{1}{2}\frac{gx}{V_o^2}\sec^2\theta = 0 \tag{4.16}$$

The value of x in Eq. 4.16 gives the range R of the projectile. Hence

$$\tan\theta - \frac{1}{2}\frac{g}{V_o^2}R\sec^2\theta = 0$$

$$R = \frac{2V_o^2 \sin\theta\cos\theta}{g} \tag{4.17}$$

$$\text{or, } R = \frac{V_o^2 \sin 2\theta}{g} \tag{4.18}$$

From Eq. 4.18, it is clear that if V_o and g are constant then

R is maximum when $\sin 2\theta$ is maximum. The maximum value of $\sin 2\theta = 1$ or $2\theta = 90^\circ$ or $\theta = 45^\circ$. The maximum range, $R_{max} = \frac{V_o^2}{g}$. Also $R = 0$ when $\theta = 0^\circ$ and $\theta = 90^\circ$. The time T of the projectile can be obtained by dividing its range with the horizontal component of the velocity of projection $V_o \cos\theta$, Hence

$$T = \frac{2V_o^2 \sin\theta \cos\theta}{gV_o\cos\theta}$$

$$= \frac{2V_o \sin\theta}{g}$$

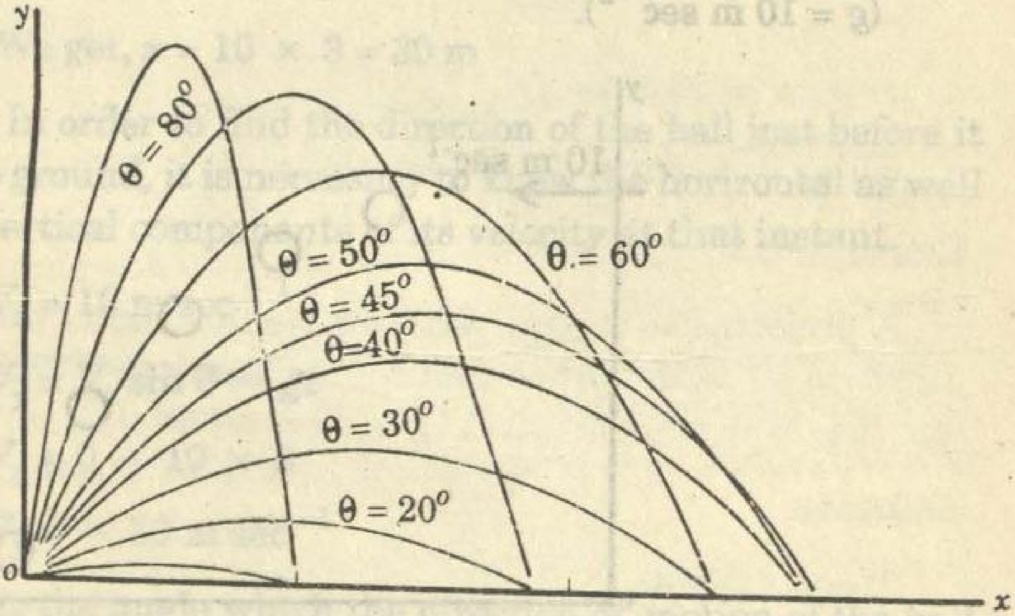


Fig. 4.5 *These parabolic trajectories are for the same initial speed 20 m sec^{-1} and launch angle at 10° intervals.*

Fig. 4.5 shows trajectories that correspond to several angles of projection, but having the same initial velocity V_o and acceleration due to gravity g. The maximum height, the distance from the projection point to the landing point, and projection angle vary from one curve to the other.

Example 4.1

A ball is thrown horizontally from the top of a cliff with a velocity of 10 m sce $^{-1}$. The height of the cliff above the ground is 45 m. Calculate

i) the time to reach the ground.

ii) the distance from the cliff to the point on which the ball hits the ground.

iii) the direction of the ball just before it hits the ground.

iv) Prove that the path of the ball in flight is a parabola ($g = 10$ m sec $^{-2}$).

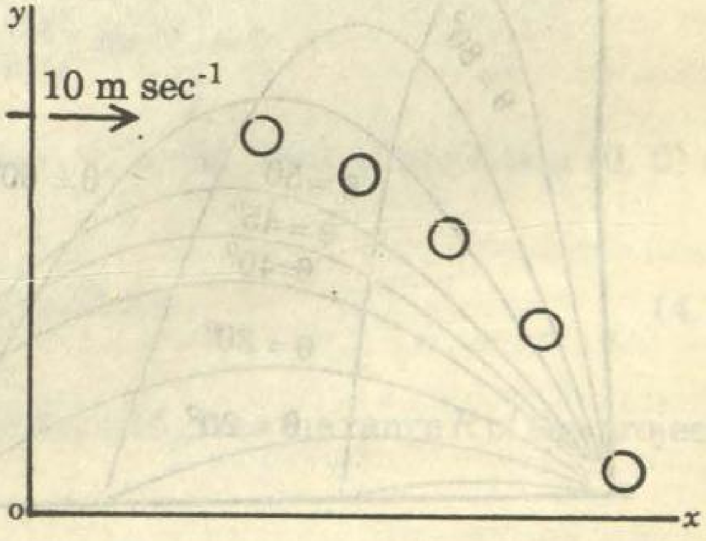


Fig. 4.6

Solution

If t is the total time of flight i.e. the time to reach the ground, then making use of the equation. 4.11 i.e.

$$\text{(i) } y = V_o \sin\theta\, t - \frac{1}{2} g t^2$$

We have

$$-45 = 0 \cdot t - \frac{1}{2} \cdot 10t^2$$

$$\text{or } t^2 = 9$$

$$\text{or } t = \pm 3 \text{ sec.}$$

Only +ve time value is to be taken into account as time cannot be negative.

(ii) If x is the distance from the cliff to the point where the stone hits the ground, then using the equation

$$x = V_o \cos \theta \, t.$$

We get, $x = 10 \times 3 = 30$ m

(iii) In order to find the direction of the ball just before it hits the ground, it is necessary to know the horizontal as well as the vertical components of its velocity at that instant.

$$V_x = 10 \text{ m sec}^{-1}$$

$$V_y = V_o \sin \theta - gt$$

$$V_y = 0 - 10 \times 3$$

$$V_y = -30 \text{ m sec}^{-1}$$

If θ is the angle which the direction of motion of the ball makes with x-axis then

$$\tan\theta = \frac{V_y}{V_x} = \frac{-30}{10} = -3$$

$$\theta = \tan^{-1}(-3)$$

$$\theta = -72^o$$

(iv) In order to establish that the path of the projected ball is a parabola we must know the position of the ball at any time

t. Hence the relation

$$x = V_o \cos\theta\, t, \text{ gives,}$$

$$x = 10t$$

$$\text{or, } t = \frac{x}{10}$$

Substituting the value of t in the Eq.4.11 we get

$$-45 + y = V_o \sin\theta\, t - \frac{1}{2} g t^2$$

$$-45 + y = -5 \cdot \frac{x^2}{100} \quad (V_o \sin\theta = 0)$$

$$\text{or } x^2 = 20\,(45 - y)$$

which is an equation of a parabola.

Example 4.2

A projectile is thrown with a speed of 100 m sec $^{-1}$ in a direction 30° with the horizontal. Find the time of flight, range and height to which it rises.($g = 10$ m sec $^{-2}$)

Solution

Time of flight:

$$T = \frac{2V_o \sin\theta}{g}$$

$$T = \frac{2 \times 100 \times \sin 30}{10}$$

$$= 10 \text{ sec.}$$

We find the range as,

$$R = V_o \cos\theta\, T$$

$$= 100 \cos 30 \times 10$$

$$= 866 \text{ m}$$

Now, we find the maximum height,

$$H = \frac{V_o^2 \sin^2\theta}{2g}$$

$$H = \frac{(100)^2 \cdot \left(\frac{1}{2}\right)^2}{2 \times 10}$$

$$H = 125 \text{ m}$$

Example 4.3

An Intercontinental Ballistic Missile (ICBM) having a maximum range of 8000 km is fired into a city from a distance of 8000 km. It is first detected at its half way point. How much warning time will have the residents of the city? How fast will the missile be travelling when detected? With what velocity will it strike the target? What will be its maximum height? Assume that the earth is flat over this 8000 km distance. Also assume that $g = 10 \text{ m sec}^{-2}$ is the same over all heights and ignore the air resistance.

Solution

The missile is fired for the maximum range, the angle of projection $\theta = 45^\circ$ and $\sin 45^\circ = \cos 45^\circ$, $R = 8000 \text{ km} = 8 \times 10^6 \text{ m}$ and $g = 10 \text{ m sec}^{-2.}$

Now using the equation of vertical motion

$$V_y = V_{oy} - gt$$

if t_1 represents the time to reach the maximum height where $V_y = 0$,

$$0 = V_{oy} - gt_1$$

$$V_{oy} = gt_1$$

Now the total duration of the journey of the missile is equivalent to twice the time taken by it to reach its maximum height. Hence the range

$$R = V_{ox}(2t_1)$$

$$\text{or } V_{ox} = \left(\frac{R}{2t_1}\right)$$

Now as $V_{ox} = V_o \cos 45^o = V_o \sin 45^o = V_{oy}$

Since $V_{oy} = g\, t_1$

$$\text{therefore, } gt_1 = \left(\frac{R}{2t_1}\right)$$

$$\text{or } t_1^2 = \left(\frac{R}{2g}\right)$$

$$t_1 = \sqrt{\frac{R}{2g}}$$

$$= \sqrt{\frac{8 \times 10^6}{2 \times 10}}\ sec.$$

$$\text{or } t_1 = 632\ sec.$$

Hence, the warning time is about 10.5 min approximately. The vertical component of velocity at the maximum height is zero, so the horizontal component ν_x is given by the relation

$$V_x = \frac{R}{2t_1} = \frac{8000\text{ km}}{2(632)\text{ sec}}$$

$$= 6.32\text{ km sec}^{-1}$$

The final velocity will have the value as,

$$V = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{2\,V_x^2} \text{ as } V_x = V_y$$

$$= \sqrt{2}\,V_x = 1.41 \cdot 6.32 \text{ km sec}^{-1}$$

$$V = 8.91 \text{ km sec}^{-1}$$

The maximum height attained by the rocket can easily be calculated by using the following formula

$$H = V_y t_1 - \frac{1}{2} g\, t_1^2$$

$$H = 6.32 \times 10^3 \times 632 - \frac{1}{2} \times 10\,(632)^2 = 2000 \text{ km}$$

The maximum height can also be calculated by considering the average vertical velocity and time.

$$H = \frac{1}{2} V_y\, t$$

$$H = \frac{1}{2} \times 6.32 \times 632 = 2000 \text{ km}$$

4.3 CIRCULAR MOTION

In two dimensional case, we have studied so far the motion of an object under the influence of gravity. Now we study the motion of a body in a circle; the universe is full of a large number of objects which move in nearly circular paths. For example, the earth and planets revolve around the sun and the moon moves around the earth in nearly circular orbits.

When an object moves with a uniform speed in a circular orbit its acceleration must be directed toward the centre, otherwise it will have a component along the direction of the motion which will change its speed. This acceleration is provided by various forces. In the case of planets it is the gravitational force which provides the necessary acceleration. For a moving car clearing a curve it is provided by the force of friction between the road and the tyres. The coulomb attraction between the nucleus and the electron is responsible for the nearly circular motion of the electron around the nucleus.

In order to study circular motion we shall have to define the following terms:

4.4 ANGULAR DISPLACEMENT

Consider a particle A which is moving on circular path of radius r. In a small interval of time t A moves to the position B (Fig. 4.7). If O is the centre of the circle along which the particle is moving, then after time t the radius OA occupies the position OB. The $\angle AOB = \theta$ is termed as the angular displacement of the particle in the interval t.

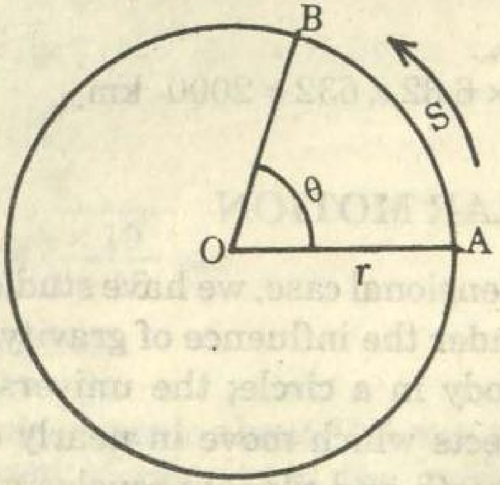


Fig. 4.7

Generally, the angular displacement is measured in terms of degrees but it is more convenient to measure it in radians. The radian is the angle subtended at the centre of a circle by an arc equal in length to its radius. If we measure the angle in radians we can easily find the length of the arc which subtends this angle at the centre. In order to understand it refer to Fig: 4.8 in which θ is the angle to be measured in radians. Arc *AB* is equal in length to the radius *r* of the circle, hence ∠*AOB* is equal to one radian. Now the arcs *AB* and *AC* are proportional to the angles which they are subtending at O

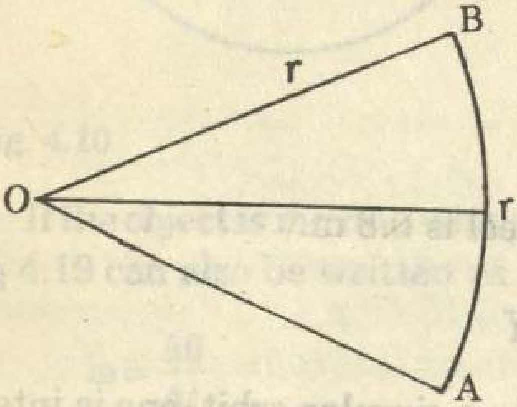


Fig. 4.8 *(One radian)*

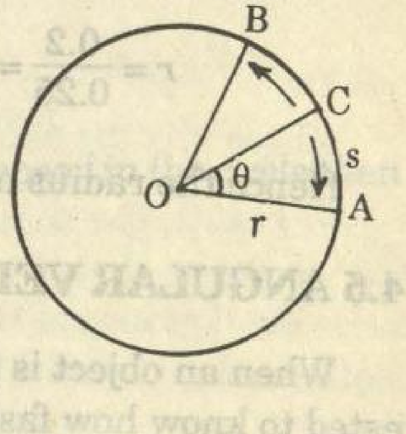


Fig. 4.9

i e.

$$\frac{\theta}{1\ \text{radian}} = \frac{\text{Arc AC}}{\text{Arc AB}} = \frac{s}{r}$$

where $s = \text{Arc}\,AC$

$$\therefore \theta = \frac{s}{r}\ \text{radian}$$

$$\text{or } s = r\,\theta \qquad (4.17\text{a})$$

i.e. length of an arc = radius of the circle × angle in radians made by the arc at the centre.

Example 4.4

A point on the rim of a wheel moves 0.2 m when the wheel turns through an angle 0.25 rad. What is the radius of the wheel?

Solution

Length of arc of a wheel = s = 0.2 m. Angle through the wheel turns = θ = 0.25 rad.

since $s = r\,\theta$,

$$0.2 = r \times 0.25$$

$$r = \frac{0.2}{0.25} = 0.8\,m.$$

Hence the radius of the wheel is 0.8 m.

4.5 ANGULAR VELOCITY

When an object is moving in a circular orbit, one is interested to know how fast it is doing so. Let us suppose that the object moves from point *B* with a uniform speed along the circumference of a circle of radius *r* in time *t*. If the arc *AB* subtends an angle θ at the centre of the circle, then the magnitude of the average angular velocity $<\omega>$ (pronounced as omega) about O is defined as the average angular displacement per second. Hence

$$<\omega> = \frac{\theta}{t} \tag{4.18a}$$

Magnitude of angular velocity is normally expressed in rad per second (rad $\sec^{-1}$). If the angular velocity is not uniform during this interval of time *t*, we find the instantaneous angular velocity (ω) which is defined as the angular displacement $\Delta\ \theta$ during a very small interval of time $\Delta\, t$.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \tag{4.19}$$

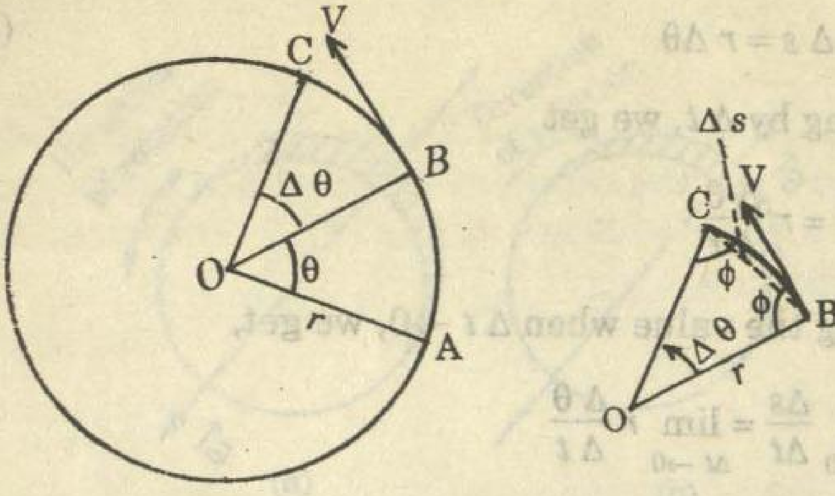


Fig. 4.10

If the object is moving with uniform speed in the circle then Eq 4.19 can also be written as

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\text{or } \Delta\theta = \omega\,\Delta t$$

Which is analogous to the formula 'distance = uniform velocity × time' for motion in a straight line. It may be noted that the time T to complete one revolution is called the period of motion and is given by

$$T = \frac{2\pi}{\omega} \tag{4.20}$$

$$\text{or } \omega = \frac{2\pi}{T}$$

Since 2 π rad is the angular displacement in one revolution.

If Δ s is the length of the arc BC subtending an angle Δθ at the centre O (Fig.4.10 b) then

$$\frac{\Delta s}{r} = \Delta\theta$$

$$\text{or } \Delta s = r\,\Delta\theta \tag{4.21}$$

Dividing by Δt, we get

$$\frac{\Delta s}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

Finding the value when $\Delta t \to 0$, we get,

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} r\,\frac{\Delta\theta}{\Delta t}$$

$$\text{or } V = r\,\omega \tag{4.22}$$

Where V is the speed of the rotating object.

To obtain the direction of $\vec{V}$ with respect to the radius $\vec{OB}$ we notice that the triangle BOC (Fig.4.10 b) is isosceles, since the distance of the object from O is constant in circular motion. Also the sum of the angles of a triangle is π we see that,

$$2\,\varphi + \Delta\theta = \pi$$

$$\text{or } \varphi = \frac{1}{2}\pi - \frac{1}{2}\Delta\theta$$

As Δt tends to zero, $\Delta\theta \to 0$, φ approaches to the value of 90°. Thus in the limit as $\Delta t \to 0$, and consequently $\vec{V} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t}$ becomes perpendicular to $\vec{OB} = \vec{r}$

We have seen earlier that the speed V is the product of ω and r and that the velocity $\vec{V}$ is at right angles to $\vec{r}$. Now we shall find a way of expressing both the magnitude and the direction of $\vec{V}$ in terms of a single equation, $\vec{V} = \vec{\omega} \times \vec{r}$. In order to do so, it is helpful to associate a definite direction to angular velocity of rotating body.

Consider the rotating disc of Fig.4.11 at a given instant,

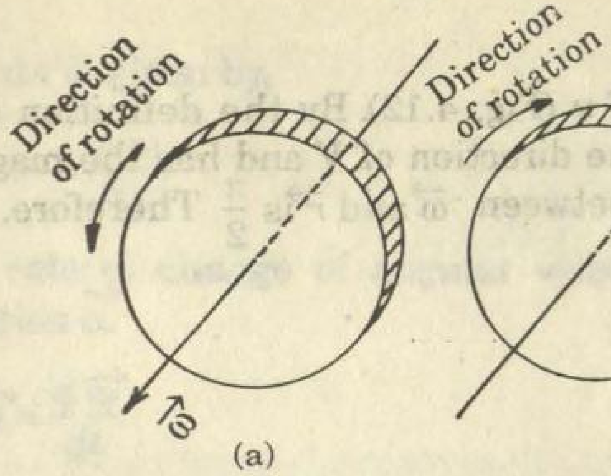


Fig. 4.11 *a) For counter-clock wise rotations ω is directed out of the page. b) ω is directed into the page for clockwise rotations.*

nearly every point on the front face of the disc is moving in a different direction from almost every other point. Hence none of the directions can be considered as the characteristic of the angular velocity of the disc. The only unique direction associated with the rotation of disc is that of the axis of rotation and so $\vec{\omega}$ is conventionally taken to be directed along the axis of rotation. It is directed out of the page, paralled to the rotation axis, if the rotation is anti-clockwise (Fig.4.11 a). If the rotation is clockwise, as in Fig.4.11 (b) $\vec{\omega}$ is directed into the page.

An alternative approach for identifying the direction of $\vec{\omega}$ is to wrap the fingers of the right hand around the rotation axis in the direction of rotation. The right thumb points

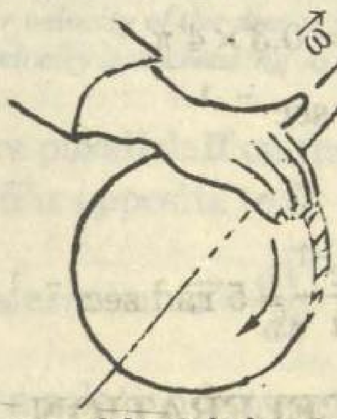


Fig. 4.12 *Curling the fingers of the right hand in the direction of rotation, the thumb points perpendicular to the disc in the direction of ω.*

in the direction of ω (Fig. 4.12). By the definition of vector product, $\omega \times r$ is the direction of V and has the magnitude $r\omega$ since the angle between $\vec{\omega}$ and $\vec{r}$ is $\frac{\pi}{2}$. Therefore, Eq. 4.22 can be written as,

$$\vec{V} = \vec{\omega} \times \vec{r} \tag{4.23}$$

Example 4.5

A model car moves round a circular track of radius 0.3 m at the rate of 2 revolutions per second what is (a) the angular velocity ω (b) the time period T (c) the speed V of the car? Find also (d) the angular velocity of the car, if it moves with a uniform speed of 2 m $\sec^{-1}$ in a circle of a radius 0.4 m?

Solution

a) For 1 revolution the angle turned $\theta = 2\pi$ rad

so $\omega = 2 \times 2\pi = 4\pi$ rad $\sec^{-1}$

b) Period T = time for 1 revolution $= \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{4\pi} = 0.5 \text{ sec.}$$

c) Velocity $V = r\,\omega = 0.3 \times 4\pi$

$= 1.2\pi = 3.8$ m $\sec^{-1}$

d) From $V = r\,\omega$

$$\omega = \frac{V}{r} = \frac{2\,m\,sec^{-1}}{0.4\,m} = 5 \text{ rad sec}^{-1}$$

4.6 ANGULAR ACCELERATION

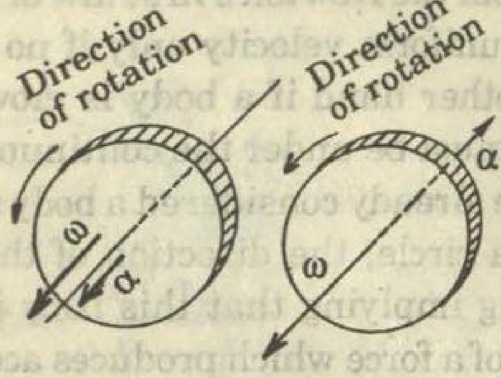
The magnitude of average angular acceleration $< \alpha >$ in an interval Δt is given by,

$$\langle \alpha \rangle = \frac{\Delta \omega}{\Delta t} \qquad (4.24)$$

The rate of change of angular velocity is the angular acceleration $\vec{\alpha}$.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} \qquad (4.25)$$

The SI units of α are rad sec $^{-2}$. If the direction of the rotation axis does not vary then the direction of α points along the axis and is either parallel or antiparallel to it. Consider the Fig. 4.13, if the rotation rate of the disc increases, the direction of $\vec{\alpha}$ and $\vec{\omega}$ are parallel. If on the other hand the disc is slowing down then $\vec{\alpha}$ is opposite to $\vec{\omega}$.



(a) (b)

Fig. 4.13 *(a) The angular velocity of the disc is increasing so $\vec{\alpha}$ and $\vec{\omega}$ are parallel. (b) The angular velocity is decreasing so α and ω are anti-parallel.*

Since linear acceleration $\vec{a} = \frac{d\vec{V}}{dt}$ and $V = r\,\omega$, hence $\vec{a} = \vec{\alpha} \times \vec{\gamma}$ This is a relation between linear and angular acceleration. The angular acceleration $\vec{\alpha}$ produces an acceleration perpendicular to the radius of the circle around which the object is moving, This is why it is called the tangential acceleration.

If the angular acceleration is uniform then the average angular velocity $\langle\vec{\omega}\rangle$ is defined as

$$\langle\omega\rangle = \frac{\omega_f + \omega_i}{2} \quad . \tag{4.26}$$

This expression has close resemblance with the expression for average linear velocity.

4.7 CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

It is well known from the Newton's first law of motion that a body moves with a uniform velocity only if no net force is acting on it. On the other hand if a body is moving along a circular path, then it must be under the continuous influence of some force. We have already considered a body moving with a constant speed in a circle, the direction of the velocity is continuously changing implying that this body is under the continuous influence of a force which produces acceleration in it.

The presence of the force needed to maintain uniform motion at constant angular speed can be demonstrated with the help of a stone tied to the end of a string and whirling around a circle. As long as the stone is moving in a circle, we can feel the tension in the string, the string must therefore be pulling the stone inward toward the hand with a force which produces an acceleration directed toward the centre on the stone. If the string breaks, the inward pull stops immediately and the stone does no longer move in a circle but flies tangentially to the circle. Let us calculate the magnitude of this acceleration.

Let the mass of the stone be m which is moving in a circle of radius r at a constant angular speed ω. It's speed remains constant, but its velocity V changes continuously. In time interval Δt, the stone moves through an angle $\Delta\theta = \omega\Delta t$ from A to B. Suppose $\overrightarrow{V_1}$ is the velocity of the stone at A, while at B it is $\overrightarrow{V_2}$ These two velocities have the same magnitude i.e.

$$|\overrightarrow{V_1}| = |\overrightarrow{V_2}| = V$$

but they differ in direction. The change in velocity ΔV is.

$$\Delta\overrightarrow{V} = \overrightarrow{V_2} - \overrightarrow{V_1} \tag{4.27}$$

In order to find the magnitude and direction of $\Delta\overrightarrow{V}$, we construct the vector diagram shown in Fig. 4.14, noting that the angle between $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ is $\Delta\theta$.

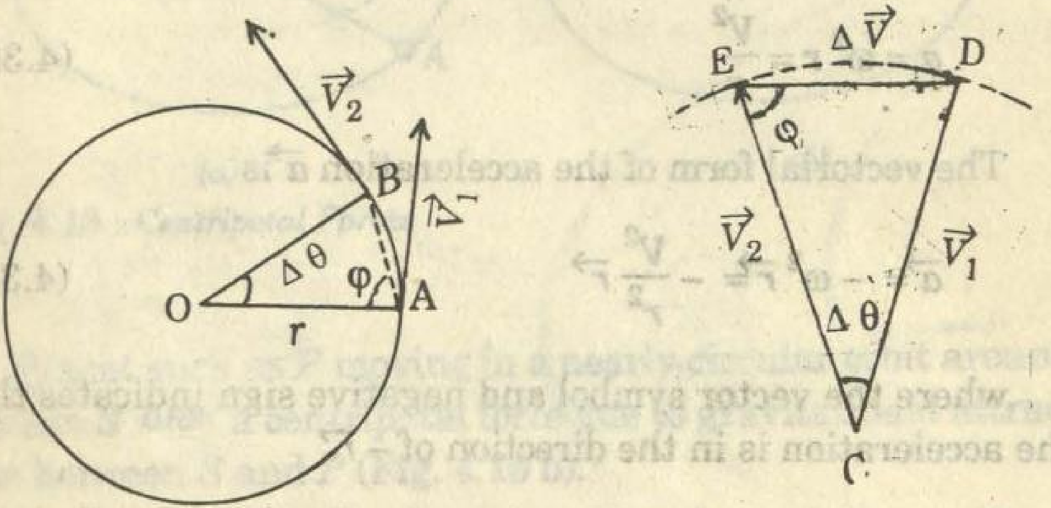

$\overrightarrow{V_2}$
B
$\overrightarrow{V_1}$
$\Delta\theta$
φ
O
A
r
$\Delta\overrightarrow{V}$
E
D
φ
$\overrightarrow{V_2}$
$\overrightarrow{V_1}$
$\Delta\theta$
C

Fig. 4.14 *Centripetal Acceleration.*

Now draw a circular arc DE with its centre at C. This arc is equal to the product of the angle $\Delta\theta$ and magnitude of the velocity V. If ΔV is small, the chord ΔV is very close to the arc DE so we have

$$\Delta V = V\Delta\theta = V\frac{\Delta\theta}{\Delta t}\Delta t = V\omega\Delta t \tag{4.28}$$

If $\overrightarrow{a}$ is the acceleration then

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{V \omega \Delta t}{\Delta t}$$

$$a = \omega V \tag{4.29}$$

We also note that in the limit $\Delta t \to 0$, the angle φ in Fig. 4.14 *b* approaches a right angle, so that $\Delta\vec{V}$ (in the limit $\Delta \theta \to 0$) is at right angles to $\vec{V}$ and is thus directed inward along the radius from the stone toward O. Thus the radial acceleration is in the direction of $-r$, that is toward the centre of the circle. From Eq. 4.23 and Eq 4.27 the magnitude of acceleration can be written as

$$a = \omega^2 r = \frac{V^2}{r} \tag{4.30}$$

The vectorial form of the acceleration $\vec{a}$ is

$$\vec{a} = -\omega^2 \vec{r} = -\frac{V^2}{r^2} \vec{r} \tag{4.31}$$

where the vector symbol and negative sign indicates that the acceleration is in the direction of $-\vec{r}$.

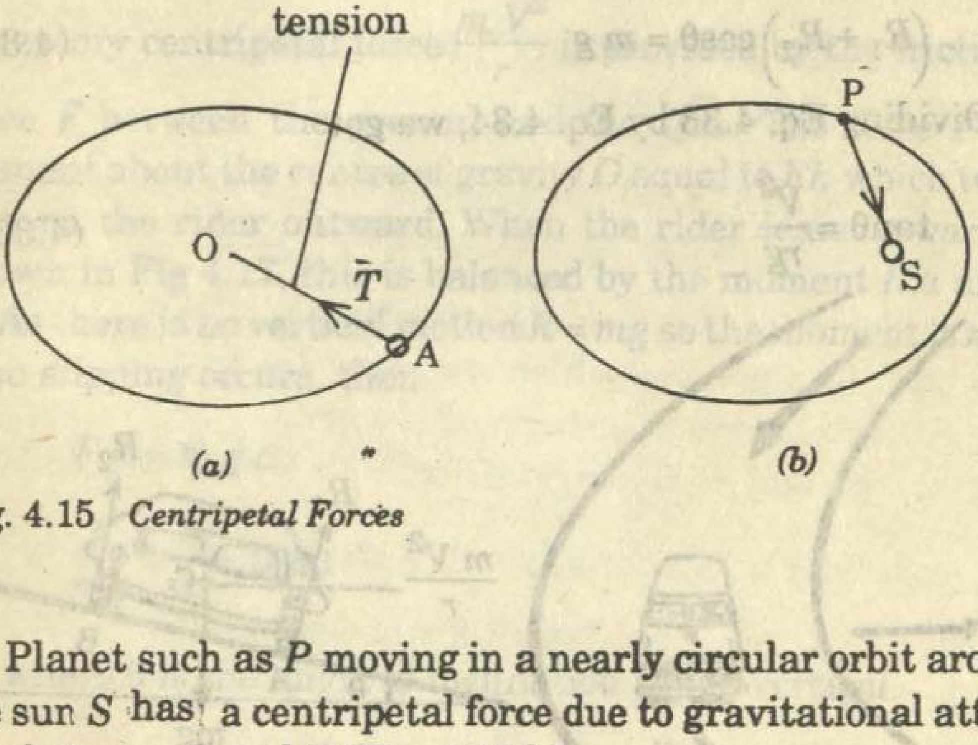
The radial acceleration given by Eq.4.31 is called the centripetal acceleration. The force that causes this acceleration is known as the centripetal force. Thus a mass m moving with a constant angular velocity in a circle must experience an inward radial force.

$$\vec{F}_r = -m\,\omega^2 \vec{r} = -\frac{mV^2}{r^2} \vec{r} \tag{4.32}$$

Whenever a body is moving in a circle, some agency must be present to provide a force directed toward the centre of rotation. The magnitude of this force depends upon the mass of the body, its distance r from the centre of the circle and the angular speed ω of the body about the axis of rotation.

Let us consider a few examples of motion in a circle from our daily life.

When a stone is whirled in a horizontal circle with centre O by means of a string the tension T provides the centripetal force (Fig. 4.15a)



(a) *(b)*

Fig. 4.15 *Centripetal Forces*

Planet such as P moving in a nearly circular orbit around the sun S has a centripetal force due to gravitational attraction between S and P (Fig. 4.15 b).

In racing car moving round a circular track the friction at the wheels provides the centripetal force.

4.8 MOTION OF A CAR (OR A TRAIN) AROUND A BANKED TRACK

Consider that a car (or train) is moving with speed V around a banked track in a circular path of radius r (Fig. 4.16). If there is no sideslip or strain on the wheels the only forces acting on the wheels A and B are the reactions R_1 and R_2

respectively. The force towards the centre of the track is $\left(R_1 + R_2\right)\sin\theta$ which is the necessary centripetal force which enables the car to move on a circular track, θ is the angle of inclination of the plane to the horizontal. Therefore,

$$\left(R_1 + R_2\right)\sin\theta = \frac{m\,V^2}{r} \tag{4.33}$$

The car does not move in the vertical direction hence for vertical equilibrium.

$$\left(R_1 + R_2\right)\cos\theta = m\,g \tag{4.34}$$

Dividing Eq. 4.33 by Eq. 4.34, we get

$$\tan\theta = \frac{V^2}{rg} \tag{4.35}$$

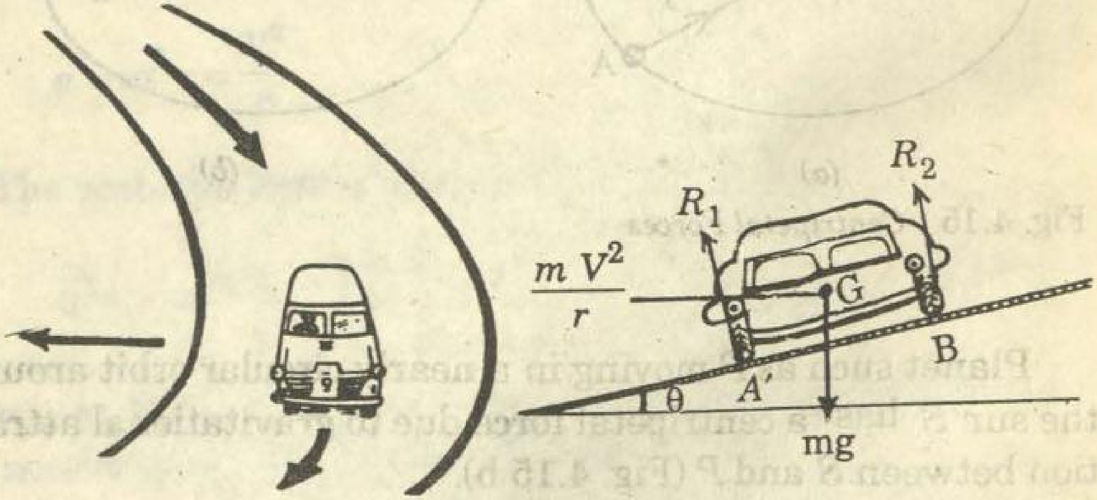


Fig. 4.16 *Car on a banked track.*

From the above equation it is quite clear that the road or a track can be ideally banked for one speed only. For such a speed the car can negotiate the turn without using steering wheel. At high speeds one has to depend upon friction to prevent the car from slipping.

A racing track is made saucer shaped because at high speed the cars may move toward a part of the track which is steeper and prevents slipping. The outer rail of a curved railway track is raised above the inner rail so that the major part of the force towards the centre is provided by the component of the reaction at the wheels.

4.9. MOTION OF A BICYCLE RIDER ROUND A CIRCULAR TRACK

When a person cycles around a circular racing track, the necessary centripetal force $\frac{mV^2}{r}$ is provided by the frictional force F between the ground and the tyre. The force F has moment about the centre of gravity G equal to Fh which tends to turn the rider outward. When the rider leans inward as shown in Fig 4.17, this is balanced by the moment $R.a$ about G. As there is no vertical motion $R = mg$ so the moment is mga. If no slipping occurs then

$$Fh = mga$$

$$\frac{F}{mg} = \frac{a}{h} = \tan\theta$$

where θ is the angle of inclination to the vertical.

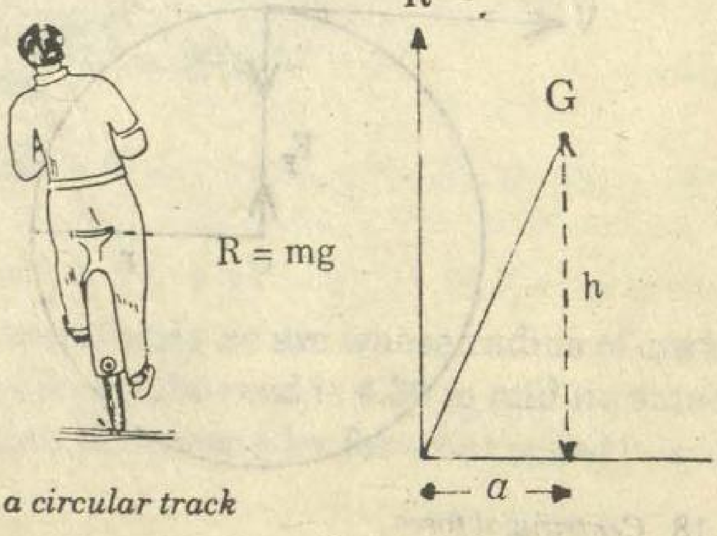


Fig. 4.17 *Rider on a circular track*

Since $F = \frac{m V^2}{r}$, therefore

$$\tan\theta = \frac{V^2}{rg}$$

When F is greater than the limiting friction, skidding takes place. In this case F is greater than μmg or $mg \tan\theta > \mu mg$. Thus $\tan\theta > \mu$ is the condition for skidding.

4.10 CENTRIFUGAL REACTION AND CENTRIFUGAL FORCE

Suppose we have a ball attached to the end of a string and we whirl it in a circle as shown in the Fig. 4.18

Since the ball is not moving in a straight line, it is being accelerated by the force which we exert by pulling on the string. This force which is directed toward the centre and is acting on the ball is centripetal force F_r which is equal to $\frac{m V^2}{r}$. According to Newton's third law, the ball exerts an equal and opposite force on us. This force is called the centrifugal reaction or centrifugal force.

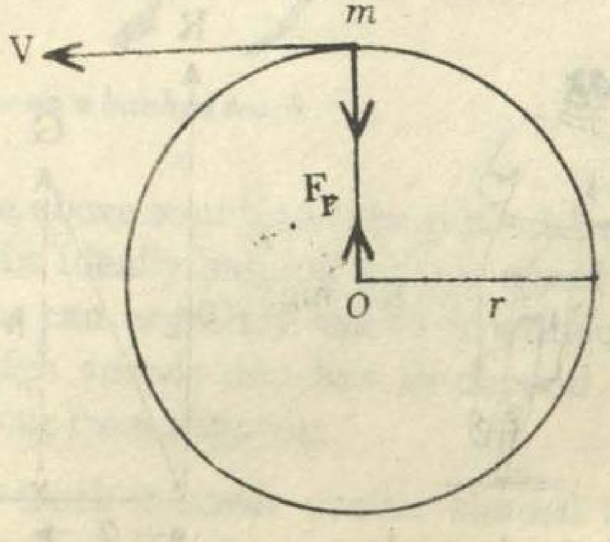


Fig. 4.18 *Centrifugal force.*

Sometime an incorrect statement is made that the ball moves under the influence of two forces centripetal and centrifugal. There is actually only one force that acts on the ball the inward centripetal force. The outward centrifugal force is only a reaction force. If the string breaks at an instant then the ball will be moving with a velocity V as shown in the Fig. 4.18, the centripetal force disappears i.e; no force is acting on the ball to make it move along the circle. According to Newton's First Law the ball must move with constant velocity. So the ball moves away along a direction tangent to the circle.

Example 4.6.

A vehicle of mass 1000 kg moves round a curve whose radius of curvature is 200 m at a constant speed of 108 km hr^{-1}. What is the centripetal force on the car.

Solution

$$\text{The centripetal force} = \frac{m\,V^2}{r}.$$

$$m = 1000\text{ kg}$$

$$V = 108\text{ km hr}^{-1} = 30\text{ m sec}^{-1}$$

$$r = 200\text{ m}$$

$$F = \frac{1000 \times (30)^2}{200} = 4500\,N$$

$$F = 4500\text{ N}$$

Example 4.7

A curve on a road forms an arc whose radius of curvature is 200 m. If the width of the road is 4.30 m and its outer edge is 0.6 m higher than the inner edge for what speed it is ideally banked.

Solution

If θ is the angle of the bank, then

$$\tan\theta = \frac{\text{difference in the level of two edges of the road}}{\text{width of the road}}$$

$$= \frac{0.6}{4.30} = 0.13$$

$$\text{But also } \tan\theta = \frac{V^2}{rg}$$

$$\therefore \frac{V^2}{rg} = 0.13$$

Since $r = 200$ m, $g = 10$ m sec $^{-2}$

$$V = \sqrt{0.13 \times 200 \times 10} = 16.12\ m\ sec^{-1}$$

Example 4.8

The curve on a level road has a radius of 75 m. A motor bike whose total mass with its rider is 120 kg has to negotiate this curve at a speed of 54 km hr $^{-1}$. If the co-efficient of friction between the tyre and ground is 0.4, will the rider be able to go round the curve without slipping? At what angle should he lean to negatiate the curve without slipping?

Solution

$$\text{The required centripetal force } = \frac{m\ V^2}{r}.$$

where m = 120 kg, V = 54 km hr $^{-1}$ = 15 m sec $^{-1}$ and r = 75 m

$$F = \frac{120 \times (15)^2}{75}$$

F = 360 Newton

The total weight of the rider and the motor bike

$= 120 \times 10 = 1200$ N

as $g = 10$ m $\sec^{-2}$

Now the maximum force of friction

F_r = coefficient of friction × reaction

$F_r = \mu \times$ weight of the rider and bike.

$= 0.4 \times 1200 = 480$ N.

Since $F_r > F$, the rider can negotiate the curve. If θ is the angle at which the rider should lean from the vertical.

$$\tan\theta = \frac{m\,V^2\!/r}{mg}$$

$$= \frac{360}{1200} = 0.3$$

$$\therefore \theta = \tan^{-1}(0.3)$$

Example 4.9

Calculate percentage difference in the weights of an object when it is measured at the poles and equator of the earth. Also calculate the angular velocity of the earth when the object becomes weightless at the equator.

Solution

Suppose an object of mass m is attached to a spring balance by a string at either of the poles. Since the polar axis passes through the poles hence the object has no rotation about the poles.

So the tension in the spring = mg. Suppose the same object is carried to the equator and attached by a string to the spring balance. Now the object has an angular velocity ω about the centre of the earth as the earth rotates about its polar axis so an acceleration $\frac{V^2}{r} = \frac{r^2\,\omega^2}{r} = r\,\omega^2$ acts toward the centre of the earth. If T_1 is the tension in the spring which acts upward on the object then centripetal force = net force toward the centre, i.e.,

$$mg - T_1 = m\,r\,\omega^2$$

$$\text{or } T_1 = mg - m\,r\,\omega^2$$

So if $m'g$ is the apparent weight then

$$mg' = mg - m\,r\,\omega^2$$

$$\text{or} \quad m(g' - g) = -m\,r\,\omega^2$$

$$\text{or} \quad m\Delta g = -m\,r\,\omega^2$$

$$\text{where} \quad \Delta g = g' - g$$

Dividing the above equation by mg, we get

$$\frac{m\Delta g}{mg} = \frac{r\omega^2}{g}$$

$$\frac{\Delta w}{w} = \frac{-r\,\omega^2}{g}$$

where $w = mg$ and $\Delta w = m\Delta g$

Since time T to complete one revolution,

$$T = 24 \text{ hr} = 24 \times 3600 \text{ sec.}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600} = 7.3 \times 10^{-5} \text{ rad sec}^{-1}$$

$$r = 6.36 \times 10^6 \text{ m}, g = 9.78 \text{ m sec}^{-2}$$

$$\text{Percentage difference,} = \frac{6.36 \times 10^6 \times \left(7.3 \times 10^{-5}\right)^2 \times 100}{9.78}$$

$$= 0.35\ \%$$

Suppose the angular velocity of the earth becomes ω' when the object is weightless on the equator, i.e

$$0 = mg - mr\omega'^2$$

$$\omega'^2 = \frac{g}{r} = \frac{9.78}{6.36} \times 10^{-6}$$

$$\text{or } \omega' = 1.2 \times 10^{-3} \text{ rad sec}^{-1}$$

4.11 EQUATIONS OF ANGULAR MOTION

In chapter II, while deriving equations of rectilinear motion of a uniformly accelerated particle only magnitudes of displacement, velocity and acceleration were considered. Similarly if the rotation of a body about a fixed axis is considered, the angular displacement, angular velocity and angular acceleration can be dealt with as scalar. The equations of motion of such rotating bodies with uniform angular acceleration can be obtained simply by replacing the corresponding quantities of linear motion of uniformly accelerated bodies. Table 4.1 illustrate the analogous equations in linear and angular motion.

Table 4.1 Equation of motion with constant acceleration

Linear motion	*Rotational Motion*
1. $S = V t$	$\theta = \omega t$
2. $V_f = V_i + at$	$\omega_f = \omega_i + \alpha t$
3. $\langle V \rangle = \frac{V_i + V_f}{2}$	$\langle \omega \rangle = \frac{\omega_i + \omega_f}{2}$
4. $V_f^2 - V_i^2 = 2 a s$	$\omega_f^2 - \omega_i^2 = 2 \alpha \theta$
5. $S = V_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
6. $F = m a$	$\tau = I \alpha$

Example 4.10

A currently available model of rotary lawn mower has a rotation rate of 3700 revolution per minute. If the blades come to a stop with a constant acceleration in 3 sec, find the number of turns it makes as it slows down.

Solution

$\omega_i = 3700$ rev. per minute

$$= 3700\,\frac{2\pi}{60} = 387 \text{ rad sec}^{-1}$$

The angle it moves through is

$$\Delta\,\theta = \frac{1}{2}\left(\omega_i + \omega_f\right)\Delta\,t$$

$$= \left(\frac{1}{2}\,387 \text{ rad sec}^{-1} + 0\right) \times 3$$

$$= 581\text{rad.}$$

$$\text{number of turns} = \frac{581}{2\pi} = 92.4$$

Example 4.11

A ship propeller shaft is turning at 100 rev/min. The order is given to the engine room to increase the angular speed at a uniform rate to 400 rev/min and to reach that angular speed in exactly 1 min. Find its angular acceleration.

Solution

$\omega_i = 100\ {}^{\text{rev}}\!/_{\text{min.}}$

$$= 100 \times \frac{2\pi}{60} = 10.47\ {}^{\text{rad}}\!/_{\text{sec.}}$$

$\omega_f = 400\ {}^{\text{rev}}\!/_{\text{min.}}$

$$= 400 \times \frac{2\pi}{60} = 41.88 \text{ rad/sec.}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{41.88 - 10.47}{60}$$

$$= 0.52 \text{ rad sec}^{-2}$$

Example 4.12

The rear wheels of an automobile are turning at 60 rad sec^{-1}. The brakes are applied for 5 sec. giving a uniform (negative) angular acceleration of 8 rad sec^{-2}. Find (a) the final angular velocity and (b) the number of revolution turned by the rear wheels during the braking period.

Solution

a) $\omega_f = \omega_i + \alpha t$

$$= 60 + (-8) \times 5$$

$$= 20 \text{ rad sec}^{-1}$$

b) Angular displacement

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$= 60 \times 5 + \frac{1}{2} \times (-8)(5)^2$$

$$= 200\text{rad} = \frac{200}{2\pi} = 31.8 \text{ rev.}$$

SUMMARY

An object launched in space without any driving power travels freely under the action of gravity without air resistance is named as projectile.

The path described by the projectile is known as its trajectory. Equation of the trajectory of a projectile is

$$y = x\tan\theta - \frac{1}{2}\frac{g x^2}{V_o^2}\sec^2\theta$$

Time of flight, $T = \frac{2\,V_o \sin\theta}{g}$

Maximum Height, $H = \frac{V_o^2 \sin^2\theta}{2g}$

Range, $R = \frac{V_o^2 \sin 2\theta}{g}$

Maximum range is obtained when the angle of projection is 45° and is, $R_{\max} = \frac{V_o^2}{g}$

Radian is the angle subtended at the centre of a circle by an arc equal in length to its radius.

1 radian = 57.3° = 0.159 rev $\qquad \theta = \frac{s}{r}$

Average angular speed, $<\omega> = \frac{\theta}{t}$ rad sec^{-1}

Instantaneous angular speed, $\omega = \lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

Period of a circular motion, $T = \frac{2\pi}{\omega}$

Relation between the speed of the rotating object and its angular speed: $V = r\,\omega$

Relation between the velocity of the rotating object and its angular velocity: $\vec{V} = \vec{\omega} \times \vec{r}$

Centripetal acceleration, $\vec{a} = -\dfrac{V^2}{r^2}\vec{r} = -\omega^2\vec{r}$

Centripetal force, $F_r = -\dfrac{mV^2}{r^2}\vec{r} = -m\,\omega^2\vec{r}$

QUESTIONS

4.1. Draw a curve to illustrate the variation of range of a projectile with angle of projection.

4.2. At what point in the path of a projectile does it have its maximum and minimum speeds.

4.3. A bombar drops its bomb when it is vertically above its target, it misses it. Explain the above statement.

4.4. In broad jumping does it matter how high you jump? What factors determine the span of the jump?

4.5. Using the trajectories for $\theta = 30^\circ$ and 60°, argue that with fixed initial speed $\theta = 60^\circ$ corresponds to the greater range if the projectile lands above its launching point, whereas $\theta = 30^\circ$ corresponds to the greater range if the projectile lands below its launching point.

4.6 What will be the advantage , if angle is measured in radians rather than in degrees?

4.7. In what direction does mud fly off the tyre of a moving cycle? Why?

4.8. Centripetal force and centrifugal reaction are equal in magnitude but opposite in direction. Do they balance each other?

4.9. Is any work done by centripetal force?

4.10. For a particle moving in a horizontal circle with constant angular velocity.

a) the linear momentum is constant but the energy varies.

b) the energy is constant but the linear momentum varies.

c) both energy and linear momentum are constant.

d) neither the linear momentum nor the energy is constant.

e) the speed and the linear velocity are both constant.

Tick ✓ the statement which is true.

PROBLEMS

4.1. A marble rolls across a horizontal table at the constant speed 2.3 m $\sec^{-1}$. When it reaches the edge of the table, the marble falls through a vertical distance of 1 m to the floor.

a) For what length of time does the marble fall?

b) What is the horizontal distance between the tables edge and the impact point of the marble on the floor?

c) Find the components of its velocity when it strikes the floor.

4.2. A dive bomber diving at an angle of 53° with the vertical releases a bomb at an altitude of 730 m. The bomb hits the ground in 5 sec, after being released.

a) What is the speed of the bomber.

b) How far did the bomb travel horizontally during its flight?

c) What are the horizontal and vertical components of its velocity just before striking the ground?

4.3. The total speed of projectile at its greatest height is $\sqrt{\frac{6}{7}}$ of its total speed when it is at half of its greatest height. Show that the angle of projection is 30 °.

4.4. In the shot put event an athlete releases the shot (7.2 kg metal ball) 2m above ground level. The athlete twice put the shot through a horizontal distance of 22 m. In one case the launch angle is 45 ° and in the other it is 42 °. Which put requires the larger initial velocity.

4.5. A gun that can fire a projectile at speed of V_o which is independent of the angle at which the gun is pointed, is aimed at a target at a height H above the gun. The gun is pointed at an angle of 45 ° with the horizontal. What is the greatest horizontal component of the displacement of the target from the gun if the projectile is to strike the target? Assume that all frictional forces on the projectile may be neglected.

4.6. Assume the monkey lets go of the tree shown in Fig:

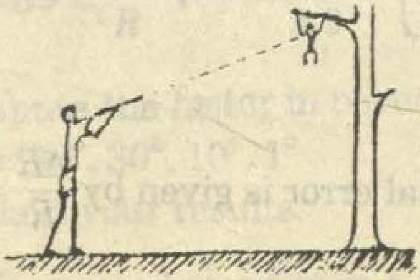


Fig. 4.19

4.19 when the gun is fired. At what angle the gun is pointed so that the bullet hits the monkey while in free fall? Show that the answer turns out to be independent of the muzzle velocity.

4.7 A stunt man runs across the roof of a building with horizontal speed V. As he leaps from the edge of the building, a truck with a net starts from rest at the bottom of the building and moves directly away from the building with an acceleration a. If the height of the building is h find an expression for V in terms of h, g and a, so that he lands in the net.

4.8 A particle is projected with speed 14 m sec^{-1} at an angle to the horizontal. Find the values of θ for which the particle just clears an obstacle 5m high and 10m from the point of projection.

4.9 Show that the range of a projectile is the same for $\theta = 45^\circ + \varphi$, as for $\theta = 45^\circ - \varphi$, provided that $\varphi < 45^\circ$.

4.10 The curvature of the earth becomes important in projectile calculations when the distance travelled x is a significant fraction of the radius of the earth R_E, ignoring any possible variation in g, show that (a) the extra time the projectile is in motion is approximately given by

$$\Delta t \cong \frac{\Delta y}{V} \cong \frac{\left(\dfrac{x^2}{2R_E}\right)}{V_{iy}}$$

(b) the fractional error is given by $\dfrac{\Delta R}{R} = \dfrac{V_{ix}^2}{gR_E}$

4.11 Using the equation in the preceding problems find the fractional error in the range of a projectile due to the curvature of the earth if the projectile is fired with launch angle of 45° and intended range of 100 km (The average radius of the earth is 6.38×10^6 m).

4.12 A wheel is revolving at a steady rate of 120 rev/min

a) What is its angular velocity in rad $\sec^{-1}$?

b) What is the linear velocity of a point on the wheel 0.25 m from the axle

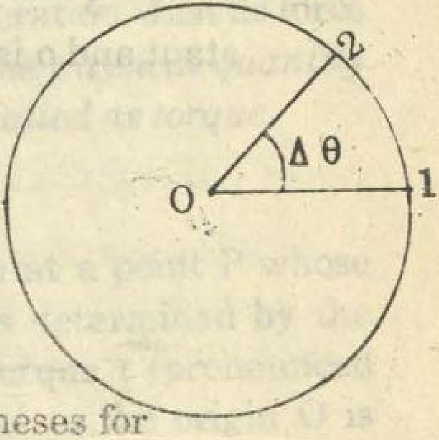
4.13 The orbit of an electron in a hydrogen atom may be considered to be a circle of radius about 50 pm and the period of motion is 0.15 fs.
Calculate,

a) the angular velocity

b) the linear speed

c) the centripetal acceleration

4.14 A particle travels at constant speed V on a circle of radius R. the particle moves from position 1 to position 2 on the circle. The angular opening between 1 and 2 is $\Delta\theta$. Show that,



a) The time, $\Delta t = \frac{R\,\Delta\theta}{V}$

b) The average acceleration,

$$\langle a\rangle = \frac{V^2}{R}\,\frac{\sqrt{2\,(1-\cos\Delta\theta)}}{\Delta\theta}$$

c) Compute the factor in parentheses for
$\Delta\theta = 90^\circ, 30^\circ, 10^\circ, 1^\circ$
Explain your results.

4.15. A stone is rotated steadily in a horizontal circle with a period T by a string of length l. If the tension in the string is constant and l increases by 1%, find the percentage change in T.

4.16. An object of mass 0.5 kg is rotated in horizontal circle by a string 1 m long. The maximum tension in the string before it breaks is 50 N. What is the greatest number of revolutions per second of the object.

4.17. Calculate the angle at which a curve must be banked to prevent side slip at 90 km $\text{hr},^{-1}$ if the radius is 100 m. If a car of mass 1000 kg travels round the banked track at 25 m sec^{-1} without sideslip. calculate the centripetal force on the car (b) the ground reaction on the car.

4.18 An object O of mass 0.2 kg is whirled in a vertical circle at the end of a string of length 1 m at a steady rate of 5 rev sec^{-1}. Find (a) the speed of the object (b) the tension in the string when O is at the top of the circle (c) tension when O is at the bottom of the circle.

4.19. A rotating star has a mass M and radius R. Prove that the critical angular velocity ω_c for the star to break up is $\omega_c^2 = \frac{4}{3}\pi G \rho$, where G is Newton's gravitation constant and ρ is the density of the star.

5

TORQUE, ANGULAR MOMENTUM AND EQUILIBRIUM

5.1 TORQUE

In previous chapters, we have studied the motion of a particle along a straight line and in a circle. We know that when a force is applied on a body it produces linear acceleration in it. The question arises which physical quantity produces angular acceleration. A familiar example of rotation of door reveals the physical quantity causing its rotation. One finds that if a force is applied at the hinge, it does not produce any angular acceleration. On the other hand, if a force of the same magnitude is applied at right angle to the outer edge of the door, it produces the maximum acceleration. Just as force produces linear acceleration in a body *the physical quantity which produces angular acceleration is called as torque.*

Torque acting on a particle

Suppose a force $\vec{F}$ acts on a particle at a point P whose position with respect to the origin O is determined by the displacement vector $\vec{r}$(Fig. 5.1). The torque $\vec{\tau}$ (pronounced tau), acting on the particle with respect to the origin O is defined as

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{5.1}$$

Torque is a vector quantity, having magnitude $rF\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$. The direction of $\vec{\tau}$ is along the normal to the plane containing $\vec{r}$ and $\vec{F}$ and its sense is obtained by using right-hand rule.

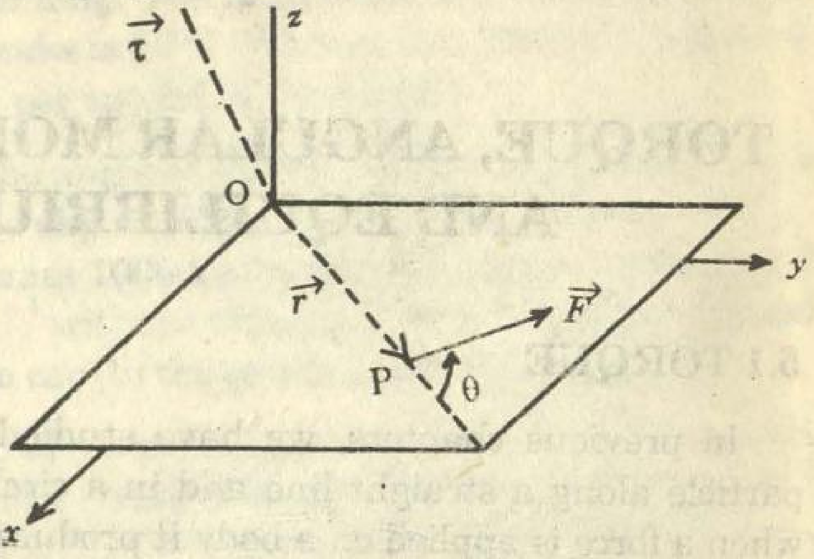


Fig. 5.1

It is evident from Eq. 5.1 that the torque produced by a force depends upon the following quantities,

a) magnitude of the force,

b) direction of the force,

c) the point of application of the force relative to the origin i.e. the position vector $\vec{r}$ relative to the point about which the turning effect is measured.

If the line of action of the force $\vec{F}$ passes through the origin, r is zero and so the torque $\vec{\tau}$ about the origin is zero. The magnitude of $\vec{\tau}$ can be written as

$$\tau = (r\sin\theta)F = r_{\perp}F \tag{5.2}$$

or as
$$\tau = r(F\sin\theta) = rF_{\perp} \tag{5.3}$$

Where $r_{\perp}$ is the component of $\vec{r}$ perpendicular to $\vec{F}$ and $F_{\perp}$ is the component of $\vec{F}$ perpendicular to $\vec{r}$ (Fig. 5.2).

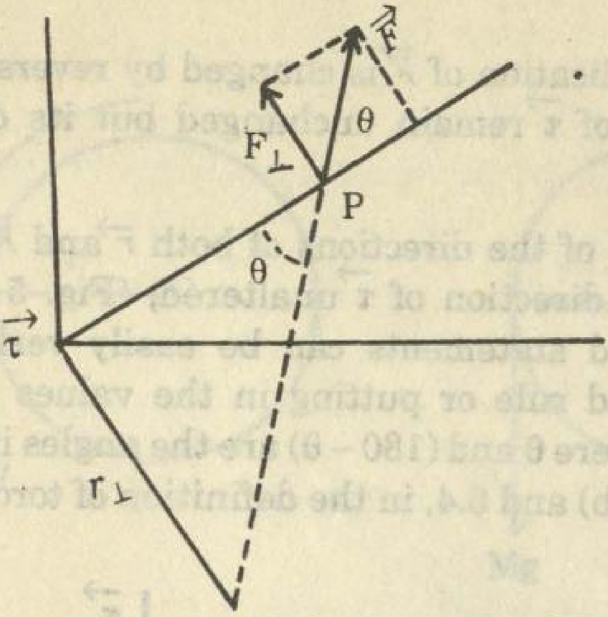

$\vec{F}$
θ
$F_{\perp}$
P
θ
$\vec{\tau}$
$r_{\perp}$

Fig. 5.2

Torque is also named the moment of force and $r_{\perp}$ is called the moment arm. Eq. 5.3 clearly implies that only the component $\vec{F}$ perpendicular to $\vec{r}$ contributes to the torque.

If $\theta = 0$ or 180, then no perpendicular component of F exists i.e. $F_{\perp} = F\sin\theta = 0$. It also means that if the line of action of force $\vec{F}$ passes through the origin and the moment arm $r_{\perp}$ about the origin is also zero, in this case both the Eqs 5.2 and 5.3 imply that the torque τ is zero.

In Fig. 5.3 the direction of $\vec{F}$ is reversed, the magnitude of τ remains the same but the direction of τ is reversed. Now if

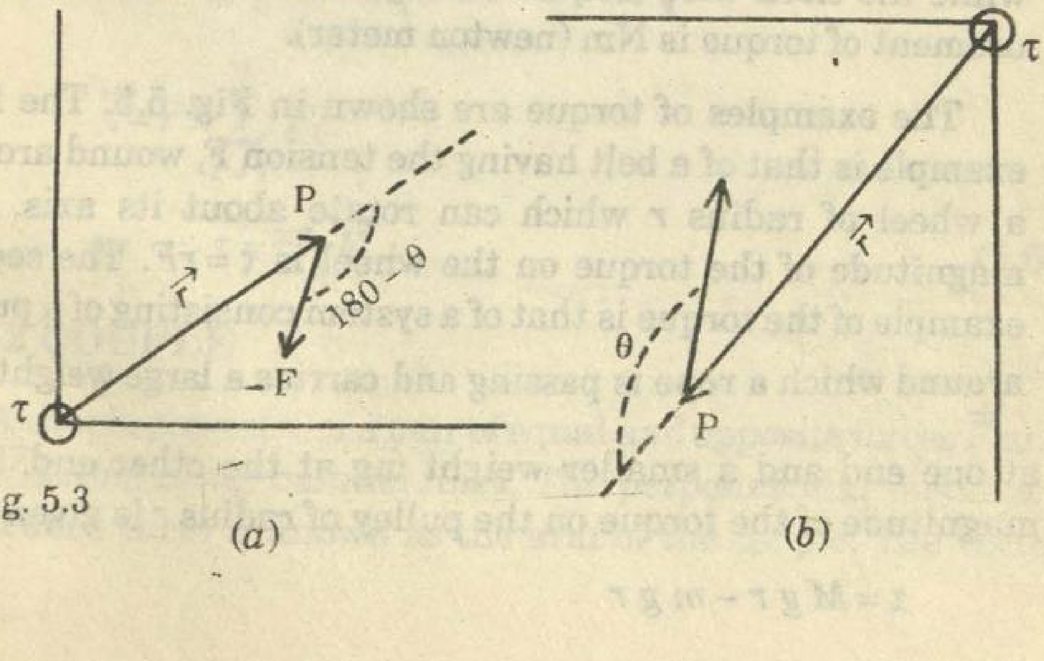

P
$\vec{r}$
$180-\theta$
$-F$
τ
τ
$\vec{r}$
θ
P

Fig. 5.3

(a) (b)

the point of application of $\vec{F}$ is changed by reversing $\vec{r}$, then the magnitude of $\vec{\tau}$ remain unchanged but its direction is reversed.

The reversal of the directions of both $\vec{r}$ and $\vec{F}$ keeps the magnitude and direction of $\vec{\tau}$ unaltered, (Fig. 5.4). All the above mentioned statements can be easily verified either using right hand rule or putting in the values of sin θ or sin (180 – θ), where θ and (180 – θ) are the angles indicated in the Figs. 5.3 (a, b) and 5.4, in the definition of torque.

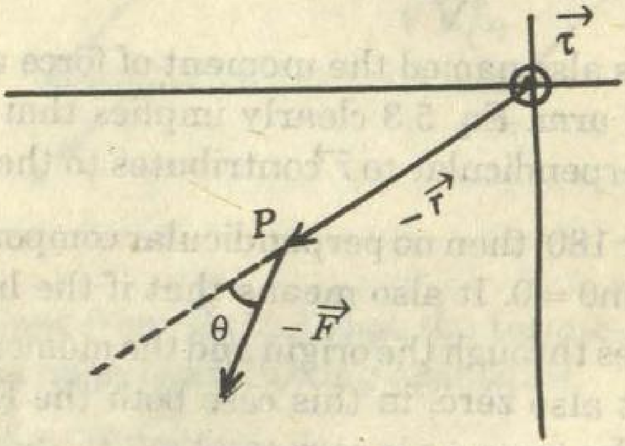


Fig. 5.4

Conventionally, anticlockwise torque are taken as positive while the clock-wise torque as negative. The unit for measurement of torque is Nm (newton meter).

The examples of torque are shown in Fig. 5.5. The first example is that of a belt having the tension F, wound around a wheel of radius r which can rotate about its axis. The magnitude of the torque on the wheel is $\tau = rF$. The second example of the torque is that of a system consisting of a pulley around which a rope is passing and carries a large weight Mg at one end and a smaller weight mg at the other end. The magnitude of the torque on the pulley of radius r is given as ,

$$\tau = Mgr - mgr$$

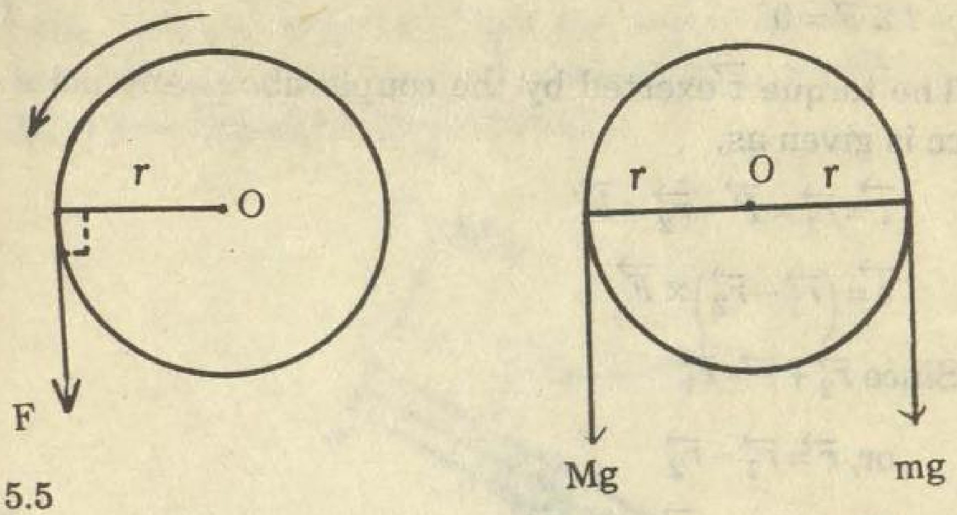


Fig. 5.5

$$\tau = (M - m)\, g\, r \tag{5.4}$$

Until now we have considered that a single force produces torque in a body. In nature a number of forces $\vec{F}_1, \vec{F}_2, \ldots\ldots \vec{F}_n$ can act on a particle. If $\vec{F}$ is the resultant of these forces then

$$\vec{F} = \sum_{i=1}^{n} F_i \tag{5.5}$$

If $\vec{r}$ is the position vector of the particle, then the torque about the origin is

$$\vec{\tau} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \ldots\ldots + \vec{r} \times \vec{F}_n$$

$$\vec{\tau} = \vec{r} \times \sum_{i=1}^{n} F_i$$

or $$\vec{\tau} = \vec{r} \times \vec{F} \tag{5.6}$$

5.2 COUPLE

A couple consists of a pair of equal and opposite forces $\vec{F}$ and $-\vec{F}$ acting along parallel lines. The perpendicular distance l between them is known as the arm of the couple. The vector sum of the forces in a couple is zero,

$$\Sigma \vec{F} = 0 \tag{5.7}$$

The torque $\vec{\tau}$ exerted by the couple about any point o in space is given as,

$$\vec{\tau} = \vec{r_1} \times \vec{F} - \vec{r_2} \times \vec{F}$$

$$\vec{\tau} = \left(\vec{r_1} - \vec{r_2}\right) \times \vec{F}$$

Since $\vec{r_2} + \vec{r} = \vec{r_1}$

or, $\vec{r} = \vec{r_1} - \vec{r_2}$

Therefore, $\vec{\tau} = \vec{r} \times \vec{F}$ (5.8)

The magnitude of $\vec{\tau}$ is $\tau = F\,r\sin\theta$, as $r\sin\theta = l$ from Fig 5.6

$\therefore \tau = F\,l$

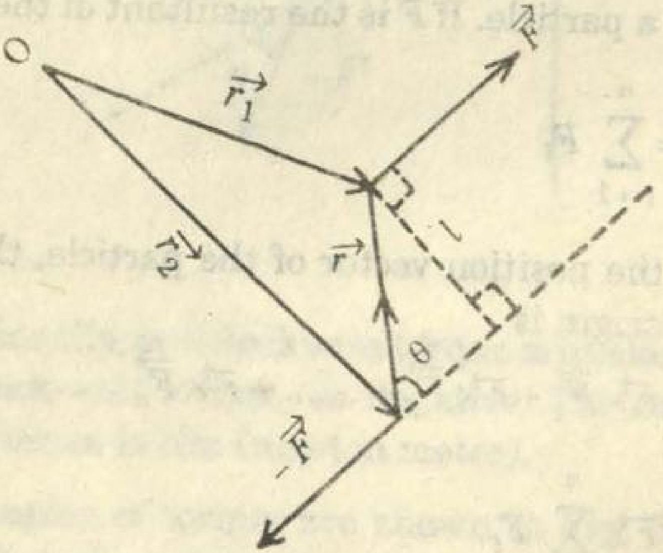


Fig. 5.6

Where F is the magnitude of either force. A very common example of couple is that of a moving coil of electrical instruments. In the instruments a coil rotates when a current I is allowed to flow through it and comes to rest after deflection through an angle θ (Fig. 5.7). The forces $\vec{F}$ on the two sides X and Y of the coil are both equal to B l I N, where $\vec{B}$ is the strength of the magnetic field, l is the length of the coil and N is the number of turns. Thus the coil is deflected by the couple having a torque of magnitude, $\tau = Fb$, where b = XY = breadth of the coil. Hence,

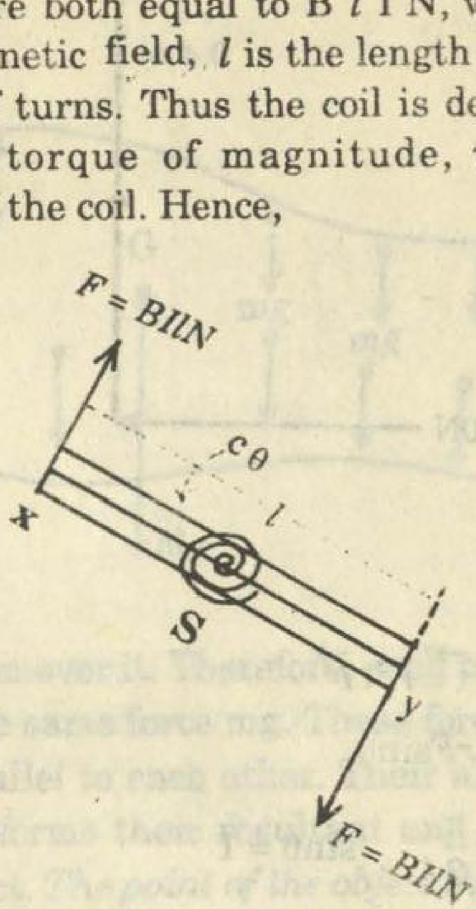


Fig. 5.7

$$|\vec{\tau}| = B\,I\,l\,N\,b = B\,A\,N\,I \qquad (5.9)$$

where A = l b = area of the coil the opposing couple is due to spring S and its torque = c θ. Hence, for equlilbrium B ANI = cθ.

$$\text{or } I = \left(\frac{c}{BAN}\right)\theta$$

Since $\left(\frac{c}{BAN}\right)$ is the constant of the instruments, the observed value of θ is proportional to the current flowing.

Example 5.1

A cross-bar wrench is used to change a tyre. The handles are 0.4m long. A force of 200 N is being exerted on each handle as shown in Fig 5.8. Calculate the torque exerted on the wrench.

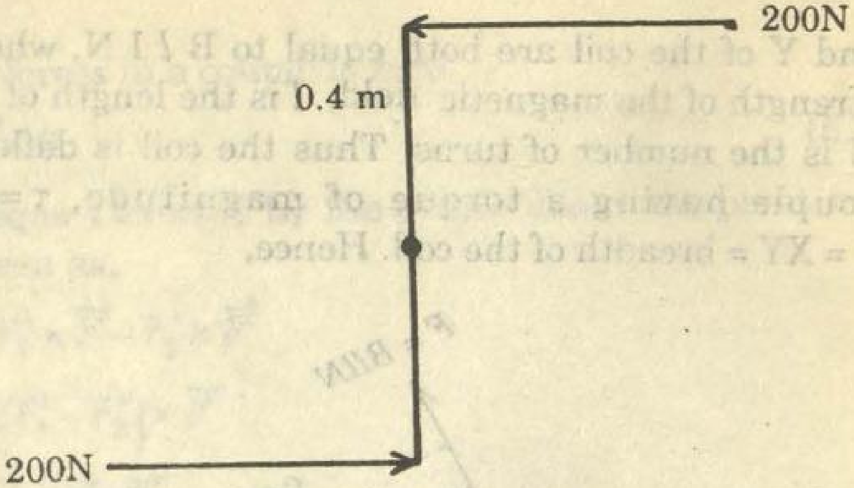


Fig. 5.8

Solution

Since $\vec{\tau} = \vec{r} \times \vec{F}$

or $\tau = rF\sin\theta$

as $\theta = \frac{\pi}{2}$, $\sin\theta = 1$

$\tau = r\,F$

Since F = 200 N, r = 0.4 m, hence

$\vec{\tau} = \tau_1 + \tau_2$

$= 200 \times 0.4 + 200 \times 0.4 = 160\ Nm$

5.3 CENTRE OF GRAVITY AND CENTRE OF MASS

Consider a small object having mass m. Its weight *mg* is acting toward the centre of the earth. An extended object (Fig. 5.9) of mass M may be considered as composed of large number of such small particle. Each constituent particle (*m*) of the extended object is acted upon by a force directed toward the centre of the earth. Since the size of the extended object is very small as compared to that of the earth, the gravitational

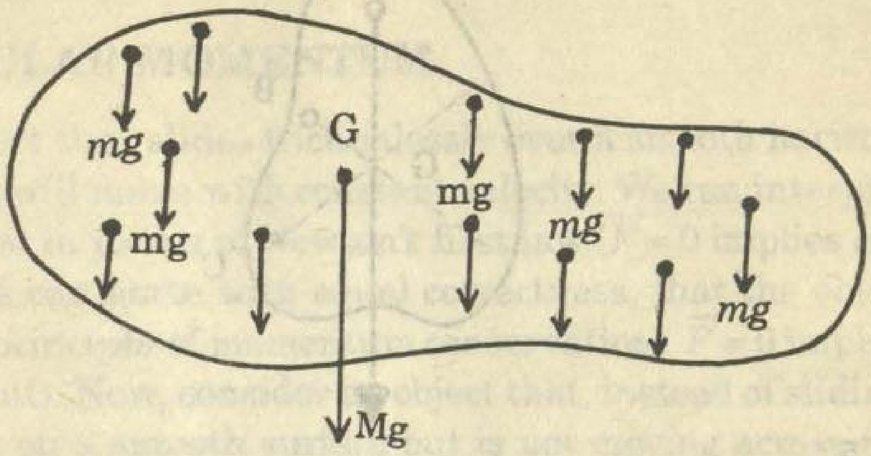


Fig. 5.9

acceleration is uniform over it. Therefore, each particle of the object experiences the same force mg. These forces acting on the particles are parallel to each other. Their algebraic sum $\Sigma\, mg = (\Sigma\, m)\, g = M\, g$ forms their resultant and is called the weight Mg of the object. *The point of the object through which the resultant Mg passes is called its centre of gravity.* In other words, the total weight of the object acts at its centre of gravity. Since g is uniform over the extended object, it can be safely concluded from equation $\Sigma\, mg = M\, g$ that the masses of the constituent particles can be replaced by the total mass of the object at its centre of gravity in a uniform gravitational field. This point is also known as centre of mass.

In a simple case of a ruler, the centre of mass is the point of support when the ruler is balanced at the sharp edge of a wedge. If the ruler is of uniform thickness and width, then its point of support is its middle point. A more precise method for determining the centre of mass of a flat plate is by suspending it in turn from points A, B, and C, as shown in Fig 5.10, so that it hangs freely in each case. If a plumbline is hung from the same support and lines are drawn to indicate the vertical of the thread then the point of intersection of the lines gives

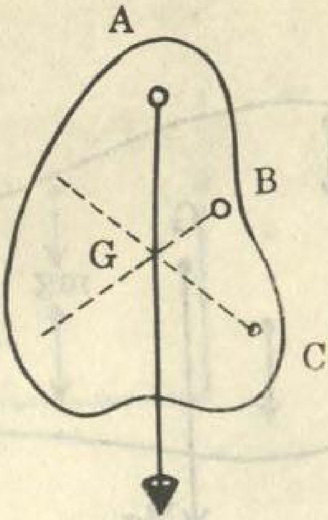


Fig. 5.10

the centre of mass of the plate. The position of the centre of mass of some familiar objects are given in Table 5.1.

Table 5.1

Name of object	*Position of centre of mass*
1. Uniform rod	centre of rod.
2. Circular plate	centre of plate.
3. Plate (triangular or parallelogram in shape) ,	Intersection of their diagonals or intersection of the lines joining the mid-points of opposite sides
4. Triangular plate	Intersection of medians
5. Rectangular block	Intersection of diagonals.
6. Sphere	Centre of sphere
7. Cylinder	Mid-point axis

The centre of mass of an object may also be defined as the point at which an applied force produces a linear acceleration but no rotation. The knowledge of the centre of mass or centre of gravity of an object is very beneficial in solving the problems of equilibrium.

5.4 ANGULAR MOMENTUM

An object that slides frictionlessly over a smooth horizontal surface will move with constant velocity. We can interpret such motion in terms of Newton's first law ($\vec{F} = 0$ implies $\vec{a}$ is zero) or we can state with equal correctness, that the object obeys the principle of momentum conservation ($\vec{F} = 0$ implies p is constant). Now, consider an object that, instead of sliding, is rotating on a smooth surface but is not moving across the surface, for example a spinning top. We know that if friction is negligible, the spinning motion will continue for a very long time. There is a definite parallel here; in the absence of outside influences, a moving object will continue in its state of uniform motion and of uniform rotation. The description of spinning motion can be made quantitative introducing the concept of angular momentum. The linear momentum $\vec{p}$ of the particle is given by

$$\vec{p} = m\vec{V}$$

The angular momentum $\vec{L}$ is defined by the equation

$$\vec{L} = \vec{r} \times \vec{p} \tag{5.10}$$

From the definition of a vector product, the angular momentum $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$ and its direction is determined by the right hand rule (Fig. 5.11b). Inserting the value of p in Eq. 5.10, we get

$$\vec{L} = m\,\vec{r} \times \vec{V} \tag{5.11}$$

From Eq. 5.11, we see that the dimensions of $\vec{L}$ are $ml^2 t^{-1}$ and its units are kg m^2 sec^{-1}. Since 1 Joule second equals 1 kgm^2 sec^{-1} (1 J sec = 1 kg m^2 sec^{-1}), so an alternate unit of angular momentum is J sec.

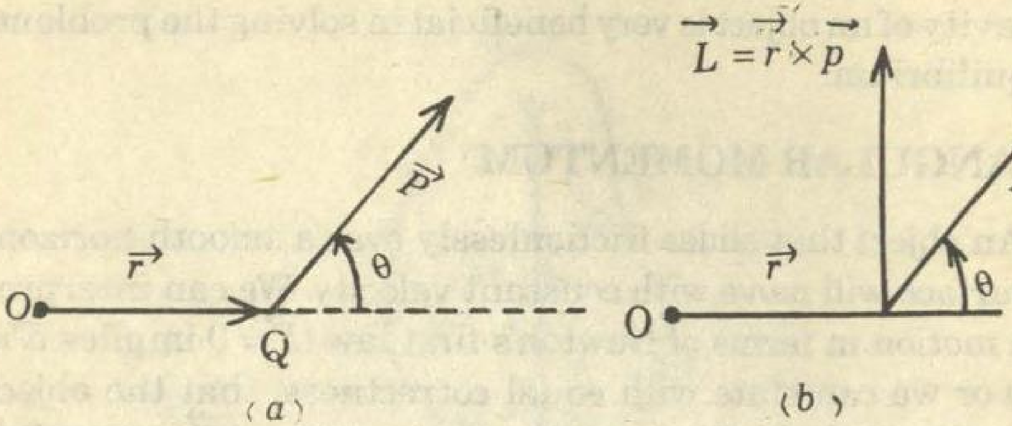


Fig. 5.11

Let $\vec{r}$ and $\vec{r'}$ be the position vectors of the point Q with respect to origin O and O' (Fig. 5.12). If the momentum of the points is p at any instant, then the angular momenta of this particle about O and O' are $\vec{L}$ and $\vec{L'}$ respectively, which are given as

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\text{and } \vec{L}' = \vec{r'} \times \vec{p}$$

$\vec{L}$ and $\vec{L'}$ will never be equal to each other unless O and O' will coincide each other. If p is directed toward or away from the origin, then the angular momentum relative to the origin will be zero. The component of $\vec{p}$ perpendicular to $\vec{r}$ is labelled $p_\perp$ and the component of $\vec{r}$ perpendicular to $\vec{p}$ is labelled as $r_\perp$ (Fig. 5.13). hence, the magnitude of L may be written as

$$L = |\vec{r} \times \vec{p}| = rp\sin\theta = (r\sin\theta) \cdot p = r_\perp p.$$

$$\text{Also, } rp\sin\theta = r\,(p\sin\theta) = r\,p_\perp \text{ or } \vec{L} = r_\perp P = r\,p_\perp$$

For a particular case when $\vec{p}$ and $\vec{r}$ are perpendicular to each other, then the magnitude of $\vec{L}$ is ,

$$L = rp = rmV$$

$$\text{since, } V = r\,\omega$$

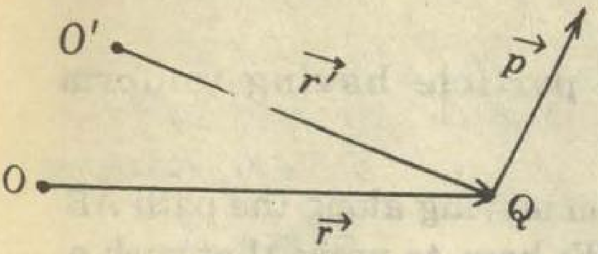


Fig. 5.12

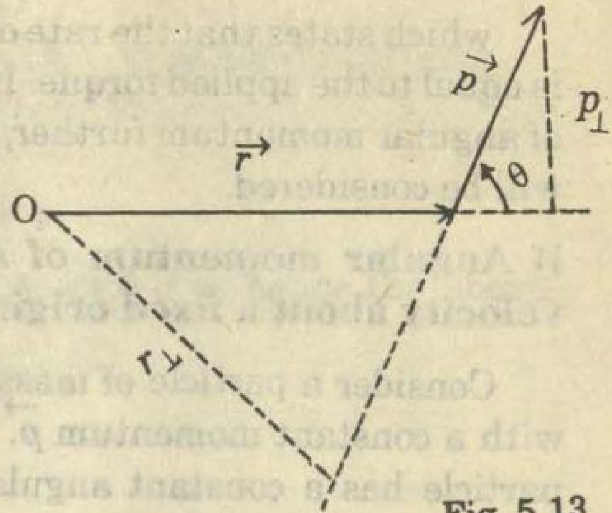


Fig. 5.13

$$L = \left(m\, r^2\right)\omega \tag{5.12}$$

The angular momentum equals the product of mr^2 and the angular velocity ω. This is similar to $p = mV$, in rectilinear motion. In rotational motion the quantity mr^2 plays the same role as the inertial mass in rectilinear motion. The quantity $I = mr^2$ is called the moment of inertia of the rotating body of mass m about the axis of rotaton. If the body is made up of n particles rigidly attached together, then $I = \sum_{i=1}^{n} m_i r_i^2$

where r_i is the distance between the mass m_i and the axis of rotation. There are numerous examples of angular momentum like the earth rotating about its axis, a satellite orbiting round the earth and the wheels of a car rotating on their axles.

We have already studied that force $\vec{F}$ and the torque $\vec{\tau}$ are analogous to each other. According to Newton's second law of motion $\vec{F} = \frac{\overrightarrow{\Delta p}}{\Delta t}$, so in rotational motion

$$\vec{\tau} = \frac{\overrightarrow{\Delta L}}{\Delta t} \tag{5.13}$$

which states that the rate of change of angular momentum is equal to the applied torque. In order to illustrate the concept of angular momentum further, following two simple examples will be considered.

i) Angular momentum of a particle having uniform velocity about a fixed origin

Consider a particle of mass m moving along the path AB with a constant momentum $\vec{p}$. We have to prove that such a particle has a constant angular momentum relative to the origin O.

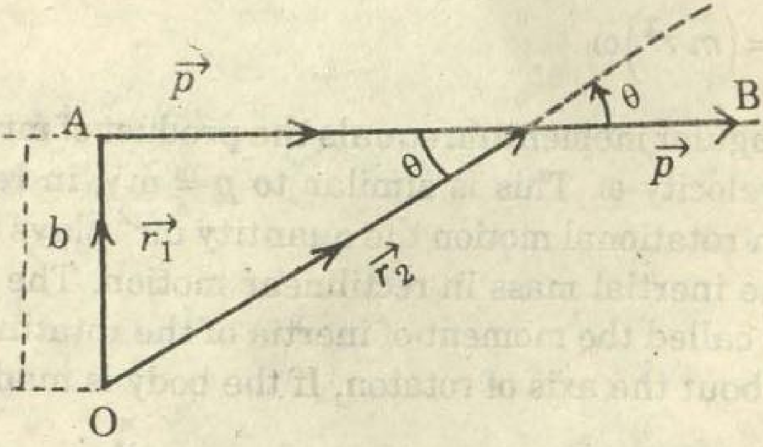


Fig. 5.14

In order to prove this we find the angular momentum of this particle at the position $\vec{r_1}$ and $\vec{r_2}$.

If $\vec{L_1}$ and $\vec{L_2}$ are the angular momentum about O, then

$$\vec{L_1} = \vec{r_1} \times \vec{p} \tag{5.13 a}$$

$$\vec{L_2} = \vec{r_2} \times \vec{p} \tag{5.13 b}$$

Using the right hand rule, we find that both $\vec{L_1}$ and $\vec{L_2}$ are directed into the paper. Which establishes that the direction of $\vec{L}$ for all particles positions is along the path directed into the paper thus having a constant direction. The magnitude of

$\vec{L_1}$ and $\vec{L_2}$ can be easily obtained with the help of Fig. 5.14.

$$L_1 = r_1 p \sin 90^\circ = mVr_1$$

$$L_2 = r_2 p \sin \theta = mVr_2 \sin \theta$$

Since from Fig. 5.14, $r_1 = r_2 \sin \theta = b$, hence the above equations become

$$L_1 = mVb$$

$$L_2 = mVb$$

$$\text{i.e. } L_1 = L_2 = mVb \tag{5.14}$$

Because both magnitude and direction of the angular momentum is constant, hence $\vec{L}$ is constant. If the linear momentum is variable then $\vec{L}$ is not constant.

ii) Angular momentum in Uniform Circular Motion

Consider a particle of mass m, having momentum $\vec{p} = m\vec{V}$ and moving along the circumference of a circle of radius r in anti-clock wise direction (Fig 5.15). The angular momentum $\vec{L}$ of such a particle is

$$\vec{L} = \vec{r} \times \vec{p}$$

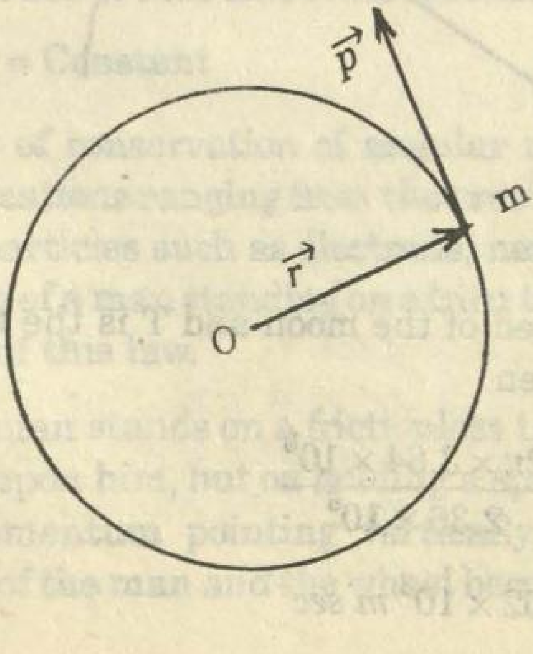


Fig. 5.15

Using right hand rule of cross-product, we find that the direction of $\overrightarrow{L}$ is out of the paper of this page. The magnitude of $\overrightarrow{L}$ is

$$L = rp \sin 90^\circ = m\,V\,r \tag{5.15}$$

Hence, if a particle moves in a circular orbit with constant speed, then its angular momentum about the centre of the circle remains constant.

Example 5.2

Compute the angular momentum of the moon relative to the earth if a distance between the centres of the earth and moon = 3.84×10^8 m, time period of a revolution = 2.36×10^6 sec, and mass of the moon = 7.35×10^{22} kg.

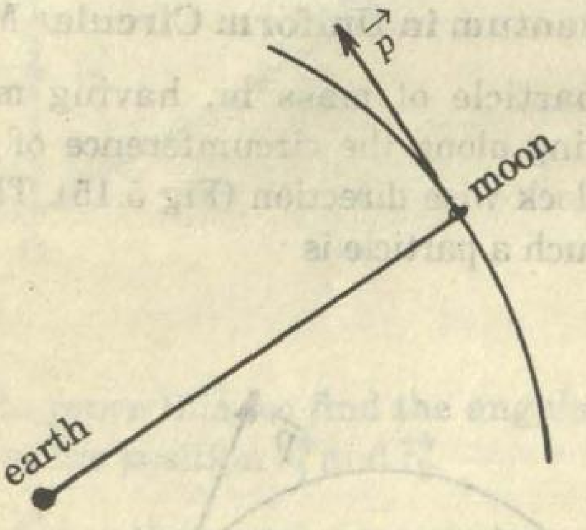


Fig. 5.16

Solution

If V is the speed of the moon and T is the time period of one revolution, then

$$V = \frac{2\pi r}{T} = \frac{2\pi \times 3.84 \times 10^8}{2.36 \times 10^6}$$

$$\text{or} \quad V = 1.02 \times 10^3\ m\ sec^{-1}$$

Thus the linear momentum $\vec{p}$ of the moon is given by.

$$p = mV$$

$$= 7.35 \times 10^{22} \times 1.02 \times 10^{3}$$

$$= 7.50 \times 10^{25} \text{ kg m sec}^{-1}$$

The angular momentum of moon relative to the centre of the earth has magnitude

$$L = rp$$

$$= 3.84 \times 10^{8} \times 7.50 \times 10^{25}$$

$$L = 2.88 \times 10^{34} \text{ kg m}^{2} \text{ sec}^{-1}$$

According to the right hand rule $\vec{L}$ is directed out of the paper, which corresponds to the direction toward the north polar star relative to the polar system.

5.5 CONSERVATION OF ANGULAR MOMENTUM

Like the law of conservation of linear momentum of system we have the law of conservation of angular momentum *which states that when the net external torque acting on a system about a given axis is zero, the total angular momentum vector of the system about that axis remain constant.* Thus,

$$\vec{L}_{total} = \text{Constant} \qquad (5.16)$$

The law of conservation of angular momentum has immense applications ranging from the creation of stars down to subatomic particles such as electrons, neutrons and protons. The example of a man standing on a turn table provides a good illustration of this law.

When a man stands on a frictionless turn table at rest, no torque acts upon him, but on holding a spining wheel with its angular momentum pointing vertically, the total vertical momentum of the man and the wheel becomes equal to L_{wheel}.

On turning the axis of the wheel away from the vertical, component of the angular momentum diminishes. Taking L_z as the vertical component,

$$L_z\,(\text{Wheel}) = L_{\text{wheel}} \cos\theta$$

where θ is the angle between the vertical and the axis of rotation of the wheel. Since no torques are acting along the vertical direction, the L_z should remain constant. To compensate for reduction in L_z the man and the turntable begins to rotate When the wheel is brought back to its original position, the rotation of the man and turntable stops.

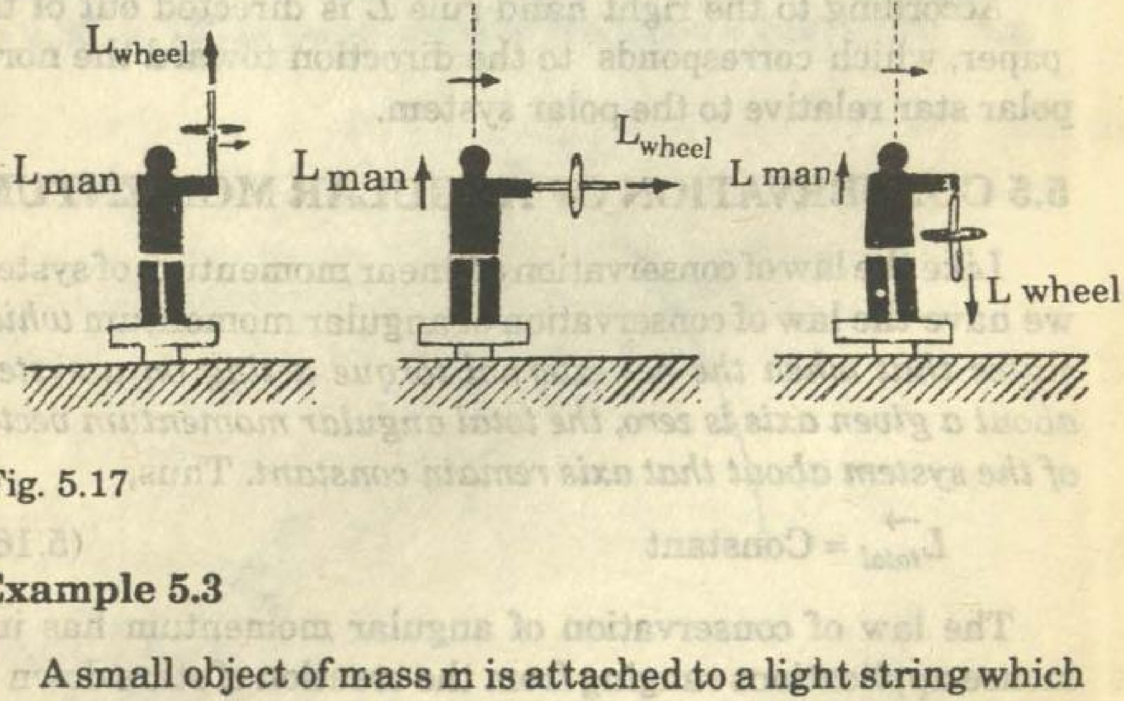


Fig. 5.17

Example 5.3

A small object of mass m is attached to a light string which passes through a hollow tube. The tube is held by one hand and the string by the other. The object is set into rotation in the circle of radius r_1 with speed V_1. The string is pulled down, shortening the radius of the path to r_2. Find the new linear speed V_2 and the new angular speed ω of the object in terms of the initial values V_1 and ω_1 and the two radii. (The pull does not exert any torque on the object.)

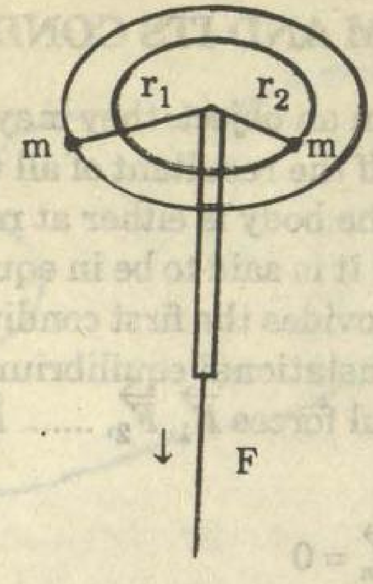


Fig. 5.18

Solution

Since the vertical of the string does not exert any torque on the mass m, hence the angular momentum is conserved i.e.

initial angular momentum = final angular momentum

$$\text{or } m\,V_1\,r_1 = m\,V_2\,r_2$$

$$\text{or} \qquad V_2 = V_1\left(\frac{r_1}{r_2}\right)$$

since $r_1 > r_2$ the object speeds up on being pulled in.

In term of angular velocity, $V_1 = \omega_1\,r_1$ and $V_2 = \omega_2\,r_2$, conservation of angular momentum gives,

$$m\,\omega_1\,r_1^2 = m\,\omega_2\,r_2^2$$

$$\text{or, } \omega_2 = \omega_1\left(\frac{r_1}{r_2}\right)^2$$

so that there is an even greater increase in angular velocity.

5.6 EQUILIBRIUM AND ITS CONDITIONS

When forces act on an object, they may change its state of rest or of its motion. If the resultant of all the forces acting on a body is zero, then the body is either at rest or moves with a constant velocity and it is said to be in equilibrium. Newton's first law of motion provides the first condition of equilibrium. A body will be in translational equilibrium if and only if the vector sum of external forces $\vec{F}_1, \vec{F}_2, \ldots\ldots \vec{F}_n$ acting on a body equal to zero i.e.

$$\vec{F}_1 + \vec{F}_2, \ldots\ldots \vec{F}_n = 0$$

$$\text{or} \sum_{i=1}^{n} \vec{F}_i = 0 \qquad (5.17)$$

In two dimensions the first condition of equilibrium leads to the following

$$\sum_{i=1}^{n} F_{ix} = 0 \quad , \quad \sum_{i=1}^{n} F_{iy} = 0 \qquad (5.18)$$

Where F_{ix} and F_{iy} are horizontal and vertical components of $\vec{Fi}$. It is possible, however, for a body to have zero net force acting upon it and yet subject to non-zero torque. An example is given in Fig. 5.19, where the net force acting on the body is equal to zero. Therefore the centre of mass of the object does not accelerate, however the object experiences an angular acceleration. An object is in rotational equilibrium if and only if the vector sum of external torque about any axis acting on it equals to zero. Thus if $\vec{\tau}_1, \vec{\tau}_2, \ldots\ldots \vec{\tau}_n$ are the torques about any axis, then

$$\sum_{i=1}^{n} \vec{\tau}_i = 0 \qquad (5.19)$$

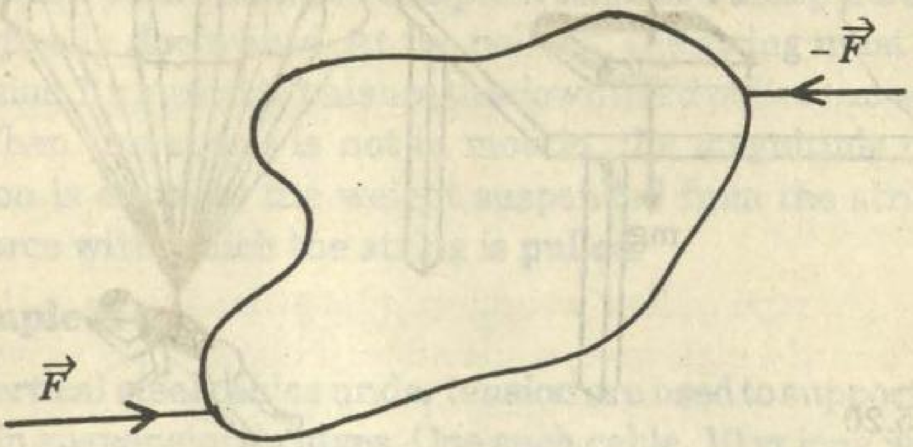


Fig. 5.19

for rotational equilibrium. This is known as the second condition of equilibrium.

A point object under the action of a number of forces remains in equilibrium, if only the first condition of equilibrium is satisfied i.e.

$$\sum_{i=1}^{i=n} \vec{F}_i = 0$$

For a rigid body to be in equilibrium under the action of many forces, both the equilibrium conditions are to be satisfied.

5.7. DAY TO DAY EXAMPLES OF OBJECT IN EQUILIBRIUM

Consider a book of mass m lying on a table. Only two forces are acting on it. One is the weight mg of the book acting vertically downward, the other is the reaction force $\vec{R}$ due to Newton's third law of motion. Since the book is at rest on the table in the vertical direction, hence R – mg = 0. The book is then considered in equilibrium under the action of these two forces.

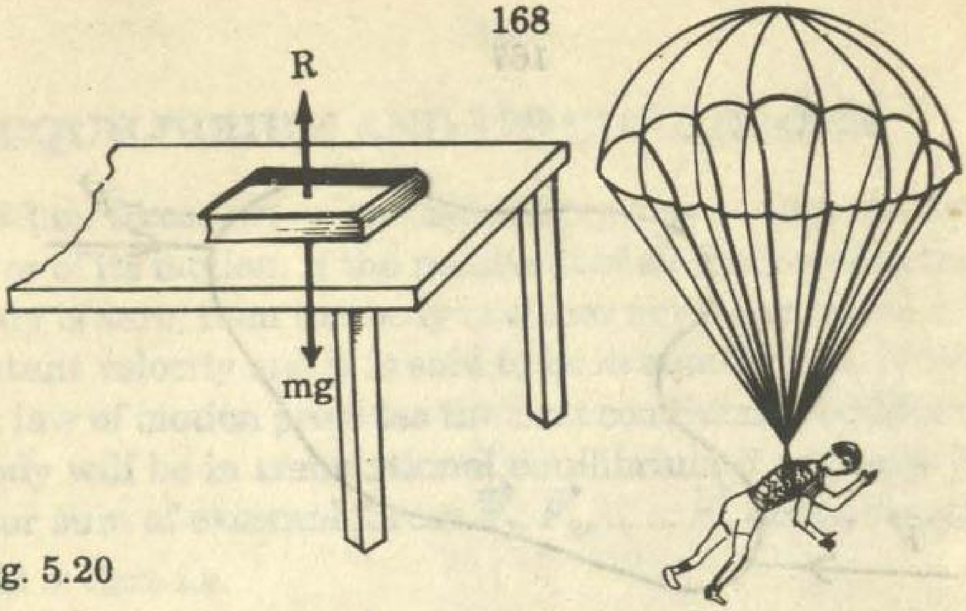


Fig. 5.20

Now consider another example i.e. of a paratrooper jumping out of an aeroplane. After a free fall, his parachute opens and a little later the paratrooper starts descending with a uniform velocity. At this instant the force of gravity acts vertically downward on the paratrooper. It is balanced by the reaction of air on the parachute which acts upward. This is an illustration of equilibrium when a body is moving with a uniform velocity.

5.8 TENSION IN A STRING

Consider a person lifting up an object of weight W with the help of a string.

In this case a force is experienced by the hand as well as

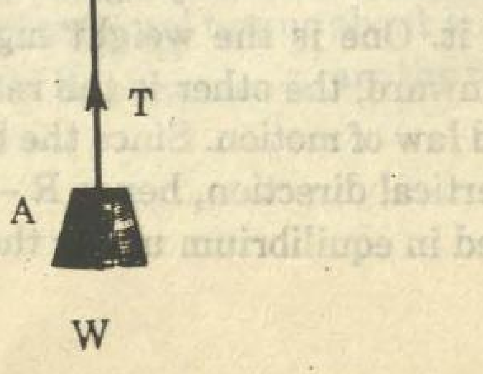


Fig. 5.21

the object through the string. At B, the hand experiences a downward pull. Hence, at this point tension T along the string is vertically downward. At the point A, the string must exert a tension T upward to balance the downward pull of the weight W. When the string is not in motion, the magnitude of the tension is equal to the weight suspended from the string or the force with which the string is pulled.

Example 5.4

Vertical steel cables under tension are used to support road beds in suspension bridges. One such cable, 10 m long weighs 500 N. How will the tension at the top compare with that at the bottom?

Solution

The forces acting on the cable are shown in the Fig. 5.22. The first condition of equilibrium requires.

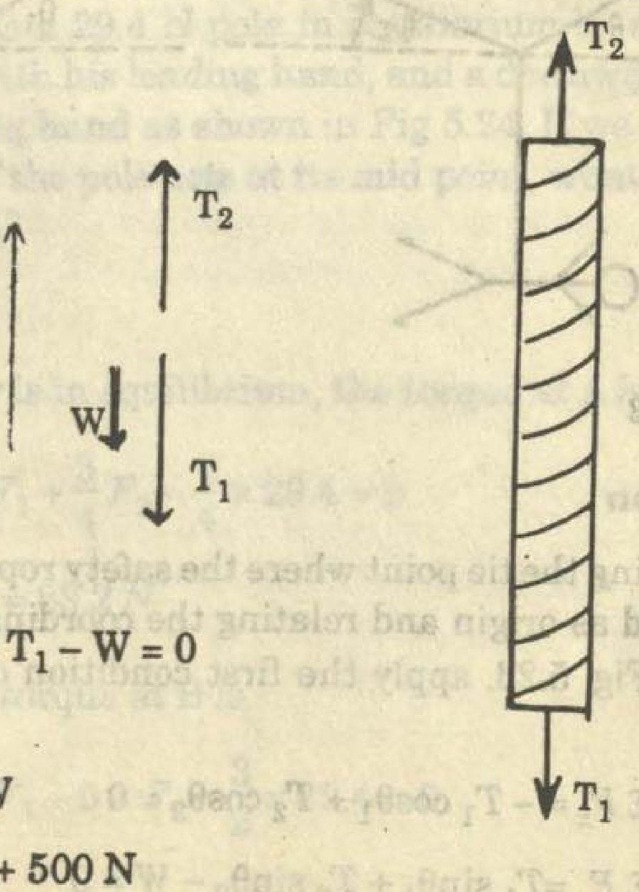


Fig. 5.22

$$\Sigma F_y = T_2 - T_1 - W = 0$$

Therefore,

$$T_2 = T_1 + W$$

$$\text{or, } T_2 = T_1 + 500\text{ N}$$

Since all the forces are acting in the vertical direction, hence only the vertical component equation of the first condition of equilibrium is applicable. Also it is worth noting that the tension in the cable will be the same if the cable were weightless.

Example 5.5

A tight rope walker weighing WNewtons shown in the Fig 5.23, falls of a high wire, while attempting to cross a river. He is hanging motionless in space by a safety rope attached to his waist and to the high wire. The wire makes angles θ_1 and θ_2 with the horizontal. Find the tension in each part of the wire where the safety rope is attached.

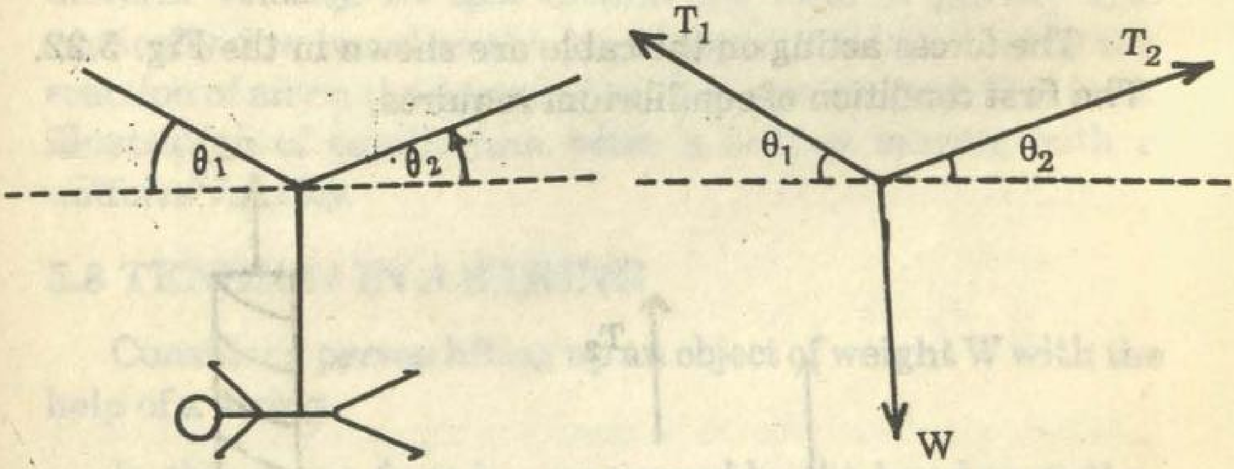


Fig. 5.23

Solution

Taking the tie point where the safety rope and the wire are attached as origin and relating the coordinate axis as shown in the Fig. 5.23, apply the first condition of equilibrium, we get,

$$\Sigma F_x = -T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0 \qquad \text{(i)}$$

$$\Sigma F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - W = 0 \qquad \text{(ii)}$$

From Eq. (i) we obtain

$$T_2 = \frac{\cos\theta_1}{\cos\theta_2} T_1$$

Substitution of this value of T_2 in Eq. ii, gives

$$T_1 \left(\sin\theta_1 + \frac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 \right) = W$$

$$\text{or } T_1 \sin\left(\theta_1 + \theta_2\right) = W \cos\theta_2$$

$$\text{or } T_1 = \frac{W\cos\theta_2}{\sin\left(\theta_1 + \theta_2\right)}$$

$$\text{Similarly, } T_2 = \frac{W\cos\theta_1}{\sin\left(\theta_1 + \theta_2\right)}$$

Example 5.6

A vaulter holds a 29.4 N pole in equilibrium by exerting upward force F_2 with his leading hand, and a downward force F_1 with his trailing hand as shown in Fig 5.24. If we assume that the weight of the pole acts at its mid point, what are the value of F_1 and F_2?

Solution

As the vaulter is in equilibrium, the torque at A is

$$\Sigma \tau_A = 0 \times F_1 + \frac{3}{4} F_2 - \frac{9}{4} \times 29.4 = 0$$

$$\text{So that } F_2 = 88.2\,N$$

Similarly, the torque at B is

$$\Sigma \tau_B = \frac{3}{4} \times F_1 - 0 \times F_2 - \frac{3}{2} \times 29.4 = 0$$

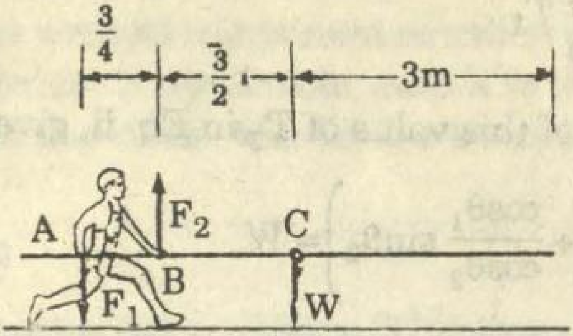


Fig. 5.24

so that $F_1 = 58.8\,N$

Now let us check whether our answer is correct or not. For equilibrium, the first conditions is

$$F_2 = F_1 + W = 0$$

$$88.2\text{ N} = 58.8 + 29.4 = 88.2\text{ N}$$

Example 5.7

Consider a 20m ladder weighing 200 N resting against a smooth wall such that it makes an angle of 60° with the horizontal. Find the reactions on the ladder due to the wall and ground.

Solution

Since the wall is smooth so the reaction of the wall at B is R. the ground has some friction force F which is resultant of normal force and force of friction acts at an angle φ with the horizontal, the forces $F_x = F\cos\phi$, $F_y = F\sin\phi$ acting on the ladder with F_x. The third force acts at its centre of gravity G. It is its weight acting vertically downward.

Applying the first condition of equilibrium, we have

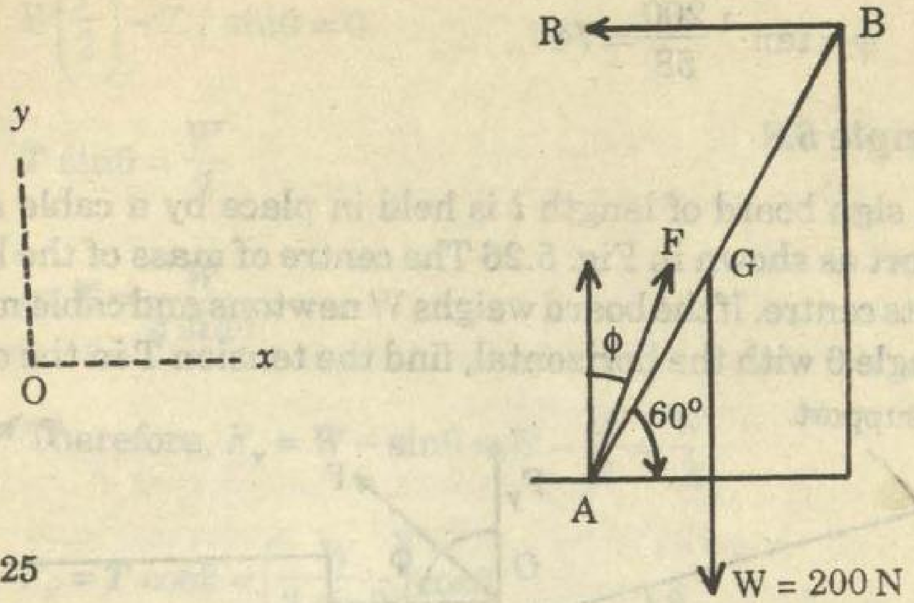


Fig. 5.25

$F_x - R = 0$ or $F_x = R$.

and $F_y - W = 0$ or $F_y = W = 200$ N

Applying the 2nd condition of equilibrium and taking the moments of the forces about A as in this case, we get

$$\Sigma\tau = W \cdot AG \cos 60 - R \cdot AB \sin 60 = 0$$

$$\text{i.e., } R \times 20 \sin 60 = 200 \times 10 \times \cos 60$$

$$R = 100 \frac{\cos 60}{\sin 60}$$

or R = 58 N

Also $F_x = R = 58\ N$

As $F_x^2 + F_y^2 = F^2$

$$\text{or, } F^2 \cos^2 \varphi + F^2 \sin^2 \varphi = F^2$$

$$F^2 = (200)^2 + (58)^2$$

$$\therefore F = \sqrt{(200)^2 + (58)^2} = 208\ N \text{ approx}$$

$$\tan\varphi = \frac{F_y}{F_x} = \frac{200}{58}$$

$$\varphi = \tan^{-1}\frac{200}{58} = 74^{\circ}$$

Example 5.8

A sign board of length *l* is held in place by a cable and a support as shown in Fig. 5.26 The centre of mass of the board is at its centre. If the board weighs W newtons and cable makes an angle θ with the horizontal, find the tension T in the cable.

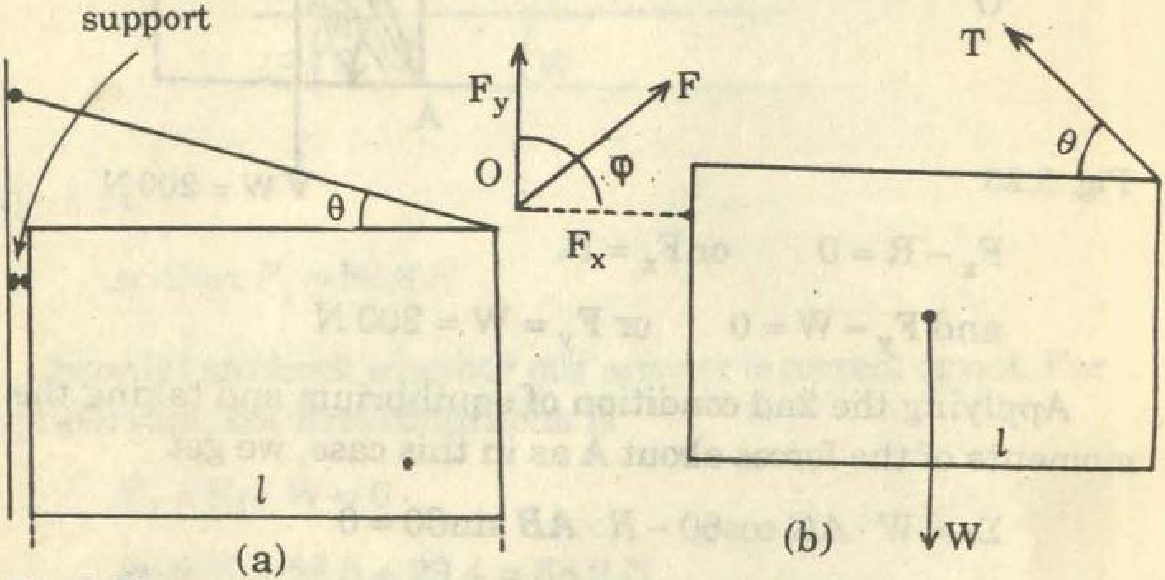


Fig. 5.26

Solution

The tension in the cable supplies the upward force to help hold up the sign, but the wall exerts outward directed force F, since the tension in the cable tries to pull the sign board into the wall. Let us call the vertical component of this force as F_y and the horizontal component F_x. All the forces are shown in Fig 5.26. Applying the first condition of equilibrium, we get

$$F_y + T\sin\theta - W = 0 \qquad \text{(i)}$$

$$F_x - T\cos\theta = 0 \qquad \text{(ii)}$$

Applying the 2nd condition of equilibrium with the wall support taken as the rotational axis, we get

$$W\left(\frac{l}{2}\right) - T\,l\sin\theta = 0$$

$$T\sin\theta = \frac{W}{2}$$

$$\text{or, } T = \frac{W}{2\sin\theta}$$

$$\text{Therefore, } F_y = W - \sin\theta = W - \frac{W}{2} = \frac{W}{2}$$

$$F_x = T\cos\theta = \left(\frac{W}{2\sin\theta}\right)\cos\theta$$

$$= \frac{W}{2}\,\frac{\cos\theta}{\sin\theta}$$

$$= \frac{W}{2}\cot\theta$$

$$F^2 = F_x^2 + F_y^2$$

$$= \frac{W^2}{4}\left(1 + \cot^2\theta\right) = \frac{W^2}{4}\operatorname{cosec}^2\theta$$

$$\therefore\ F = \frac{W}{2}\operatorname{cosec}\theta$$

$$\text{and } \tan\varphi = \frac{F_y}{F_x}$$

$$\frac{W/2}{W/2\cot\theta} = \tan\varphi$$

$$\text{or } \frac{1}{\cot\theta} = \tan\theta$$

$$\text{or } \tan\theta = \tan\phi \text{ or } \phi = \theta$$

SUMMARY

The torque $\vec{\tau}$ associated with a force $\vec{F}$ is defined by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is a vector directed from the axis of rotation to the point of application of the force.

Couple is a pair of equal and opposite forces $\vec{F}$ and $-\vec{F}$ acting along parallel lines displaced by a perpendicular distance l known as the arm of the couple. Torque $\vec{\tau}$ exerted by the couple is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is a vector directed from the point of application of $-\vec{F}$ to that of $\vec{F}$

The centre of mass of an extended body is the point where the total mass of the body may be assumed to act.

The angular momentum of a particle is defined by

$$\vec{L} = \vec{r} \times \vec{p}$$

where $\vec{r}$ is the position vector and $\vec{p}$ is the linear momentum of the particle.

The rate of change of angular momentum $\frac{\Delta \vec{L}}{\Delta t}$ is equal to the applied torque, i.e.,

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

The necessary and sufficient conditions for a body to be in a state of equilibrium are,

$\sum_{i=1}^{i=n} F_i = 0$, for translational equilibrium, known as first conditon of equilibrium.

$\sum_{i=1}^{i=n} \tau_i = 0$, for rotational equilibrium known as second condition of equilibrium.

QUESTIONS

5.1. Consider a door which has the handle on the right and hinges on the left. To open it, a torque must be applied. Is the torque clockwise or counter clockwise when viewed from above? Does your answer depend on whether the door opens toward or away from you?

5.2. A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

5.3. Two moon of identical mass are in stable circular orbits of different radii about the same planet. Which of the two moons has larger angular momentum, the one closer to the planet or the one farther away. Explain.

5.4. At various seasons of the year the earth's atmosphere has a net average motion with or against the direction of the earth's rotation. Explain how this affects the rotation period of the earth.

5.5. Explain why a particle experiencing only one force cannot be in equilibrium.

5.6. A particle is in equilibrium under the influence of two forces. Explain why the forces must be qual in magnitude and oppositely directed.

5.7. A uniform board attached to the ceiling by a string is shown in three positions in Fig. 5.27. Assume that the weight acts at the centre of the board, determine for each orientation whether equilibrium is possible or not.

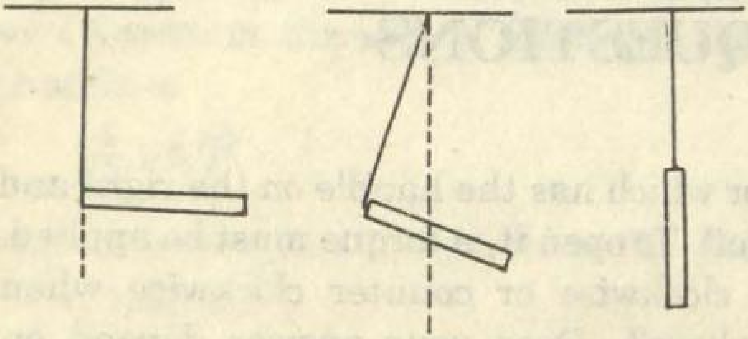


Fig. 5.27

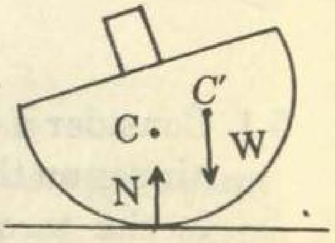


Fig. 5.28

5.8. The tippic-top, when placed in the position shown in Fig 5.28, automatically rights itself. The floor exert a normal force, N directed through the centre of curvature C and the weight W acts at the centre of gravity 'C. If N = W, explain why the top is not in equilibrium in the position shown.

PROBLEMS

1. A particle P with mass 2.0 kg has position vector $\vec{r}$ and velocity $\vec{V}$ in Fig. 5.29. It is acted on by the force $\vec{F}$. All three vectors lie in a common plane. Assume that

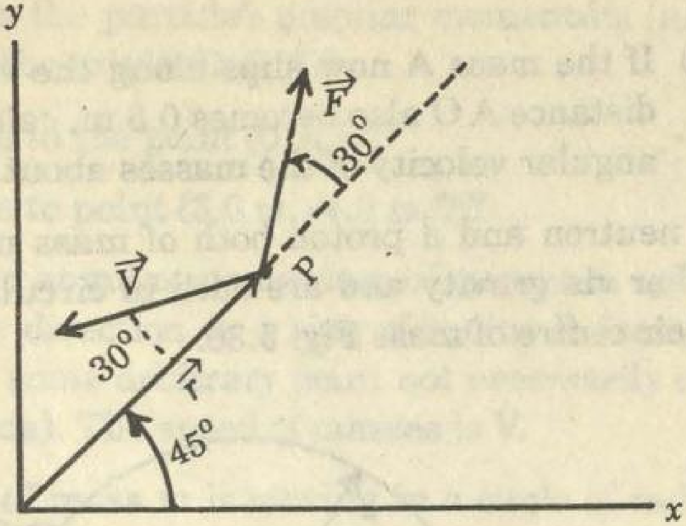


Fig. 5.29

$\vec{r} = 3.0\ m$, $\vec{V} = 4.0\ msec^{-1}$ and $\vec{F} = 2.0\ N$

Compute (a) the angular momentum of the particle and (b) the torque acting on the particle. What are the directions of these two vectors?

5.2. The electron in hydrogen atom rotates at a frequency of 6.6×10^{15} rev sec^{-1} in a circle of radius 5.3×10^{-11} m. The mass of the electron is 9.1×10^{-31} kg. Calculate the angular momentum of the electron about an axis through the centre of the circle perpendicular to its plane

5.3. Two masses A and B, on a light horizontal rod rotate in a horizontal plane in the same direction about the mid point O of AB with an angular velocity of 10 rad sec^{-1}, A has a mass of 0.2 kg and B has a mass of 0.1 kg.

i) Calculate their total angular momentum about O if the distance AB is 0.4m.

ii) If the mass B slips along the rod so that the distance BO becomes 0.3m, calculate the new angular velocity of the masses about O.

iii) If the mass A now slips along the rod so that the distance A O also becomes 0.3 m, calculate the new angular velocity of the masses about O

5.4. A neutron and a proton both of mass m attract each other via gravity and are each in circular orbit about their centre of mass Fig. 5.30.

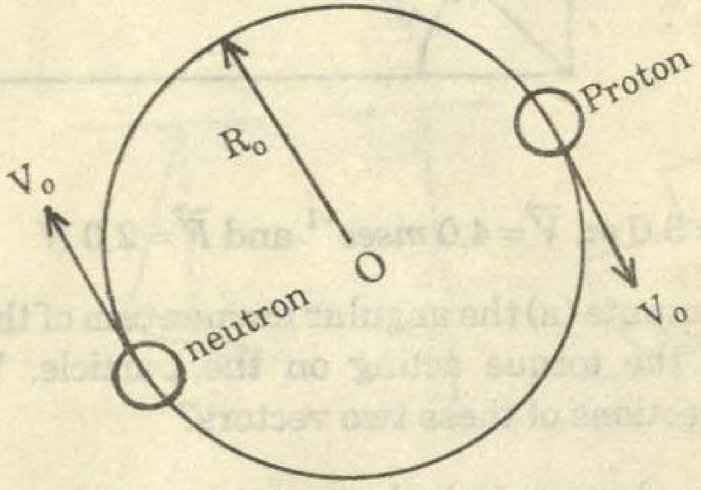


Fig. 5.30

i) If R_0 is the radius of the circular orbit and Vo the velocity, what is the total angular momentum of the system about the centre of mass in terms of m R_0 and Vo?

ii) What is the force on the neutron in terms of G. m and R_0?

iii) What is the force on the neutron in terms of m R_0 and V_0?

iv) If the total angular momentum L = h where h is called Planck's constant, what is R_0 in terms of h, m_0 and G?

5.5. A 1. kg particle at the location 3. m along the positive *x*-axis, has a velocity of 2.0 m sec^{-1} along the positive *y*-axis.

a) What is the particle's angular momentum (a) relative to the origin (0, 0, 0)?

b) relative to the point (0, 4, 0)?

c) relative to point (3.0 m, -4.0 m,0)?

5.6. What is the angular momentum of masses m spinning in opposite direction on a ring of radius R. (computed relative to some arbitrary point not necessarily on the rotation axis). The speed of masses is V.

5.7. A particle of mass m is moving in a circle of radius r_0 about an axis through point O, as shown in Fig. 5.31. Just as it is at the position P, the string holding it in the circle breaks, so that the particle moves off with a velocity $\vec{\omega} \wedge \vec{r_o}$. Show that the angular momentum of the particle about O remains the same as it was before.

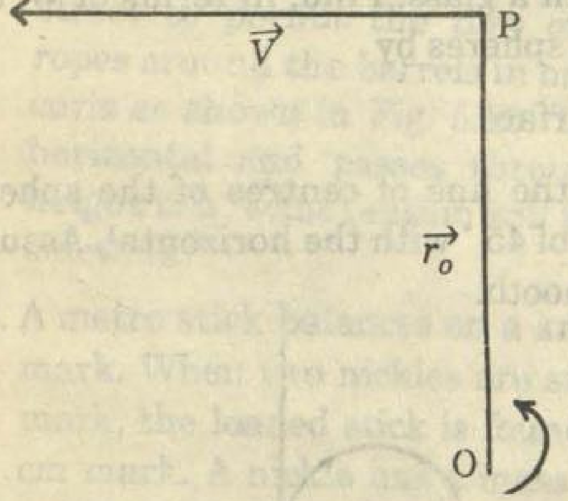


Fig. 5.31

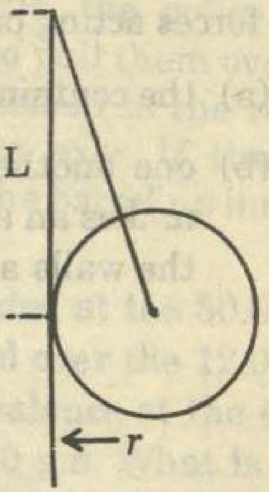


Fig. 5.32

5.8 A uniform sphere of weight W and radius *r* is being held by a rope attached to a frictionless wall at a distance L above the centre of the sphere, as in Fig. 5.32. Find (a) The tension in the rope , and (b) The force exerted on the sphere by the wall.

5.9. A uniform sphere of weight W lies at rest, wedged between two inclined planes of inclination θ_1 and θ_2 Fig. 5.33.

Assume that no friction is involved and determine the forces (directions and magnitude) that the planes exerts on the sphere.

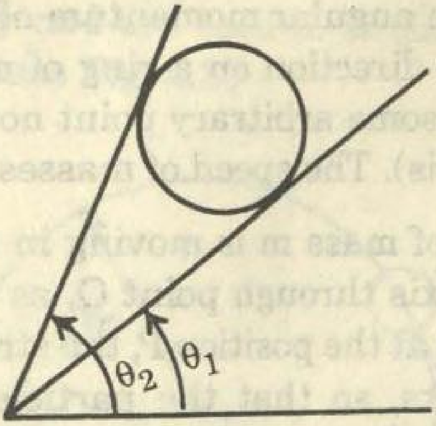


Fig. 5.33

5.10. Two smooth billiard balls, each of weight W, rest as shown in Fig. 5.34 in a glass. Find, in terms of W, the forces acting on the spheres by

(a) the container surface

(b) one another, if the line of centres of the spheres makes an angle of 45° with the horizontal. Assume the walls are smooth.

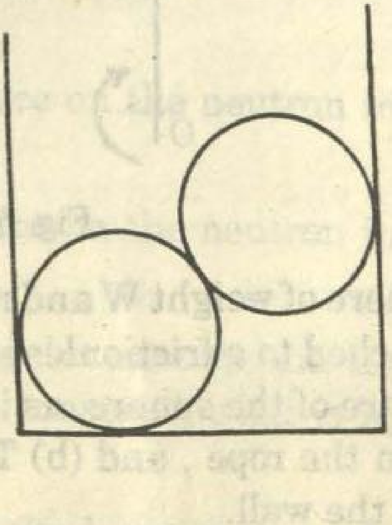


Fig. 5.34

5.11. An 800 N painter is shown in Fig. 5.35 at a distance x from the left end of a uniform 5. m board weighing 500 N (a) Determine the maximum and minimum values of the tensions T_1 and T_2, for values of x, $0 \leq x \leq 5$ m. (b) For what value of x will $T_1 = 410$N and $T_2 = 890$ N.

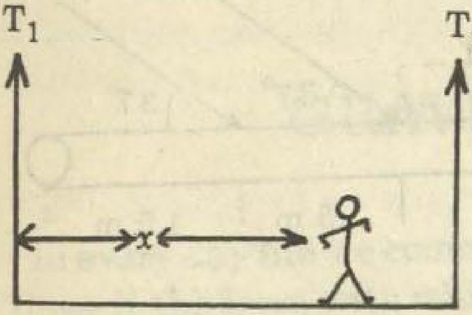


Fig. 5.35

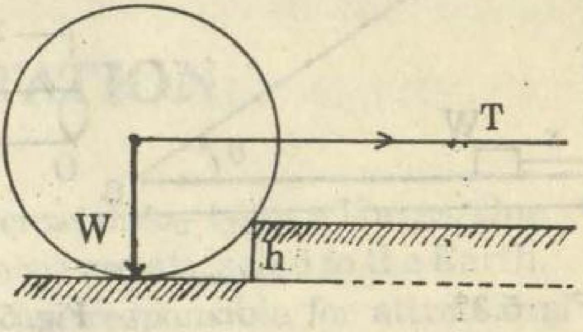


Fig. 5.36

5.12. To set some heavy barrels each of weight W off the street to permit the flow of traffic, the police wrap ropes around the barrels in order to pull them over the curls as shown in Fig. 5.36. The tension in the rope is horizontal and passes through its axle. If the curl height is h, what tension will pull the barrel up into the sidewalk?

5.13. A metre stick balances on a knife edge at the 50.0 cm mark. When two nickles are stacked over the 12.0 cm mark, the loaded stick is found to balance at the 45.5 cm mark. A nickle has a mass of 5.0 gm. What is the mass of the metre stick?

5.14. A thin horizontal bar AB of negligible weight and length l is pinned to a vertical wall at A and supported at B by a thin wire BC that makes an angle θ with the horizontal. A weight W can be moved anywhere along the bar as defined by the distance x from the wall (Fig. 5.37). (*a*) Find the tension T in the thin wire as function of x.

(b) the horizontal and (g) the vertical components of the force exerted on the bar by the pin at A.

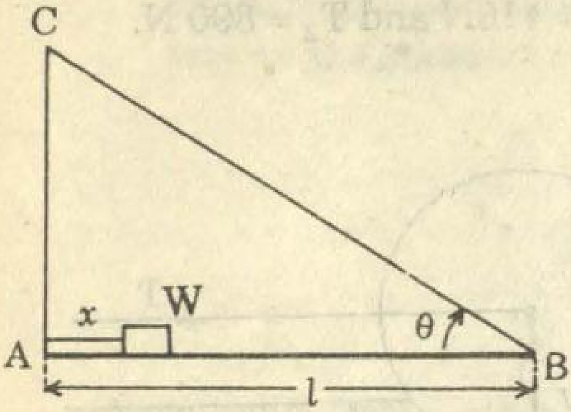


Fig. 5.37

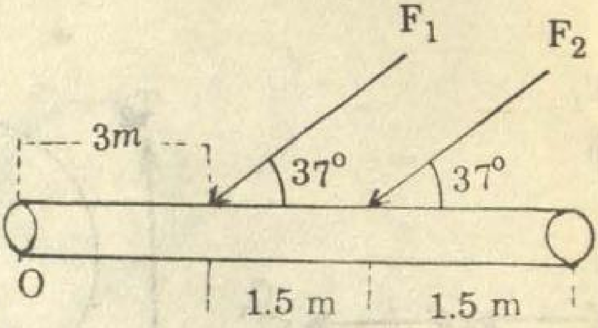


Fig. 5.38

5.15. Two forces of magnitudes $F_1 = F_2 = 3N$ are applied to a rod as shown in Fig. 5.38.

(a) Calculate the net torque about point O due to these two forces by calculating the torque due to each separate force.

(b) Find the moment of the couple to keep the rod horizontal

5.16. A force $\vec{F}$ is given by $\vec{F} = (2\hat{i} - 3\hat{j})\,N$ and the vector r from the axis to point of application of the force is $\vec{r} = (-0.3\,\hat{i} + 0.5\,j)\,m$. Calculate the torque produced by it.

5.17. The tidal forces causes a slowing down in the rotation of the earth. Show that the law of conservation of angular momentum requires that the moon slowly increases its distance from the earth i.e the orbital angular momentum of the moon increase with increasing radius.

6

GRAVITATION

In every day life we come across many types of forces. One of these is the force with which we are attracted to the Earth. There are some forces which are responsible for attraction between electric charges, electric currents and magnets. Some forces keep the nucleus stable. The remarkable fact about all of these forces is that they were thought to be four different kind of basic forces:

1. Gravitational force

.2. a. Electric Force } Electromagnetic force
b. Magnetic force }

3. Strong (nuclear) force.

4. Weak (β - decay) force.

One of the most important achievement of the 19th century physics was to show that electric and magnetic forces are two different aspects of the same force called electromagnetic force. In 1969, Prof. Abdus Salam of Pakistan, alongwith Weinberg and Glashov showed that electromagnetic force and weak force were the same. For this discovery they were awarded Nobel Prize in 1979. So now there are three basic forces in nature.

The gravitational force is responsible for the motion of falling objects, the motion of the planets and galaxies around the sun. To explain the origin of the gravitational force, Newton put forward the law of gravitation based on the empirical laws of planetory motion. The gravitational theory of Issac Newton was published at the end of 17th century and successfully explained the cause of the regular motion of the Moon around the Earth and of the planets around the sun.

6.2 NEWTON'S LAW OF GRAVITATION

The conclusion drawn by Newton may be summed up in his famous law of gravitation which states that everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Thus if two bodies of masses m_1 and m_2 respectively, are placed with their centres at a distance r from each other, the magnitude of the force of attraction between them is given by

$$F \propto m_1 m_2 \qquad \text{(i)}$$

$$F \propto \frac{1}{r^2} \qquad \text{(ii)}$$

Combining (i) and (ii) we get

$$F \propto \frac{m_1 m_2}{r^2},$$

$$F = \text{constant} \frac{m_1 m_2}{r^2},$$

$$F = G \frac{m_1 m_2}{r^2}, \qquad \text{(iii)}$$

where G is a constant of proportionality and is called the constant of gravitation or universal gravitational constant.

The gravitational forces between two bodies are an action and reaction pair. The first body exerts a force on the second body that is directed toward the first body along the line joining their centres and vice versa. These forces are equal in magnitude but opposite in direction.

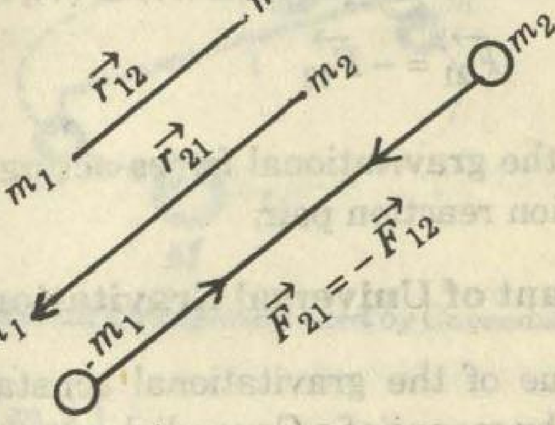


Fig. 6.1 *The gravitational forces between bodies shown as action and reaction pair.*

The law of universal gravitation can be expressed in vector form. Let the displacement vector $\vec{r}_{12}$ point from first body of mass m_1 to second body of mass m_2 as shown in Fig. 6.1. The gravitational force $\vec{F}_{21}$, exerted on m_2 by m_1 is given by the vector relation.

$$\vec{F}_{21} = -G\frac{m_1 m_2}{r_{12}^3} \cdot \vec{r}_{12} \tag{iv}$$

$$= -G\frac{m_1 m_2}{r_{12}^2} \frac{\vec{r}_{12}}{|r_{12}|}$$

where $|\vec{r}_{12}|$ is the magnitude of $\vec{r}_{12}$. The minus sign shows that the gravitational force is attractive.

The force exerted on m_1 by m_2 is given by

$$\vec{F}_{12} = -G\frac{m_2 m_1}{r_{21}^3}\cdot\vec{r}_{21}$$

$$= G\frac{m_1 m_2}{r_{12}^3}\cdot\vec{r}_{12} \quad \text{Since } \vec{r}_{12} = -\vec{r}_{21} \qquad \text{(v)}$$

Thus from above equations (iv) & (v), it is clear that

$$\vec{F}_{21} = -\vec{F}_{12}$$

that is, the gravitational forces acting on the two bodies form an action reaction pair.

The Constant of Universal Gravitation

The value of the gravitational constant G is found by experiment by means of a Cavendish apparatus. A light rod is suspended from its centre by means of a fine quartz fibre as shown in Fig. 6.2. Two identical balls each of mass m are suspended from its ends. When heavy balls each of mass M are brought near them on opposite sides of the small balls, they exert forces of attraction on the suspended balls. Each of these forces is $\frac{GMm}{r^2}$, where r is the distance between the centres of the balls M and m. These forces produced torque which causes the fibre to twist. The twist produced in the fibre is proportional to the magnitude of the torque that is,

$$\tau \propto \theta$$

$$\tau = c\,\theta \qquad (6.1)$$

If l is the length of the rod then the magnitude of the torque due to gravitation is given by

$$\tau = \frac{GMm}{r^2}\frac{l}{2} + \frac{GMm}{r^2}\frac{l}{2}$$

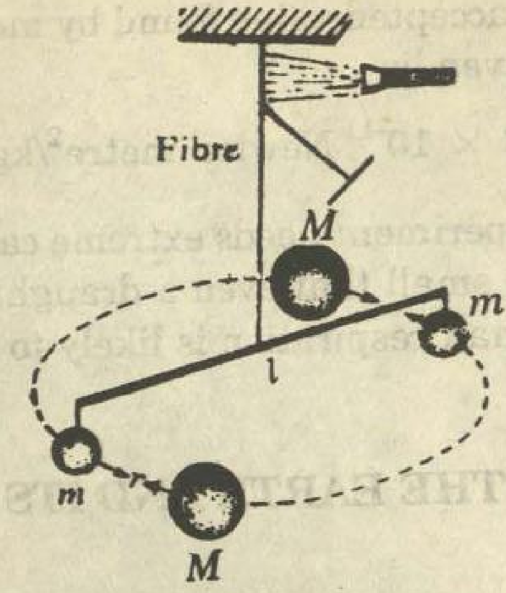


Fig. 6.2 *The experimental arrangement used by Cavendish*

$$= 2\left(\frac{G\,M\,m}{r^2}\,\frac{l}{2}\right)$$

$$= G\,\frac{M\,m}{r^2}\,l \qquad (6.2)$$

Equating Eq. 6.2 and Eq. 6.1, we get

$$G\,\frac{M\,m\,l}{r^2} = c\,\theta$$

$$G = \frac{c\,\theta r^2}{M\,m\,l}$$

where c is a torsion constant of the fibre. Its value depends upon the material of the fibre, θ is the angle through which the fibre is twisted and it is measured by observing the deflection of beam of light reflected from the small mirror attached to the fibre. The lamp and scale arrangement is used to measure the angle of twist accurately.

All the quantities in the above equation except G are known. The value of G found from this experiment was within

1 percent of the accepted value found by most modern equipment which is given by

$$G = 6.673 \times 10^{-11} \text{ Newton metre}^2/\text{kg}^2$$

The above experiment needs extreme care as the forces of attraction are so small that even a draught of air such as is produced by human respiration is likely to affect the results very badly.

6.3. MASS OF THE EARTH AND ITS AVERAGE DENSITY

We can determine the mass of the earth by using the law of universal gravitation and the value of G. Consider a body of mass m placed on the surface of the earth.

If M_e is the mass of the earth and R_e its radius, then the magnitude of the force with which the earth attracts the body toward its centre is given by

$$F = \frac{GmM_e}{R_e^2} \tag{6.3}$$

But the force with which the earth attracts a body on its surface towards its centre is equal to its weight W

Hence F = W

As W = mg.

$$\therefore F = mg \tag{6.4}$$

where g is acceleration due to gravity.

Thus from Eq. 6.3 and Eq. 6.4 we get

$$\frac{GmM_e}{R_e^2} = mg$$

$$\therefore M_e = \frac{R_e^2 g}{G}$$

Since $R_e = 6.4 \times 10^6$ m, $g = 9.8$ m sec^{-2}

and $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}

$$\text{Hence } M_e = \frac{9.8 \times \left(6.4 \times 10^6\right)^2}{6.67 \times 10^{-11}}$$

$$= 6 \times 10^{24} \text{ kg}$$

And since its volume $\left(\frac{4}{3}\pi R_e^3\right)$ is 1.08×10^{21} m^3, it follows that the mean density or mass per unit volume of Earth is 5.5 gm/cm^3. or about 5.5 times the density of water. As the mean density of the material in the surface layers of the Earth is 2.65 gm/cm^3, we conclude that the interior of the earth contains materials of density greater than 5.5 gm/cm^3.

6.4 VARIATION OF g

(a) Variation of g with altitude.

In the last article we found that the mass of the earth M_e is given by

$$M_e = \frac{R_e^2 g}{G}$$

$$\text{or } g = \frac{M_e G}{R_e^2} \qquad (6.5)$$

Since the values of M_e and G are constant

$$g \propto \frac{1}{R_e^2}$$

As Earth is nearly a sphere, the value of the radius of the earth R_e is almost constant on the surface of earth. Therefore, the value of g is almost the same all over its surface. As we go up in space, the distance from the centre of the Earth increases as is shown in Fig. 6.3. If station A is located at a height h from the surface of the Earth, its distance from the centre of the Earth is $(R_e + h)$ so the value of g at A is

$$g_h = \frac{M_e G}{(R_e + h)^2} \qquad (6.6)$$

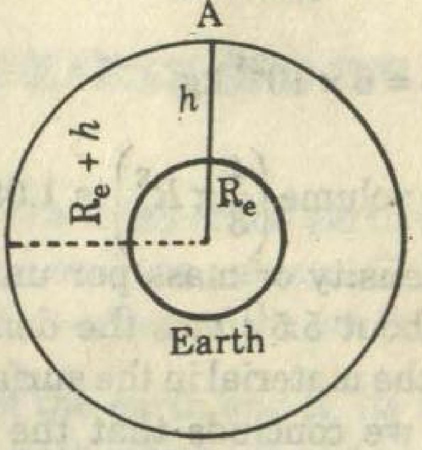


Fig. 6.3 *Variation of g with altitude*

Dividing equation (6.6) by equation (6.5), we get

$$\frac{g_h}{g} = \frac{R_e^2}{(R_e + h)^2}$$

$$g_h = g\,\frac{R_e^2}{(R_e + h)^2}$$

$$= g\,\frac{1}{\left(1 + \frac{h}{R_e}\right)^2}$$

$$= g\left(1 + \frac{h}{R_e}\right)^{-2}$$

Using Binomial expansion and retaining only the first two terms, we get

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

$$g_h = g - \frac{2gh}{R_e} \qquad (6.7)$$

Eq. (6.7) tells us that as we go up the value of g decreases by an amount $\frac{2gh}{R_e}$. For example in going up by 16 km from the earth's surface, the value of $\frac{h}{R_e}$ is $\frac{1}{400}$

$$\text{or } g_h = 9.8 - \frac{9.8 \times 2}{400} = 9.75\ m\ sec^{-2}$$

Near the earth's surface the value of g is almost constant. However, at higher altitudes such as those for a typical satellite orbits or for the Moon's orbit g drops as shown in Table 6.1.

Table 6.1. Variation of g with altitude.

Altitude (m)	*g, (m sec^{-2})*	*Altitude (m)*	*g, (m sec^{-2})*
0	9.806	32,000	9.71
1,000	9.803	100,000	9.60
4,000	9.794	500,000	8.53
8,000	9.782	*1,000,000	7.41
16,000	9.757	**380,000,000	0.00271

*Typical satellite orbit altitude. **Radius of moon's orbit

b) Variation of g with Depth below the Surface of the Earth

In the last article, we proved that the value of g decreases as the distance from the earth increases. What about the value of g in a mine dug below the surface of the earth? The immediate answer would be that the value of g should increase as the distance from the centre of the earth has decreased. But as we shall see presently, this is not the case.

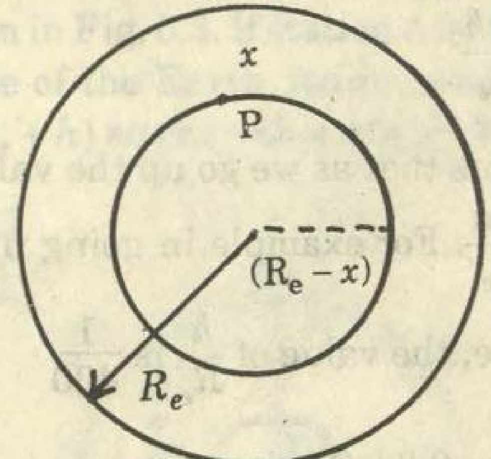


Fig. 6.4 *Variation of g below the Surface of Earth.*

Taking the earth to be a uniform sphere, an observer P at depth x below the surface is inside a sphere of radius R_e and thickness x, so that the attraction due to this part of the Earth's mass is zero. The effective Earth's mass is a sphere of radius (Re - x). If d is the density of the Earth supposed to be uniform everywhere, then

$$M_e = \frac{4\pi}{3} R_e^3 d$$

While the mass of the sphere effective for attraction at a depth x from the surface is

$$M_e' = \frac{4\pi}{3} (R_e - x)^3 d,$$

The value of 'g' at the surface is then

$$g = \frac{GM_e}{R_e^2}$$

$$= \frac{G\frac{4\pi}{3}R_e^3 d}{R_e^2}$$

$$= \frac{4}{3}\pi R_e dG$$

While at depth x, the value of g the acceleration due to gravity g_x is

$$g_x = \frac{GM_e'}{(R_e - x)^2}$$

$$= \frac{4}{3}\pi (R_e - x)\, d.G$$

$$= \frac{4}{3}\pi R_e\, d\, G\left(1 - \frac{x}{R_e}\right)$$

$$\therefore \qquad g_x = g\left(1 - \frac{x}{R_e}\right) \quad \text{As } g = \frac{4}{3}\pi R_e dG$$

$$\text{or, } g_x - g = -\frac{x}{R_e}g \tag{6.7a}$$

Hence as we go below the surface of the earth, the value of g decreases.

The variation of g in the earth's interior is given in the Table 6.2. The earth's radius is 6400 km.

Table 6.2 Variation of g with depth

Depth (km)	*g . (m sec^{-2})*	*Depth (km)*	*g . (m sec^{-2})*
0	9.82	1400	9.88
33	9.85	1800	9.85
100	9.89	2000	9.86

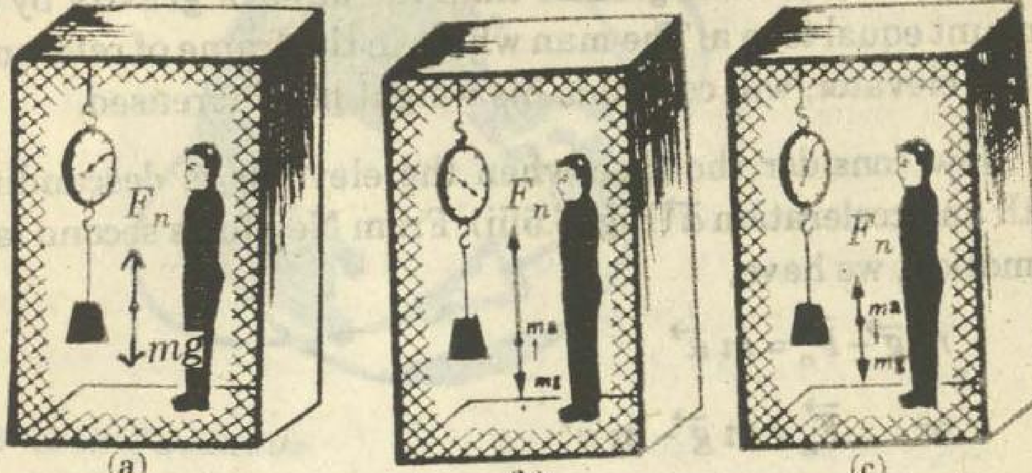
Depth (km)	*g . (m sec^{-2})*	*Depth (km)*	*g . (m sec^{-2})*
200	9.92	2200	9.90
300	9.95	2400	9.98
413	9.98	2600	10.09
600	10.01	2800	10.26
800	9.99	2900	10.37
1000	9.95	4000	.00
1200	9.91		

The variation of g in this table is not exactly as it should be according to the equation. Eq. 6.7a was derived on the assumption that the density was uniform throughout the interior of the earth. Since it is not so in actual practice the values of g given in this table do not agree with those predicted by the said eq. 6.7a.

6.5 WEIGHTLESSNESS IN SATELLITES

In chapter 3 we found that in any frame of reference, the weight of a body is equal and opposite to the force required to prevent it from falling in the Earth's gravitational field. In an inertial frame of reference (one which is not accelerated) the weight of a body is equal to the force of gravity experienced by the body, but in an accelerated frame of reference, the weight of the body depends upon the acceleration of the frame of reference as well.

A passenger in an elevator is acted upon by two forces. There is the force, $m\vec{g}$, his weight acting downward and a force $\vec{F}_n$ of the elevator floor pressing upward upon the bottom of his feet. (Fig. 6.5a). This upward force $\vec{F}_n$ can be conveniently measured by interposing a bathroom scale between his feet



(a) (b) (c)

Fig. 6.5 *(a) Forces acting upon a passenger in an elevator.(b) The weight of a body in an elevator appears, to a man in it to vary depending upon the motion of the elevator.*

and floor. The sum of these forces is $\vec{F}_{net}$,

i.e., $\vec{F}_{net} = \vec{F}_n - m\,\vec{g}$

When the elevator is at rest (Fig. 6.5a) $\vec{F}_n$ is equal and opposite to the force of gravity experienced by the passenger i.e. $\vec{F}_{net} = 0$, which gives

$$\vec{F}_n = m\,\vec{g}$$

Thus the weight of the passenger is equal to the force of gravity acting on him. When the elevator is ascending with an acceleration $\vec{a}$, (Fig. 6.5b), then by Newton's second law of motion, we get

$$\vec{F}_n - m\,\vec{g} = m\,\vec{a}$$

$$\vec{F}_n = m\vec{g} + m\vec{a}$$

Hence in this case the weight of the man as indicated by the bathroom scale is greater than the force of gravity by an amount equal to m a. The man who is in the frame of reference of the elevator, will conclude his weight has increased.

Now consider the case when the elevator is descending with an acceleration $\vec{a}$ (Fig. 6.5iii). From Newton's second law of motion, we have,

$$m\vec{g} - \vec{F}_n = m\vec{a}$$

$$\text{or} \quad \vec{F}_n = m\vec{g} - m\vec{a}$$

The weight of the passenger in this case is less than $m\vec{g}$ by an amount equal to $m\vec{a}$. The man in the elevator will conclude that his weight has decreased.

If the cable carrying the elevator breaks, the elevator will fall down with an acceleration equal to $\vec{g}$, so that

$$\vec{F}_n = m\vec{g} - m\vec{g} \quad (\because a = g)$$

$$\text{or} \quad \vec{F}_n = 0$$

Hence the scale will read zero and the man in the elevator will conclude that he has no weight i.e. he is in the state of "weightlessness". Actually the force of gravity still acts upon him and is given by W = mg. If it were not present he would not be accelerating rapidly down toward an unpleasant fate. However, he is used to living under the balance of various forces. The floor presses up on his feet, his body presses down on his legs, and his knee joints are squeezed together by these opposing forces. During free fall the absence of the force exerted on the man upsets this balance and the person experiences exactly the same sensations as one who is truly weightless, that is, far-removed from gravitational objects like

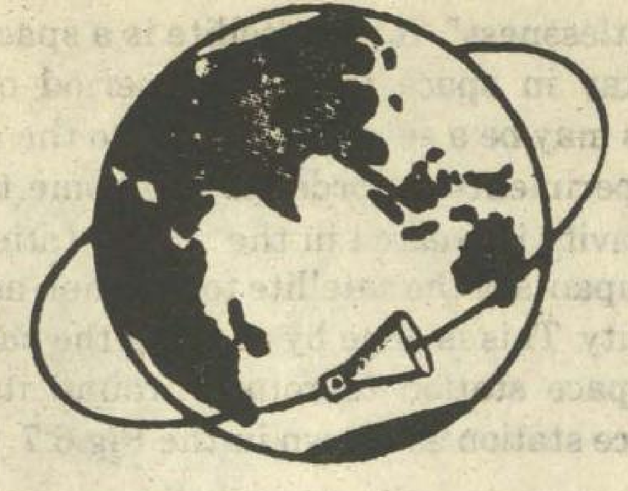


Fig. 6.6 *Orbit of a satellite*

the Earth.

Now we consider the state of weightlessness experienced by an astronaut and other objects in a satellite when it is in an orbit around the Earth.

While putting the satellite into orbit it is given a suitable tangential (orbital) velocity. The force of gravity just keeps the satellite in orbit around the earth providing it necessary centripetal force. This means that the astronaut, the satellite and every object in it are essentially freely falling bodies and they fall together at the same ate. Thus no force is required to hold an object from falling in the frame of reference of the satellite. This does not mean that no force is acting on the astronaut and his vehicle. Earth is still pulling him and his vehicle with a force only slightly less than that before the take-off. It is the absence of restraining force, which gives the impression of weightlessness.

6.6 ARTIFICIAL GRAVITY

In the last article we have seen that an astronaut in a satellite orbiting the earth will fall toward the earth at exactly the same rate as his satellite and consequently will be in the state of "weightlessness". If the satellite is a space laboratory intended to stay in space for a long period of time, this weightlessness may be a severe handicap to the astronaut in performing experiments. In order to overcome this problem an artificial gravity is created in the space station which will permit the occupants of the satellite to function as they would in earth's gravity. This is done by causing the satellite being used as the space station to rotate around its own axis. Consider a space station as shown in the Fig.6.7. Let T be the time for one revolution and 'v' the frequency, then $T = \frac{1}{v}$

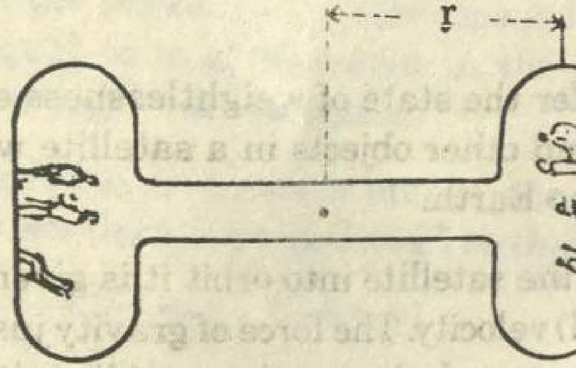


Fig. 6.7

Any point on the outer rim experiences a centripetal acceleration a_c is given by

$$a_c = \frac{V^2}{r}$$

But $V = r\omega$, where ω is the angular velocity

$$\therefore a_c = \frac{r^2\omega^2}{r} = r\omega^2$$

$$\text{But } \omega = \frac{2\pi}{T}$$

$$\therefore a_c = r\left(\frac{2\pi}{T}\right)^2$$

$$a_c = r\frac{4\pi^2}{T^2}$$

$$\text{But } \nu = \frac{1}{T}$$

$$\text{so } a_c = r\left(4\pi^2\nu^2\right)$$

$$\therefore \nu^2 = \frac{a_c}{4\pi^2 r}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}}$$

We have seen earlier that in the case of satellite orbiting around the earth, the force of gravity provides the requird centripetal accleration. In this case the centripetal acceleration a_c provides the force of gravity.

$$\text{i.e. } a_c = g$$

$$\therefore \nu = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

When the space ship is rotating with this frequency, the artificial gravity like that of the Earth will be provided to the inhabitants of the space ship

Example 6.1

Consider a space craft consisting of two chambers connected by tunnel of length 20 m, as shown in Fig. 6.8. How many revolutions per second must the space craft make in order to supply normal (Artificially) gravity for the passengers.

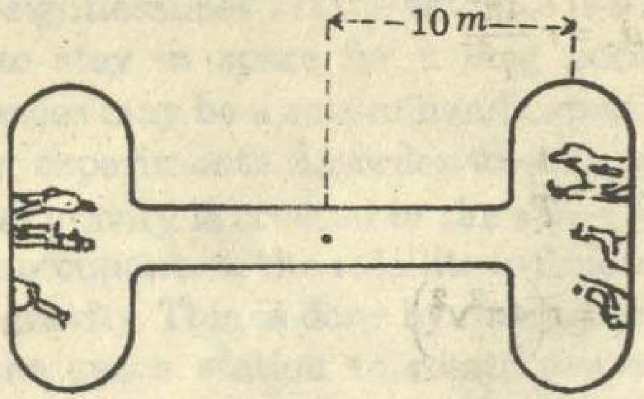


Fig. 6.8

Solution

$$\text{Here } r = \frac{20}{2} = 10\, m$$

$$\therefore \text{v} = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.8}{10}}$$

$$= 0.16 \text{ rev sec}^{-1}$$

Example 6.2

A ball of mass m is tied to one end of a cord and is whirled from the other end in a vertical circle of radius *r* with a constant speed V. Find the tension in the cord, when the ball is at the highest point.

Solution

(a) If the ball is to ramain in its circular path, the resultant forces acting upon it when it is at the highest point are:

(i) the weight mg of the ball acting downward.

(ii) the tension T in the cord also acting downward.

Hence, $T + mg = \frac{mV^2}{r}$

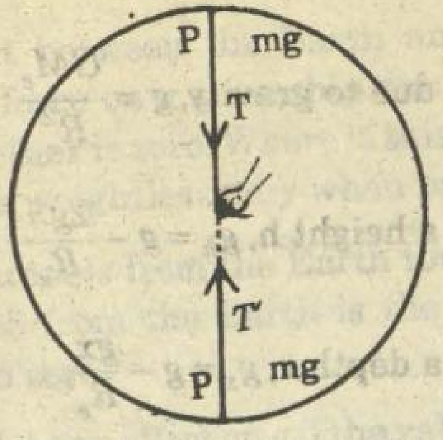


Fig. 6.9 *When* $\frac{V^2}{r} > g$ *or equal to g, the ball completes the circle, but when* $\frac{V^2}{r} < g$*, it will fall vertically down.*

or
$$T = \frac{mV^2}{r} - mg\ ,$$

now if $\frac{V^2}{r} = g$ then

$$T = \frac{mV^2}{r} - \frac{mV^2}{r} = 0$$

and the ball will just pass through the point P. But if $\frac{V^2}{r} < g$ the ball will fall down vertically.

SUMMARY

Newton's law of gravitation, $F = G\,\frac{m_1 m_2}{R^2}$

Mass of the Earth, $M_e = \frac{R_e^2\, g}{G}$

Acceleration due to gravity, $g = \frac{GM_e}{R_e^2}$

Value of g at a height h, $g_h = g - \frac{2gh}{R_e}$

Value of g at a depth x, $g_x = g - \frac{gx}{R_e}$

Free falling bodies are in a state of weightlessness and so are the occupants of space ship.

Artificial gravity is produced by spinning the space ship around its own axis with a frequency ν, given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

QUESTIONS

6.1 If the force of gravity acts on all bodies in proportion to their masses, why does not a heavy body fall correspondingly faster than a light body?

6.2 Would we have more sugar to the pound at the pole than at the equator? What about sugar to the kilogram?

6.3 How can one determine the mass of the Moon?

6.4 How is the orbital speed of a planet related to its orbital radius?

6.5 An elevator starts up from rest at the basement with an initial acceleration of 16 m $\sec^{-2}$. Will the passenger's apparent weight will be increased, decreased or remains the same.

6.6 At some point between the Earth and the Moon, the gravitational force on a spaceship due to the Earth and the Moon together is zero. Where is this point? Would the passengers be weightless only when passing this point?

6.7 A spaceship travels from the Earth towards the Sun. At what distance from the Earth is the net gravitational force equal to zero?

6.8 Would there be any effect on g if the value of G is doubled?

6.9 How would the values of g ad G be affected if the mass of the Earth becomes four times?

6.10Is the gravitational force between the Earth and the Sun be same at all times of the year? Explain!

6.11 How would the weight of a body vary as it is taken from the Earth to the Moon? What will be the effect on its mass?

6.12How would the weight of a body vary as it is taken to the centre of the Earth?

PROBLEMS

6.1 How far from the Earth must a body be along a line toward the sun so that the Sun's gravitational pull balances the pull of the Earth? The sun is 14.88×10^7 *m* away and its mas is 3.24×10^5 M_e.

6.2 At what altitude above the Earth's surface would the acceleration of gravity be 4.9 m sec^{-2}? The mass of Earth is 6.0×10^{24} kg and its mean radius is 6.4×10^6 m.

6.3 A 60 kg man in an elevator that is accelerating upward with $a = 9.8$ m sec^{-2}. What is the net force in newtons acting on the man.

6.4 A 60 kg man in an elevator that is accelerating downward with a = 9.8 m sec^{-2}. What is the net force in newtons acting on the man?

6.5 The centres of two identical spheres are 1 m apart. What must be the mass of each sphere if the gravitational force of attraction between them is 1 N?

6.6 What is "g" at the surface of Mars? Its radius is 3.43×10^6 m and its density is 3.95×10^3 kg m^{-3}.

6.7 The Moon has approximately one eighteith the mass of Earth. Its diameter is approximately one fourth that of the Earth. How does the acceleration due to gravity on the Moon compare with that of g on the Earth.

6.8 A man of mass 100 kg on the ground is standing in an elevator. Find the force exerted by the man on the floor of the elevator, when it (a) in at rest. (b) moves up with an acceleration of 4 m sec^{-2}. (c) moves down with an acceleration of 4 m sec^{-2}.

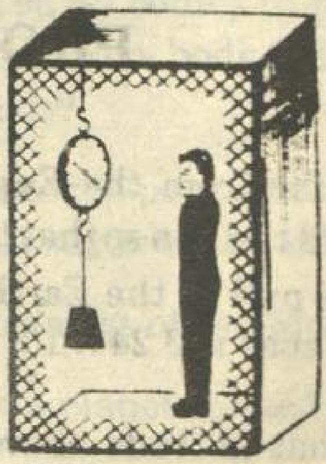


Fig. 6.10

6.9 A spring balance attached to the ceiling of a moving elevator indicates the weight of a body which weighs 980 N on the ground as 1470 N. Find both the direction and magnitude of the acceleration of the elevator.

6.10A 10 kg mass is at a distance of 1 m from 100 kg mass. Find the gravitational force of attraction which

(a) 10 kg mass exerts on the 100 kg mass.

(b) 100 kg mass exerts on the 10 kg mass. If the masses are free to move, find their respective accelerations.

6.11 What is the acceleration of a meteor at a distance of 8×10^6 m from the centre of the earth? Take mass of the earth = 6×10^{24} kg.

7

WORK, POWER AND ENERGY

7.1. WORK

We have seen the effect of forces on the motion of material objects. In this chapter we shall discuss another very important concept which is associated with the problems of motion in matter i.e., of energy. It is seen that energy is transferred from one object to another or transferred from one form to another during the motion of the matter due to the application of certain forces. In order to understand the concept of energy we shall first discuss the concept of work.

Work is a very commonly used word associated with "doing something" in our daily life. But in physics it has very precise meaning which shows that not all kinds of "doing something" would be called work. Thus in physics, we define work in the following way:

The work is done on a body when it is moved or stopped through a certain distance by the action of an applied force. Thus here we are assuring that the force is either in the direction of the motion or in the opposite direction. It is seen that during the process of doing work on the body or by the body the energy is either transferred to the body in the form of kinetic energy or taken out of the body, respectively. If a force $\vec{F}$ acting on the body displaces it through a distance d in the direction of application of the applied force, then the magnitude of work done by the force is given by

$$Work = Force . displacement$$

$$= \vec{F} . \vec{d} \qquad (7.1)$$

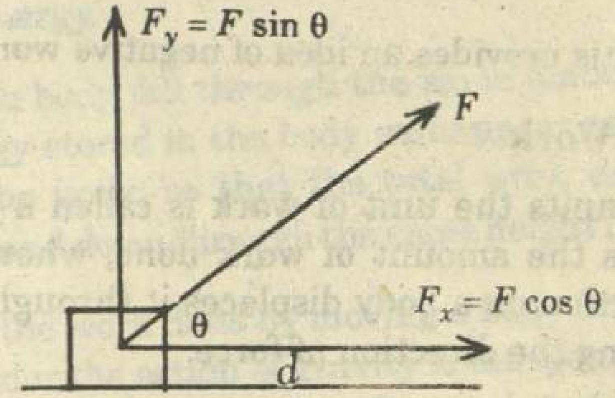


Fig. 7.1

Sometimes the displacement is not parallel to the direction of the applied force. For example, when a child pulls a toy car with a string, the displacement of the toy car may not be necessarily in the direction of force but may make an angle θ with it. When this situation occurs, we use the component of force that is along the direction of motion and not the force itself. As it is clear from Fig. 7.1. the component of the force $\vec{F}$ in the direction of the displacement is F cos θ so the work done W is given by,

$$W = F \cdot d \cos \theta$$

$$= (\vec{F} \cdot \vec{d}) \qquad (7.2)$$

i.e., work is a scalar quantity and measured as the scalar product of force and displacement. Equation 7.2 correctly implies that the value of work is maximum if a force is parallel to the direction of motion of the body ($\theta = 0$). No work is done by the force on the body when the direction of the force is perpendicular to the direction of the motion of the body ($\theta = 90^o$). In case, when the angle between the force and the displacement is 180°, the amount of work done is

$$W = F \cdot d \cos\theta$$

$$= F\,(-1) \cdot d = -F \cdot d$$

and this provides an idea of negative work.

Units of Work

In SI units the unit of work is called a joule. A joule is defined as the amount of work done, when a force of one newton action on a body displaces it through a distance of 1 metre along the direction of force.

$$1 \text{ Joule} = 1 \text{ newton} \times 1 \text{ metre} \qquad (7.3)$$

The dimensions of work are $\mathrm{ml}^2 t^{-2}$.

Example 7.1

A man pulls a trolley through a distance of 50m by applying a force of 100 N which makes an angle of 30° with the horizontal. Calculate the work done by the man.

Solution

$F = 100$ N, $d = 50$ m, $\theta = 30^\circ$

The work done

$$W = \vec{F} . \vec{d}$$

$$= Fd \cos\theta$$

$$= 100 \times 50 \times \cos 30^o$$

$$= 5000 \times 0.866 = 4330 \text{ Joule}$$

7.2. WORK DONE AGAINST THE GRAVITATIONAL FORCE

Another example of doing work on the body is by lifting it through a certain height h against the force of gravity. In order to do so we have to exert a force on it equal to its weight. In this process the amount of work done on the body is $(W \cdot h)$ which is stored as energy.

Now if we let the body fall through the same distance h, the amount of energy stored in the body will appear as work $= Wh$ done by the body; so that the total work done in moving the body up and down through the same height is zero.

It is found that the work done by moving a body from one place to another under the action of gravity is independent of the path followed by the body. Consider three points A,B,C in a gravitational field. (The space or region around the earth within which a body can experience a force of attraction due to the earth is called gravitational field). The force acting on the body in the gravitational field at any point of its path is equal to its weight W. Let us compute the work done in taking the body from A to C. There are two paths (i) one is from point A to point C (direct). (ii) other is from point A to point B and then from point B to point C.

The work done in moving it from A to C direct is given by

$$W_{A\to C} = \vec{F} \cdot \vec{d}$$

$$= \vec{W} \cdot \vec{d}, \qquad \because \vec{F} = \vec{W}$$

$$= W \cdot d \cos\theta$$

Now in right angled Δ CAB

$$\frac{AB}{AC} = \frac{d_1}{d} = \cos\theta$$

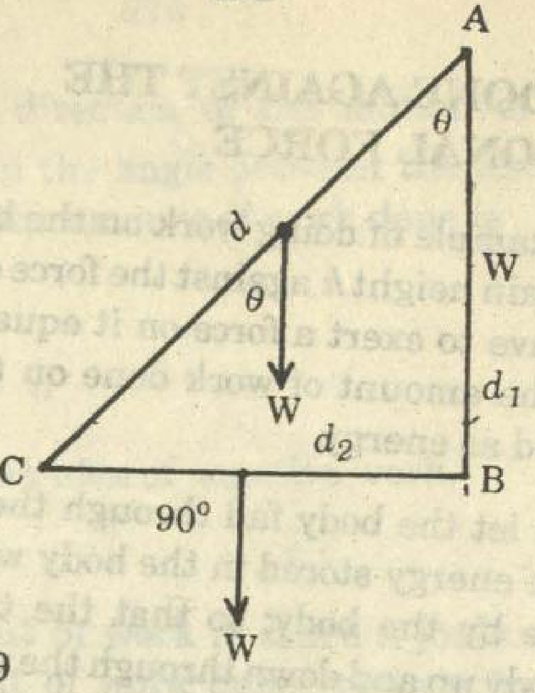


Fig. 7.2

$$d_1 = d\cos\theta$$

$$W_{A\to C} = W\,d_1 \tag{7.4}$$

Now the work done from A to B is given by

$$W_{A\to B} = \vec{F}\cdot\vec{d_1}$$

$$= \vec{W}\cdot\vec{d_1}$$

$$= W\cdot d_1\cos\theta$$

Since W and d_1 are along the same direction the angle between them is zero i.e., $\theta = 0$, $\cos 0 = 1$

$$W_{A\to B} = W\,d_1$$

Also, the work done in moving body from B to C is given by

$$W_{B\to C} = \vec{F}\cdot\vec{d_2}$$

$$= W\cdot d_2\cos\theta$$

Since, the angle θ between W and d_2 is 90°, $\cos 90^\circ = 0$

$$\therefore W_{B\to C} = 0$$

The work done in moving the body along the path ABC is given by

$$W_{A \to B} + W_{B \to C} = W d_1 + 0$$

$$W_{A \to B} + W_{B \to C} = W d_1 \qquad (7.5)$$

Which is the same as the work done in moving the body directly from A to C. Thus, the work done in a gravitational field is independent of the path followed by the body.

Now we calculate the total work done in moving the body around a closed path ABCA, i.e., from A to B, B to C and from C to A. Now,

$$W_{A \to B} = W d_1$$

$$W_{B \to C} = 0$$

$$W_{C \to A} = - W d_1$$

The work done in moving the body from C to A has a negative sign because the motion is in the opposite direction.

The total work done in moving the body around the closed path ABCA

$$W_{total} = W_{A \to B} + W_{B \to C} + W_{C \to A}$$

$$W_{total} = W d_1 + 0 + (- W d_1)$$

$$= W d_1 - W d_1$$

$$W_{total} = 0$$

Thus, the total work done in moving a body along a closed path in a gravitational field is always equal to zero. A field satisfying this condition is known as conservative field. Other example of such fields are electric and magnetic field.

7.3 POWER

In practice one is more interested in the rate at which the work is done rather than the total amount of work done. The rate of doing work is known as power and it is a derived quantity.

If the work is done at a constant rate, then the power P can be obtained as:

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{W}{t}$$

But work

$$W = \vec{F} \cdot \vec{d}$$

$$\therefore\ P = \frac{\vec{F} \cdot \vec{d}}{t}, \quad \text{as } \vec{V} = \frac{\vec{d}}{t}$$

$$P = \vec{F} \cdot \vec{V} \tag{7.6}$$

As power is the scalar product of force $\vec{F}$ and velocity $\vec{V}$ and thus it is a scalar quantity.

If the rate of doing work by the agent is not uniform, the power at any instant will not be equal to the average power. The power at any instant t, the instantaneous power may be *defined as the ratio of work done to the time interval, which is extremely small.* If ΔW denotes the amount of work done in a small interval of time Δt, then the instantaneous power, is given by

$$\text{Instantaneous power} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \tag{7.7}$$

The average power and instantaneous power become equal if the work is done at a uniform rate.

Units of Power

In SI units, the unit of power is called a watt, after James Watt who helped to develop the steam engine. *The watt is defined as the rate of doing one joule of work in one second i.e.,*

$$1\ watt = \frac{1\ J}{1\ Sec.}$$

The dimensions of power are $m\ l^2\ t^{-3}$

In British Engineering System, the unit of power is horse power (hp) and

$$1\ \text{hp} = 746\ \text{Watts}$$

The power of generating stations and grid stations are expressed in Mega-watts or Kilo-watts. The power of the electric bulbs and fans used in the house are in the range of 25 watts to 250 watts. Electric heaters, electric irons and air conditioners use 500 watts to 2 kilowatts of power. If a weight lifter uses 500 joules of energy to raise the weights in a time of 2 seconds, then his rate of doing work is $\frac{500\ \text{joules}}{2\ \text{sec}} = 250$ watts. So his muscle power is 250 watts. If a second weight lifter performs the same task in one second, his muscular power is then 500 watts. Clearly the second weight lifter is more powerful than the first one. Work may be expressed in terms of power.

$$\text{Since power} = \frac{\text{work}}{\text{time}}$$

$$\text{work} = \text{power} \times \text{time}.$$

Thus, if an agent consumes a power of 1 kilowatt in one hour, the work done is 1 kilowatt hr.

7.4 ENERGY

We have discussed in the preceding paragraphs that either we put energy into a body or take out from it when we do work on it or the body does work. *Thus the energy of a body is defined as its ability to do work.* It may be mentioned that the energy cannot be associated with either the height or speed alone. In fact, energy is found in more than one form. One form, called kinetic energy depends on the velocity of the body. Another form, called gravitational potential energy, depends on the height of the body above the surface of the earth. There are other types of energy such as nuclear, electrical, magnetic and mechanical etc.

Kinetic Energy

The kinetic energy (K.E.) of a body is the energy possessed by a body due to its motion.

Consider a body of mass m moving with initial velocity V. Let a constant force F be applied to produce an acceleration, $a = -\frac{F}{m}$ in it.

The body comes to rest after covering some distance say d.

As $V_f = 0$, use of relations

$$2ad = V_f^2 - V^2 \qquad \text{and} \quad a = -\frac{F}{m}$$

gives,

$$2Fd = m\,V^2$$

$$\text{or } F\,d = \frac{m\,V^2}{2}$$

But $F \,.\, d$ = work done, on the body to bring it to rest, which must be equal to the energy possessed by the body due to its motion.

$$\therefore K.E. = \frac{1}{2} m\, V^2 \tag{7.8}$$

$$i.e., K.E. = \frac{1}{2}\, mass \times (velocity)^2$$

$$K.E. = \frac{1}{2} m\, (\vec{V} \cdot \vec{V})$$

and is thus a scalar quantity. The units of energy are the same as those of work.

Work-Energy Relation

The work done on a body by the force acting on it is equal to the change in its kinetic energy caused by the force. If a force $\vec{F}$ acts on a body of mass m increases its velocity from V_i to V_f through a distance d, we have

$$V_f^2 - V_i^2 = 2\, a\, d$$

$$\text{or } d = \frac{V_f^2 - V_i^2}{2\, a}$$

The work done by the force is,

$$Work = F \cdot\ d$$

$$= m\ a \cdot d \qquad (F = m \cdot a)$$

$$= m\ a \left(\frac{V_f^2 - V_i^2}{2\, a}\right)$$

$$Work = m\, \frac{\left(V_f^2 - V_i^2\right)}{2}$$

$$= \frac{m\ V_f^2 - m\ V_i^2}{2}$$

$$= \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 \qquad (7.9)$$

Which is known as work energy relation

Work done = final K.E. – initial K.E.

= increase in K.E. of the body

Thus, the work done on a body is equal to the change in the kinetic energy of the body.

Example 7.2

A 1000 kg car is moving at a speed of 10 m sec $^{-1}$ and comes to rest after covering a distance of 50 m. How large was the decelerating force.

Solution

As the work done by the decelerating force is equal to the change in kinetic energy i.e.,

$$F \cdot d = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$$

As $d = 50$ m, then

$$F \cdot 50 \text{ m} = \frac{1}{2} \times 1000 \text{ kg} \times (0)^2 - \frac{1}{2} \times 1000 \text{ kg} \times (10)^2$$

$$F \cdot 50\, m = 0 - 500 \text{ kg} \times 100 \text{ m sec}^{-2}$$

$$F \cdot 50\, m = 0 - 500 \times 100$$

$$F = \frac{500 \times 100}{50}$$

$$F = 1000\, N$$

Potential Energy (P.E)

Potential energy of a body is the energy possessed by it due *to its position*. As we have discussed earlier, if a body is raised to a certain height in the gravitational field; the work is done on it. This work is stored in the body as potential energy. Similarly, if an elastic spring is compressed work is done against the elastic force, such a potential energy is called elastic potential energy. Here we shall restrict our discussion only to gravitational potential energy. Let a body of mass m be raised to a height h from the ground. In order to keep the body at a certain height, the weight of body is balanced by the force applied i.e., $F = W$, thus, the work done against the force of gravity is $W h = m g h$. This work done is due to the change in position of the body, so it is called gravitational potential energy or simply potential energy. Mathematically,

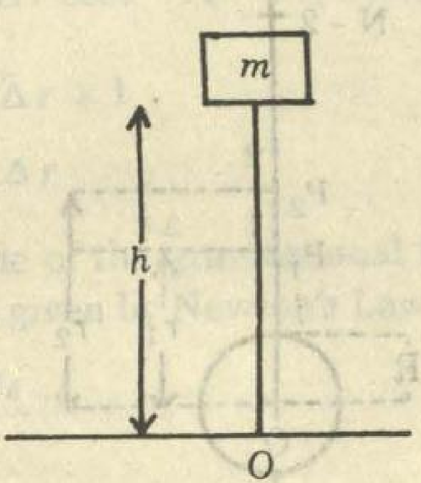


Fig. 7.3

$$\text{P.E} = mgh \tag{7.10}$$

Absolute Potential Energy

The force of gravity on a body is constant and equal to mg within small heights above the surface of the earth. The equation P.E. = mgh, is based on the assumption that the height h above the earth is very small as compared to the radius of the Earth. However, force of gravity actually decreases as the height increases, so the absolute potential energy of a body will be taken as zero when the body is at infinite distance from the centre of the earth. Let us calculate the work required to move a body from the surface of the earth up to an infinite distance. This work will be a measure of the absolute potential energy of the body on the surface of the earth.

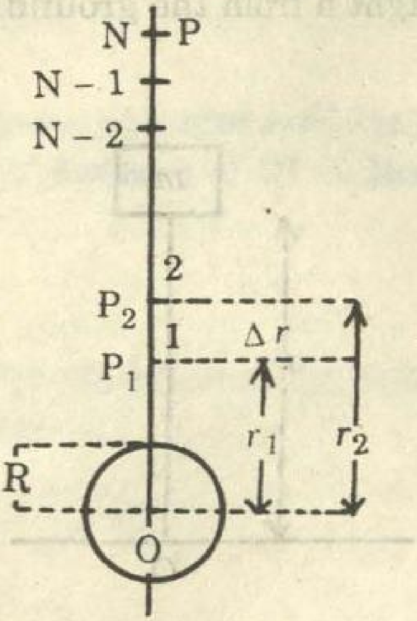


Fig. 7.4

Let the body be lifted from the point P_1 to some very far point P. The potential energy of the body will still be measured by the work done against gravity but the fact that the gravitational force varies with the height must now be taken into account. Since the distance of the body from the centre of the earth varies from point to point the gravitational force acting on the body changes constantly. However, if we divide the total distance into small equal elements each equal to Δr at points 1, 2, 3, 4, ... $N-2, N-1, N$ or P.

The average gravitational force may be assumed to remain constant for each element. Let r_1 = distance of point 1 from the centre of the earth, r_2 = distance of point 2 from the centre of earth then the distance between point 1 and 2 will be $r_2 - r_1$

$= \Delta r$ (say).

The work required to raise the body from point 1 to point 2 is given by

$$W_{1\to 2} = Force \cdot distance$$

$$= \vec{F} \cdot \vec{d}$$

$$= \vec{F} \cdot \overrightarrow{\Delta r} \qquad (\vec{d} = \overrightarrow{\Delta r})$$

$$= F\,\Delta r \cos\theta \qquad (\because \theta = 0, \cos 0 = 1)$$

$$= F\,\Delta r \times 1$$

$$W_{1\to 2} = F\,\Delta r \tag{7.11}$$

The magnitude of the gravitational force $\vec{F}$ between the body and earth is given by Newton's Law of gravitation as,

$$F = \frac{G\,m\,Me}{r^2}$$

Where *Me* denotes the mass of earth, *G*, the universal gravitational constant and *m* the mass of the body and *r* the average distance of point 1 and point 2 from the centre of the earth. i.e.,

$$r = \frac{r_1 + r_2}{2}$$

as $r_2 = r_1 + \Delta r$

$$\text{so, } r = \frac{2\,r_1 + \Delta r}{2}$$

Squaring both the sides,

$$r^2 = \left(\frac{2\,r_1 + \Delta r}{2}\right)^2$$

$$r^2 = \frac{4\,r_1^2 + \Delta\,r^2 + 4r_1\,\Delta r}{4}$$

$$r^2 = r_1^2 + \frac{\Delta\,r^2}{4} + r_1\,\Delta r$$

$\Delta\,r$ is very small so $\frac{\Delta\,r^2}{4}$ can be neglected as compared to r, thus,

$$r^2 = r_1^2 + r_1\,\Delta\,r$$

$$r^2 = r_1^2 + r_1\left(r_2 - r_1\right)$$

$$r^2 = r_1^2 + r_1\,r_2 - r_1^2$$

$$r^2 = r_1\,r_2 \qquad (7.12)$$

So that the magnitude of the gravitational force is,

$$F = \frac{G\,mMe}{r_1\,r_2}$$

Putting the values of F and Δr in the Eq. 7.11,

$$W_{1\rightarrow 2} = F\,\Delta\,r$$

$$= \frac{G\,mMe}{r_1\,r_2}\left(r_2 - r_1\right)$$

$$W_{1\rightarrow 2} = G\,m\,Me\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly, the work required to lift the body from point 2 to point 3

$$W_{2\rightarrow 3} = G\,m\,Me\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

and so on.

Work done in last step is

$$W_{N-1 \to N} = G\,m\,Me\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Hence the total work required to lift the body from point 1 to N.

$$W_{1 \to N} = W_{1 \to 2} + W_{2 \to 3} + \dots + W_{N-2 \to N-1} + W_{N-1 \to N}$$

$$W_{1\to N} = GmMe\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \dots + \left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right] \quad (7.13)$$

if the point N is situated at infinity $r_N = \infty$ and $\frac{1}{r_N} = 0$. Hence the work done to raise the body from point 1 to an infinite distance is,

$$= \frac{G\,m\,Me}{r_1}$$

If the body lies on the surface of earth i.e., $r_1 = R_e$ the work required to lift a body of mass m from surface of the earth to an infinite distance is absolute potential energy of the body

$$= \frac{G\,m\,Me}{Re}$$

Escape Velocity

If we want to lift a body of mass m from the surface of the earth to infinity, this energy has to be supplied to the body in the form of kinetic energy i.e.,

$$\frac{1}{2} m\,V^2 = \frac{G\,m\,Me}{Re}$$

$$\text{or } V = \sqrt{\frac{2\,G\,Me}{Re}} \quad \text{(a)}$$

Now, $mg = \frac{G\,m\,Me}{R_e^2}$ on the surface of the earth

$$\therefore \frac{GMe}{R_e} = R_e\,g$$

Putting this value in Eq.(a), we get

$$V = \sqrt{2\,R_e\,g}$$

$$g = 9.8\ m\ sec^{-2},\ R_e = 64 \times 10^5\ m$$

$$V = \sqrt{2 \times 9.8 \times 64 \times 10^5}$$

$$= 11 \times 10^3\ m\ sec^{-1}$$

This velocity should be given to a body to enable it to escape from the earth's gravitational field, therefore, this velocity is called the *escape velocity*. This much velocity is provided to interspace ships by multi-stage rockets.

Example 7.3

The moon's radius is 1600 km and $g = 1.6\ \mathrm{N\ kg^{-1}}$ on its surface. Calculate the escape velocity from the lunar surface.

Solution

Given $g = 1.6\ \mathrm{N\ kg^{-1}}$, $r_m = 16 \times 10^5$ m

$$V_{escape} = \sqrt{2\,g\,r_m}$$

$$V_{escape} = \sqrt{2 \times 1.6 \times 16 \times 10^5}$$

$$= 2.3 \times 10^3\ m\ sec^{-1}$$

7.5 INTERCONVERSION OF KINETIC ENERGY AND POTENTIAL ENERGY

Kinetic energy can be converted into potential energy and vice versa i.e. they are interchangeable, when a body falls freely, its potential energy goes on changing into kinetic energy. This loss of potential energy must be equal to gain in kinetic energy. Let a body of weight *W* be raised to a height *h* from the ground to the point *P*. The energy possessed by the body will be all potential and given by P.E. = $m\,g\,h$

Its kinetic energy is zero, since it is at rest. Therefore the total energy at $P = m\,g\,h$.

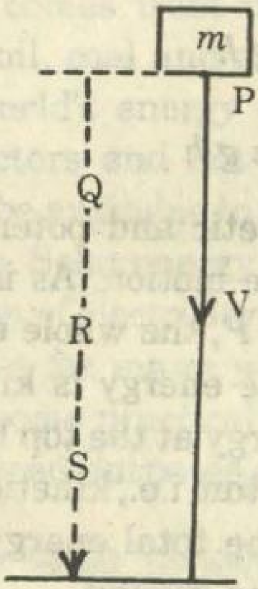


Fig. 7.5

The potential energy at *Q* when it has fallen to a distance $\frac{h}{4}$ is $\frac{3}{4}\,m\,g\,h$, its velocity at *Q* is $V = \sqrt{\frac{2gh}{4}}$.

and K.E. $= \frac{1}{2} \times \frac{mgh}{2} = \frac{mgh}{4}$

Thus the total energy at $Q = \frac{3}{4}mgh + \frac{1}{4}mgh = mgh$

Similarly when it has fallen through a distance $\frac{h}{2}$, at point *R*, potential energy $= mg\frac{h}{2}$ and K.E.

$$= \frac{1}{2} m \left(\sqrt{2g \frac{h}{2}} \right)^2 = \frac{1}{2} mgh = mg \frac{h}{2}.$$

Again the total energy $= mg \frac{h}{2} + mg \frac{h}{2} = mgh.$

At the bottom, potential energy is zero, but the K.E.

$$= \frac{1}{2} m \, (\sqrt{2gh}\,)^2$$

$$= \frac{1}{2} m \, 2 \, g \, h = m \, g \, h$$

The total energy $= m \, g \, h$

The sum of the kinetic and potential energies remains constant throughout the motion. As is clear from the above discussion. At the point P, the whole energy is potential and at the bottom the whole energy is kinetic in form, in other words the potential energy at the top has been converted into kinetic energy at the bottom i.e., kinetic and potential energies are interconvertible. The total energy is converted into the form of heat as it hits the ground.

7.6 VARIOUS SOURCES OF ENERGY

Present day society consumes energy at a fantastic rate which is increasing continuously. It has been estimated that the present worldwide consumption of energy amounts to about 6 × 10^{13} kilowatt hours annually. From where this energy comes from? The ultimate source of almost all the energy that we use today is the Sun. All our primary fuels - wood, coal, oil and natural gas are derived from plant life and animal life that grow because of the action of sunlight. The water that drives hydroelectric generating plants is lifted to highland through evaporation and precipitation processes that result from solar heating. The most important source of energy other than Sun is the atomic nucleus which releases energy through fission and fusion. The source of geothermal energy is the heat produced in the Earth's interior by radioactive decay. The tidal energy is due to the rotation of the Earth relative to the Moon. Until about 100 years ago, the primary sources of energy were wood, water, wind and the Sun's direct rays. They are still in use, but only water power in the form of electricity generated by huge hydroelectric plants is a significant factor in the world energy supply. Most of the energy used today comes from the burning of the various chemical fuels like oil, coal and natural gas. In the future a major portion of world's energy demand will be met from nuclear fission reactors and not in the too distant future, fusion reactors will be available to take over the major burden of energy production. Solar energy in the form of solar heating and in the generation of electricity from solar cells may not be economically feasible for many years, improved techniques may soon result in some practical system for the generation of energy for specialized purposes directly from the Sun rays.

We shall discuss these various sources in the following pages:

7.7. WATER POWER

Historically water wheels of many kinds have long been used in various parts of the world to produce mechanical power for grain milling, sugar crushing and the like ones. Today, we no longer use water power directly but instead, use the potential energy in water stored behind dams to generate electricity. In a modern hydroelectric plant, water is allowed to pass through tunnels and drive huge turbines whose rotating shafts are connected to electric generators that produce powers for consumers. The largest dams produce electric power in the range of thousands of Mega watt., Tarbela Dam on the Indus river produces 1750 MW of electricity. Mangla Dam on river Jhelum generates 900 MW. There are other dams like Warsak on river Kabul which have considerably smaller output. At present the total installed hydroelectric generating capacity in Pakistan amounts to about 3000 MW. The hydroelectric power generation is affected adversly by the lowering of the water level in the artificial lakes behind the dams especially in winter and early summer. The reduction in the power supply is met by rotational loadshedding with which everybody is familiar.

7.8 FOSSIL FUELS

Since the beginning of the 20th century most of the world's energy has been derived from the burning of fossil fuels. The fossil fuels are mainly coal, petroleum and natural gas.

Most of the world's coal began forming in the carboniferous period of the geological time scale from 345 to 280 million years ago. The remains of swamp forests were subjected to great pressure under the weight of sands and other materials laid on them. The dead vegetables turned into coal.

Oil was formed under great heat and pressure and is found in certain types of rock formation. It is held in porous rocks which act as sponges. These rocks are trapped between non porous layers which neither hold any liquid nor let it pass through. Oil in its natural or crude form is useless, it is refined and separated into various components which are used as fuels in aircrafts and motor vehicles. Natural gas also occurs, underground usually lying above a layer of oil bearing rocks.

Coal occurs widely throughout the world. The largest deposits are in the U.S.S.R. whereabout 60 percent of the world's coal reserves are located. In Pakistan the coal is mined in Baluchistan and is transported to the various parts of the country by trucks for use in brick kilns. Because of high contents of sulphur in it, Pakistan coal is not used in electric generating plants.

Oil deposits are widely distributed in coastal areas beneath the continental shelves. Oil is found in island regions that were once submerged, such as the south western United States, parts of Sahara deserts, and in the regions bordering the Persian Gulf i.e., Kuwait, Iran, U.A.E. and Saudi Arabia. Large deposits of oil, probably more than that occur beneath land areas are found beneath the continental shelves. The most significant deposits occur off the eastern southren and south western coasts of the United States, in the North sea, in the Persian Gulf, and in the waters of southeast Asia and Indonesia. In Pakistan oil is being produced from Pothohar area and also in Sind. About fifty percent of the demand for oil is being met by our own resources. The natural gas called Sui gas is being supplied for household and industrial use from huge reservoirs of gas located at Sui in Baluchistan. Some smaller deposits have been found in Sind.

7.9 NUCLEAR ENERGY

The most concentrated form of energy that is available to man is stored in nuclei. This energy can be released in the processes of fission (the breaking apart of heavy nuclei) and fusion (the fusing together of light nuclei) (Fig. 7.6).

Fission reactors have been producing electricity in commercial quantities for about 30 years. In Pakistan we have only one nuclear power reactor located at Karachi which generates 900 MW of power. We have fully exploited our hydroelectric sources. To meet our future domestic and industrial need, we will have to generate electricity primarily from nuclear power plants using uranium and plutonium in fission reactions. Recently Pakistan has signed two different

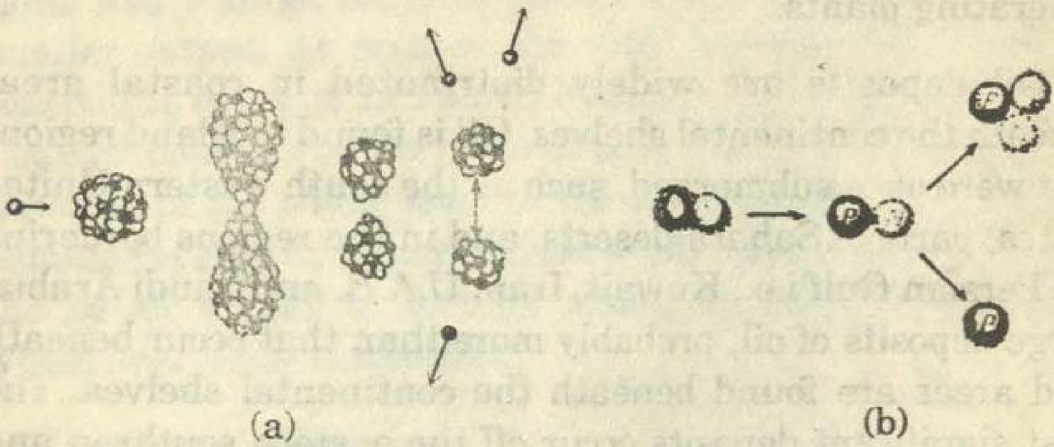


Fig. 7.6 *When a uranium nucleus is struck by a neutron a fission reaction starts that releases energy and produces several neutrons that cause the fission reactions. (b) When two deuterium nuclei collide at high energy, a reaction takes place, that produces a He nucleus and a neutron and releases energy.*

agreements with France and China under which Pakistan will get two nuclear power reactors from these countries. So far no feasible fusion reactor has been developed. When it happens heavy hydrogen (deuterium) will become the principal fuel.

7.10 GEOTHERMAL ENERGY

Geothermal energy is generated by tapping the energy of the hot, molten areas inside the earth crust. There is one operating geothermal power plant in the United States and others in Iceland, Mexico, Italy, Japan and New Zealand. Geothermal power is used in two ways. One involves, the piping of steam from naturally occuring reservoirs of steam deep in the earth, and using that steam to turn turbines that generate electricity. The other way involves the tapping into very hot water which is sometimes found deep in the earth, and using the steam from the water to run turbines.

A technique is being developed for pumping water down to hot, dry rock, thereby creating steam that is then piped back up to turn turbines. In the near future it appears that geothermal power may be an important source of energy but only at particularly favourable locations where reservoirs of steam or hot water are found. In Pakistan we do not have such reservoirs.

7.11 WIND ENERGY

The non uniform distribution of heat due to solar energy in different regions causes the movement of hot and cold air over the earth's surface. Winds blow from area of high pressure to areas of low pressure. At sea the winds are even stronger than on the land surface. An equivalent of 100 billion watts per year of power in the shape of wind energy is available on the earth. In the windy regions, wind mills are installed to produce mechanical energy. This mechanical energy may be used in tube wells or flour mills. In Pakistan places like Chitral, Gilgit and other high mountain valleys are suitable for the exploitation of wind power.

7.12 TIDAL ENERGY

The water rises along coasts due to the gravitational pull of the moon on water twice daily forming tides. In certain parts of the world, tides rise very high. This back and forth surging of water can be tamed in narrow tunnels and used for power generation. Tidal power stations have been installed in France, Alaska, Argentina and in the U.S.S.R.

7.13 SOLAR ENERGY

The source of energy most easily available to us is sunlight. Mirrors and lenses have been used to concentrate the energy of the Sun's rays into small spots. Every year the Earth absorbs about 4×10^{17} KWh of solar energy. All of this energy plus a small additional amount that is conducted to the Earth's surface from the interior is radiated into space. If we could utilize only 0.1 percent of the incident solar energy, that would be more than enough to satisfy the entire world's energy requirements. The diffusion of the solar energy supply is a great handicap in terms of Earth bound uses of this energy. Enormous areas of absorbers are necessary to produce any substantial amount of useful energy. On a cloudless day the rate at which the solar energy reaches the Earth is about 0.8 KW m^{-2} during 8 hours. The most practical way to utilize this energy is to allow it to vaporize water into steam which can then be used to drive conventional electricity producing turbines. There are formidable problems in this type of development, however, such as finding the proper surface for absorbing the sun light and devising an efficient method for converting the energy in the absorbing materials into steam energy.

A more efficient use of solar energy is to produce electricity by the use of solar cells. Solar cells are thin sandwich of semiconductors. The diagram of a solar cell is given in Fig. 7.7.

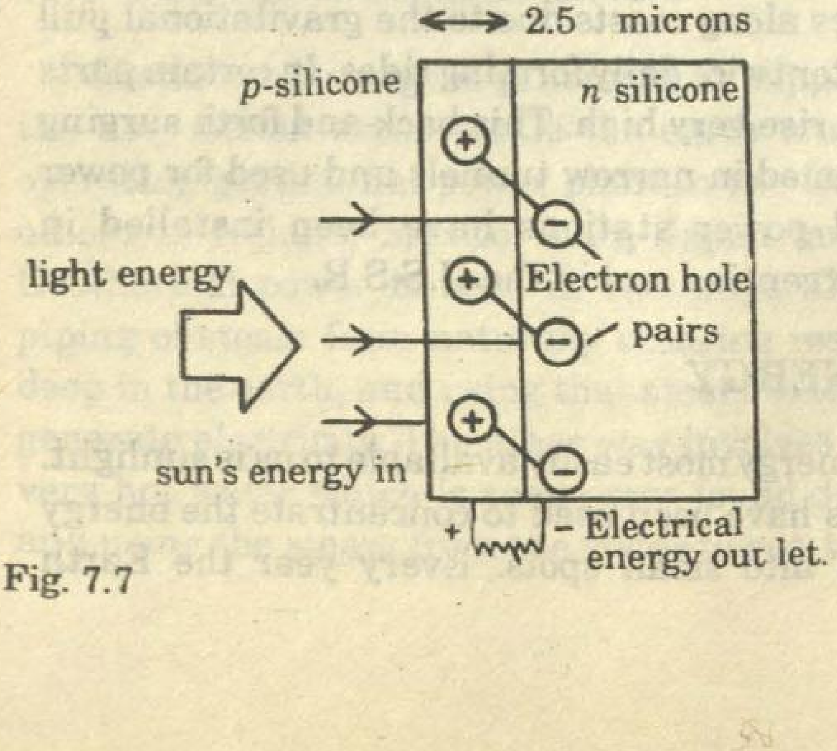


Fig. 7.7

Each cell consists of a disc of silicon about 2 - 3 cms in diameter and about 2 mm thick which is called n-layer. A second substance such as boron is allowed to diffuse into the surface of the silicon disc. This surface is called a *p*-layer and gives the disc the characteristics of a semiconductor. Now the electrons can travel only in the direction from the *p*-layer to the n-layer. Electrons cannot flow in the reverse direction. When this disc is placed in sunlight, photons of sunlight knock out electrons out of the *p*-layer making it positively charged. The electrons are forced into the n-layer making it negatively charged. The electrons can return to the *p*-layer only by flowing through a circuit as shown in the Fig. 7.8. The voltage produced by a single solar cell is very small, in order to get a large voltage, a number of solar cells are connected in series forming a panel of solar cells. This solar panel can be used to drive radio, T.V., etc. This solar panel provides the electrical power directly from the sunlight.

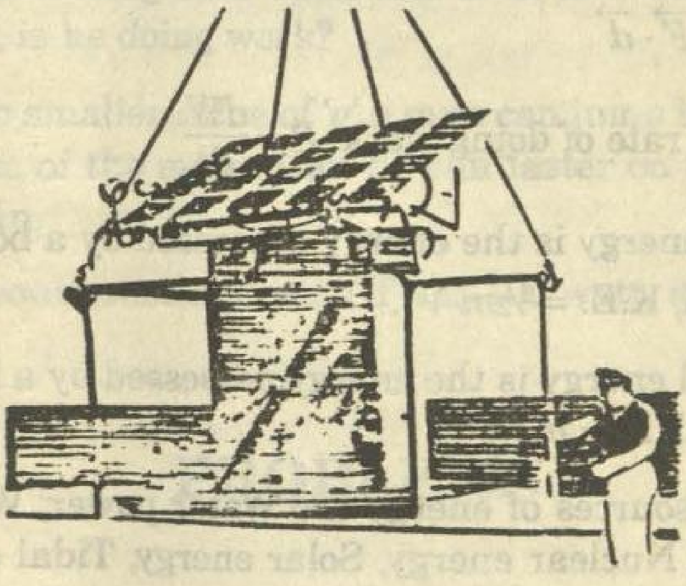


Fig. 7.8 *Solar panel prepared to power navigational lights on an offshore oil platform.*

Nickel-cadmium batteries, according to our power requirements, are connected in series with solar panels in sunshine to store this energy. Then, these batteries can be used to supply power to our electrical appliances at night or on a cloudy day. In Pakistan the solar energy electricity may be used for operating tube wells, health clinic in far bledged areas.

SUMMARY

Work done is force × distance moved in the direction of force

i.e., $W = \vec{F} \cdot \vec{d}$

Power is rate of doing work, $P = \frac{dW}{dt}$

Kinetic energy is the energy possessed by a body because of its motion, K.E. = $\frac{1}{2}m\,V^2$.

Potential energy is the energy possessed by a body due to its position, P.E. = mgh

Various sources of energy are Water power, Wind power, Fossil fuels, Nuclear energy, Solar energy, Tidal energy and Geothermal energy.

QUESTIONS

7.1. Does the work done in raising a box on to a platform depends upon how fast it is raised up? If not, why.

7.2. When an arrow is shot from its bow, it has K.E. From where does it get the K.E.?

7.3. What type of energy is stored in the spring of the watch?

7.4. A bucket is taken to the bottom of a well, does the bucket possess any potential energy?

7.5. A block of mass 1 kilogram is placed on a shelf located at a height of 5 m from the top of a table which is 1.5 m high. What is its potential energy?

7.6. The force between pair of protons is that of repulsion. Does the P.E. increase as the pair of protons is brought together.

7.7. A meteorite when enters into Earth's atmosphere burns. What happens to its energy and momentum.

7.8. A man rowing boat upstream is at rest with respect to shore, is he doing work?

7.9. Due to smaller value of '*g*' a man can jump higher on the surface of the moon. Can he run faster on this account. Explain.

7.10. Find your muscle power. (If it is 100 watt, it is not bad!).

PROBLEMS

7.1. A spanner of length 0.2 m is applied to tighten a nut. A force of 25 N at right angle to the spanner is applied at the end and the nut turns through 180°, calculate the total work done.

7.2. A ball of mass 100 gm is thrown vertically upward at a speed of 25 m sec^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25 m, calculate the work done against air resistance.

7.3. Water falls over a waterfall of height 30 m at a rate of 45000 cubic per minute. Determine the power generated (density of water is 1000 kg m^{-3}).

7.4. The velocity of 50 kg mass is changed from 500 to 200 cm sec^{-1}. What is its loss of kinetic energy?

7.5. What force is required to stop a bullet of mass 100 gram and velocity 30 m sec^{-1} in a distance of 10 cm?

7.6. What horse-power is required to pump 2500 kg of water up 100 m in 5 minutes?

7.7. What horse-power is used when an engine pulls a train with a force of 2000 N at 96 km hr^{-1}.

7.8. How much work does a man do in holding a 10 kg mass at a height of 5 m for four hours? How much work does he do in carrying it horizontally for 10 metres?

7.9. A force of 2000 N is exerted in lifting a 100 kg mass straight up to a height of 10m (a) How much work is done? (b) What are the kinetic and potential energy of the object when it gets to that height.

7.10. If it takes a force of 10N to depress a typewriter key through a distance of 10 cm in time of 0.1 sec. how much average power does it take?

7.11. A ball with a mass of 50 gm is dropped to the floor from a height of 1 m and bounces back to a height of 0.8 m. Using the conservation of energy law find its velocity just before it struck the floor?

8

WAVE MOTION AND SOUND

8.1 VIBRATORY MOTION

You must have seen the swinging bob of a pendulum. It moves to and fro about its position of equilibrium. Such type of motion is known as *vibratory motion*. Similarly, if the tight string of a violin is plucked, it also performs a vibratory motion. This type of motion is quite different from the translatory or the rotatory motion of a body - the types of motion which we have already studied. In this chapter we shall consider the various characteristics of the vibratory motion.

8.2 MOTION UNDER AN ELASTIC RESTORING FORCE

Consider a mass m attached to one end of a spring and placed on a frictionless horizontal surface as shown in Fig. 8.1. When the mass is pulled towards right through a distance x, the force F exerted on the spring is proportional to the displacement x. Mathematically

$$F = \mathrm{k}x$$

where k is a proportionality constant, usually referred as spring constant. Due to elasticity the spring opposes the applied force which produces the displacement. This opposing force is called the *restoring force* and is equal to $-\mathrm{k}x$.

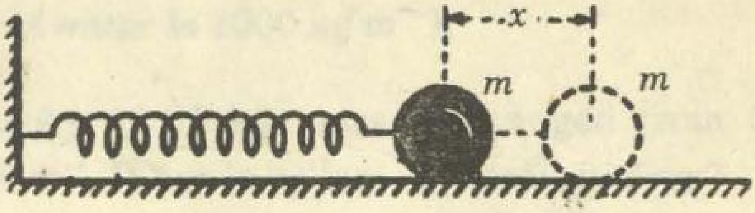


Figure 8.1 *When a spring is pulled or pushed, the restoring force is proportional to the change in length of the spring.*

We shall now consider the motion of a mass m attached to one end of a spring (Fig. 8.2). There is no force acting on the mass m in the position (a) of this figure since the spring suffers no extension in this case. Let the mass m be now pulled towards right with some force. The extension of the spring gives rise to a restoring force which is, at all instants, equal and opposite to the applied force, provided the elastic limit is not violated. If the mass is displaced through a distance x_0 to the right, the restoring force F will be given by $-kx_0$.

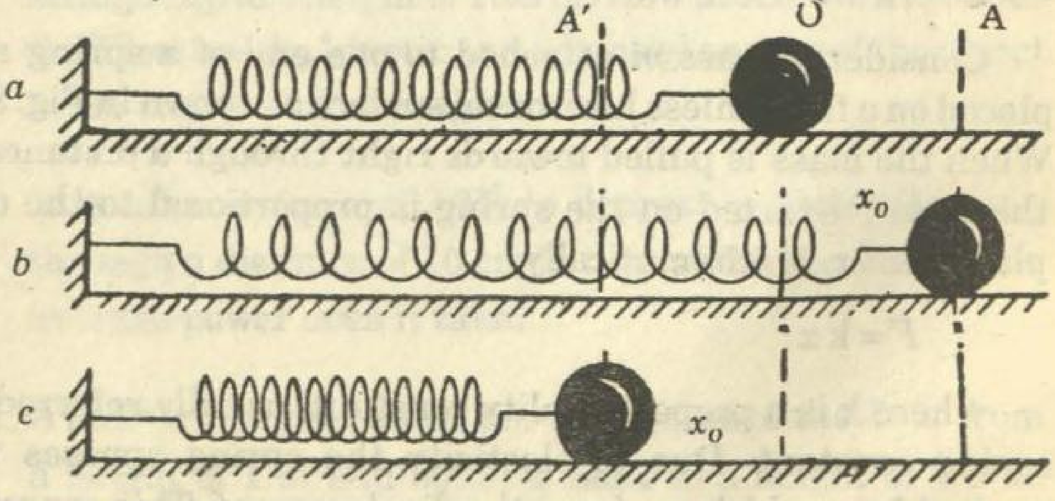


Figure 8.2 *When a mass attached to one end of a spring is pulled to one side and released, it begins to oscillate.*

Some work shall have to be done against this force in displacing the mass from its equilibrium position. This work will be stored in this spring as its potential energy. If the mass is now released, the spring will do work on the mass and will exert a force in order to restore it to its equilibrium position. The mass moves towards left under the action of this force and when it arrives at the equilibrium position, all its potential energy is converted into kinetic energy. Due to inertia the mass continues to move towards left. The spring is now compressed and it retards the motion of the mass. The mass comes to rest when the whole of the kinetic energy is converted into the potential energy of the compressed spring. The process is repeated and the mass continues to oscillate between A and A′. Note that the energy oscillates back and forth between kinetic energy and potential energy but as we shall see later, the total energy remains the same.

We may now find the acceleration of the mass by applying the second law of motion. consider the position of the mass at any instant during its to and fro motion about its mean position. If x is its displacement from the mean position at this instant as shown in Fig. 8.1 the restoring force F, acting on it, is given by

$$F = -\mathrm{k}x = ma$$

where a is the acceleration at the instant t.

$$\therefore a = -\frac{\mathrm{k}}{m}x \qquad (8.1)$$

$$= -\,(\text{constant})\,x$$

$$\therefore a \propto -x$$

$$\text{or } a \propto -\text{ displacement}$$

This type of vibratory motion which is characterised by the fact that when the mass is at a distance x away from its equilibrium position, it has an instantaneous acceleration of magnitude proportional to x and always directed towards the equilibrium position, is called *simple harmonic motion.*

8.3 CHARACTERISTICS OF SIMPLE HARMONIC MOTION

We shall now define some important terms used in connection with simple harmonic motion (SHM).

(i) Periodic Motion

A motion which repeats itself in equal intervals of time is called periodic motion.

(ii) Vibration

A vibration means one complete round trip of the body. In Fig. 8.2 it is the motion of mass m from A back to A, or from A′ back to A′, or from O to A, A to A′ and back to O.

(iii) Time period (T)

It is the time required to complete one vibration.

(iv) Frequency (ν)

It is the number of vibrations executed by a body in one second and is expressed as vibrations $\sec^{-1}$, cycles $\sec^{-1}$ or hertz (Hz). The frequency ν and the time period T are obviously related by the equation

$$\nu = \frac{1}{T} \qquad (8.2)$$

(v) Displacement

Displacement of a vibrating body at any instant is its distance from the equilibrium position at that instant.

(vi) Amplitude

It is the maximum displacement of the body on either side of its equilibrium position. In Fig. 8.2 x_0 is the amplitude as it is the maximum displacement on either side of its equilibrium position. In order to get an insight into SHM we shall correlate it with uniform circular motion which we have already studied.

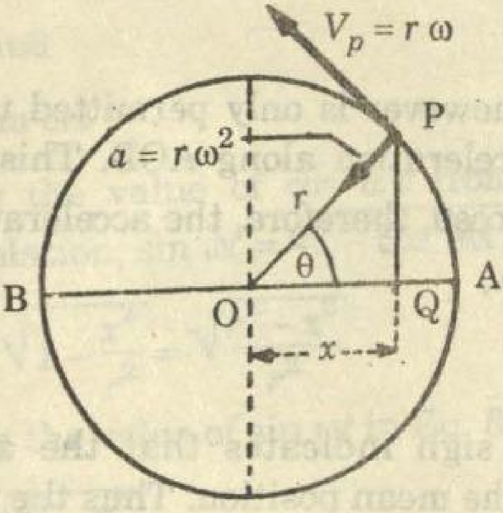


Figure 8.3 *As point P moves over a circle with uniform speed, the point Q which is projection of point P on the diameter AB, describes simple harmonic motion.*

Consider a point P moving along a circular trajectory around the centre O with angular speed ω (Fig. 8.3). The radius of the circle is r, so the speed of the moving point P is $V_p = r\omega$. If A is the initial position of the point P, the angle θ which the radius OP sweeps out in the time t is $\theta = \omega t$.

Consider the motion of the point Q, the projection of P on the diameter AB. As P describes uniform circular motion with a constant angular speed ω, Q oscillates to and fro along the diameter. At some given instant t, Q is at a distance x from the centre O. Its maximum displacement from O is OA or OB which is the radius r. Does the point Q execute SHM? How can we tell that? We should determine its acceleration. If it is always directed towards O and has a magnitude proportional to x, then the motion of Q would be simple harmonic motion.

It can be seen that as Q moves away from O, it slows down, and as it moves towards O it speeds up, that is, it has an acceleration directed towards O. What is the magnitude of its acceleration? The acceleration of the point P is directed inwards along PO and is of magnitude

$$\frac{V_p^2}{r} = r\,\omega^2$$

The point Q, however is only permitted to have a component of this acceleration along AOB. This component is $r\,\omega^2\cos\theta$. As $x = r\cos\theta$, therefore, the acceleration a of Q has magnitude $\omega^2 x$ i.e.,

$$a = -\,\omega^2 x \qquad (8.3)$$

The negative sign indicates that the acceleration is directed towards the mean position. Thus the point Q has an acceleration proportional to displacement and directed towards the mean position, which is the characteristic of *SHM*. Hence we can say that, the motion of the point Q is also *SHM*. The time period T of Q is obviously the time in which the point P completes its one round of the circle.

$$\therefore\ T = \frac{2\pi}{\omega} \qquad (8.4)$$

It may be noted that ω is the square root of the proportionality constant that occurs in the expression of acceleration of SHM and it represents the angular speed of the point P. Eq. 8.4 also shows that the time period is independent of the amplitude r of the motion. Referring to Fig. 8.3, it can be seen that the instantaneous displacement of the point Q is

$$x = r\cos\theta$$

$$= r\cos\omega t \qquad (8.5)$$

The angle θ or ωt which specifies the displacement of the point executing *SHM* is usually referred as the *phase.*

The speed of the point Q is the component of the speed of P directed along AB. If V be the speed of the point Q when the speed of the point P is V_p, then from Fig. 8.3 we have

$$V = V_p \sin\theta$$

$$= \omega r \sin\theta$$

$$= \omega r \sin \omega t \tag{8.6a}$$

Substituting the value of $\cos \omega t$ from Eq. 8.5 in the trigonometric relation, $\sin \omega t = \sqrt{1 - \cos^2 \omega t}$, we have

$$\sin \omega t = \sqrt{1 - \frac{x^2}{r^2}} = \sqrt{\frac{r^2 - x^2}{r^2}}$$

Substituting the value of $\sin \omega t$ in Eq. 8.6(a) we have

$$V = \omega r \sqrt{\frac{r^2 - x^2}{r^2}} \tag{8.6b}$$

$$= \omega \sqrt{r^2 - x^2} \tag{8.6c}$$

Using equations (8.4), (8.5) and (8.6) we can evaluate the time period, instantaneous displacement and instantaneous velocity of the mass attached to one end of a spring. Comparing equations (8.1) and (8.3),

$$\omega = \sqrt{\frac{\mathrm{k}}{m}} \tag{8.7}$$

Therefore the time period of the mass attached to the end of spring is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{\mathrm{k}}} \tag{8.8}$$

Replacing the amplitude r by x_0 the instantaneous displacement of the mass m is

$$x = x_o \cos\omega t$$

$$x = x_o \cos\sqrt{\frac{k}{m}} \times t \tag{8.9}$$

Similarly the instantaneous velocity

$$V = \omega\sqrt{r^2 - x^2}$$

$$= \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$$

$$= x_o\sqrt{\frac{k}{m}}\sqrt{1 - \frac{x^2}{x_o^2}} \tag{8.10}$$

Eq. (8.10) shows that velocity of the mass is maximum when $x = 0$. Representing the maximum velocity by v_o, we have

$$V_o = x_o\sqrt{\frac{k}{m}} \tag{8.11}$$

$$V = V_o\sqrt{1 - \frac{x^2}{x_o^2}} \tag{8.12}$$

Knowing the instantaneous velocity, we can compute the kinetic energy of the mass at the instant t.

$$\therefore KE = \frac{1}{2}mV^2$$

$$= \frac{1}{2}m \times x_o^2\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)$$

$$KE = \frac{1}{2}k\,x_o^2\left(1 - \frac{x^2}{x_o^2}\right) \tag{8.13}$$

Eq. (8.13) shows that kinetic energy is maximum when $x = 0$ i.e., when the mass m is passing through its mean position.

$$\therefore (KE)_{\max} = \frac{1}{2}\,\mathrm{k}\,x_o^2 \qquad (8.14)$$

The kinetic energy will have a minimum value of zero when $x = x_0$ i.e., when the mass is at its extreme position.

According to equation F = kx the force on the mass is zero when $x = 0$ and it is kx when the displacement of the mass is x. Hence the average force acting on the mass during the displacement x is ½ kx. Therefore the work done in displacing the mass through x is $\frac{1}{2}\,\mathrm{k}x \times x = \frac{1}{2}\,\mathrm{k}x^2$. This will appear as the potential energy of the mass. Thus the potential energy of the mass when it has a displacement x is

$$PE = \frac{1}{2}\mathrm{k}x^2 \qquad (8.15)$$

Equation (8.15) shows that the potential energy is maximum when the mass is at its extreme position i.e. when $x = x_o$.

$$\therefore (PE)_{\max} = \frac{1}{2}\,\mathrm{k}\,x_o^2 \qquad (8.16)$$

The potential energy has a minimum value of zero when $x = o$ i.e., when the mass is at the mean position. At any displacement x, the energy of the mass is partly kinetic and partly potential. So the total energy at the displacement x

$$= PE + KE$$

$$= \frac{1}{2}\,\mathrm{k}x^2 + \frac{1}{2}\,\mathrm{k}x_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$= \frac{1}{2}\,\mathrm{k}x_o^2$$

Thus we see that the total energy of the mass remains constant everywhere. At mean position the potential energy is zero and the energy is wholly kinetic. At extreme positions, the energy is wholly potential. Note that energy oscillates back and forth between kinetic energy and potential energy but the total energy remains the same.

In this way equations (8.4), (8.5) and (8.6) can be used to determine the various characteristics of any system performing *SHM* just by expressing ω and *r* in terms of suitable parameter of the system.

Example 8.1

A force of 0.04 N is requird to displace a body attached to a spring through 0.1 *m* from its equilibrium position. Calculate the spring constant of the spring.

We know that the applied force

$$F = kx$$

$$\text{or } k = \frac{F}{x}$$

since $F = 0.04$ N

and $x = 0.1\ m$

$$\therefore k = \frac{0.04}{0.1}$$

$$= 0.4\ \text{Nm}^{-1}$$

Example 8.2

A body of mass 0.025 kg attached to a spring is displaced through 10 cm to the right of its equilibrium position. If the spring constant of the spring is 0.4 Nm^{-1} and its velocity at the end of this displacement be 0.4 *m* sec^{-1} calculate: (i) time period T (ii) the frequency (iii) the angular speed ω (iv) the total energy (v) the amplitude of its motion (vi) the maximum velocity (vii) the maximum acceleration.

(i) Time Period

since $T = 2\pi\sqrt{\frac{m}{k}}$

and $m = 0.025$ kg, $k = 0.4$ Nm^{-1}

$$\therefore T = 2\pi\sqrt{\frac{0.025}{0.4}}$$

$$= 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}$$

or T = 1.57 sec

(ii) Frequency

$$\nu = \frac{1}{T} = \frac{2}{\pi}$$

$\nu = 0.638$ vib sec^{-1}

(iii) Angular Speed (ω)

$$\omega = 2\pi\nu = 2\pi \times \frac{2}{\pi}$$

$= 4$ rad sec^{-1}

(iv) Total Energy

Total energy is equal to the sum of potential energy and kinetic energy

$$\therefore \text{Total energy} = \frac{1}{2}kx^2 + \frac{1}{2}mV^2$$

But $k = 0.4$ Nm^{-1}, $x = 0.1\,m$ and $V = 0.4\,m$ sec^{-1}

$$\text{Total energy} = \frac{1}{2} \times 0.4 \times (0.1)^2 + \frac{1}{2} \times 0.025 \times (0.4)^2$$

$$= 4 \times 10^{-3}\ \text{joules}$$

(v) Amplitude of Maximum Displacement

Total energy is equal to the maximum potential energy.

$$\therefore \text{ Total energy} = \frac{1}{2}\text{k}x_o^2$$

$$= 4 \times 10^{-3} \text{ joules}$$

As $\text{k} = 0.4 \text{ N}/m$

$$\therefore x_o^2 = \frac{2 \times 4 \times 10^{-3}}{4 \times 10^{-1}} = 0.02$$

$$\therefore x_o = \sqrt{0.02} = 0.1414 \text{ m}$$

(vi) Maximum Velocity V_0

$$V = V_o\sqrt{1 - \frac{x^2}{x_o^2}}$$

$$V_0 = \frac{V}{\sqrt{1 - \frac{x^2}{x_o^2}}}$$

But $V = 0.4\, m \text{ sec}^{-1}$ $x = 0.1\, m$, $x_o = \sqrt{0.02}$

$$\therefore V_0 = \frac{0.4}{\sqrt{1 - \frac{0.01}{0.02}}} = 0.57 \text{ m sec}^{-1}$$

(vii) Maximum Acceleration;

$$\text{Now } a = -\frac{\text{k}}{m}x$$

$$\therefore a_{\max} = -\frac{\text{k}}{m}x_o$$

$$= -\frac{0.4}{0.025} \times \sqrt{0.02}$$

$$= -2.26 \text{m sec}^{-2}$$

8.4 WAVEFORM OF SHM

We have seen that the displacement of a body executing *SHM* is given by

$$x = r \cos \omega t$$

$$\text{where } \omega = \frac{2\pi}{T}$$

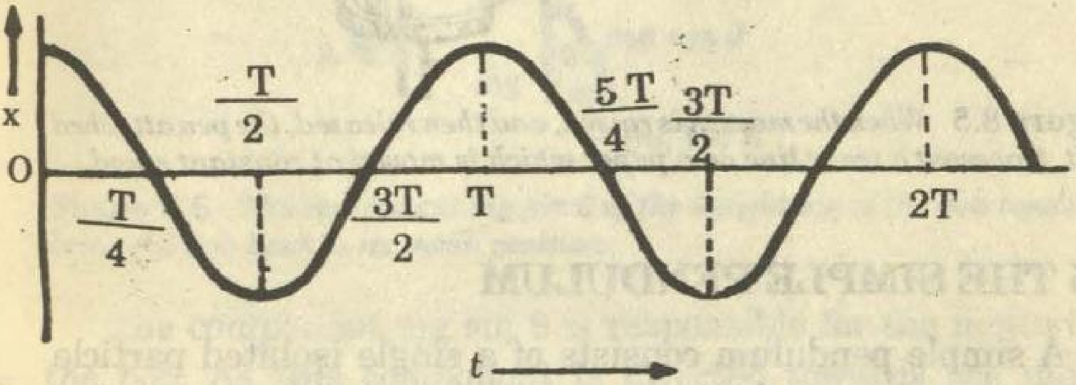


Figure 8.4 *Waveform of simple harmonic motion.*

If we determine the value of x at different instants and plot it against time, we get a curve as shown in Fig. 8.4. This curve which gives the displacement as function of time is known as the waveform of *SHM*. The waveform of *SHM* can also be experimentally traced by the following experiment.

Suppose, a small, mass m is hung from a vertical spring. A sheet of paper is placed behind the mass and there is an arrangement to move the paper at a constant speed (Fig. 8.5). The mass is provided with a pen which slightly touches the paper. The pen would mark the position of the mass m on the paper at each instant. The mass is raised a distance y and then released. It executes simple harmonic motion of amplitude y and the pen will trace out its waveform on the paper as shown in Fig. 8.5.

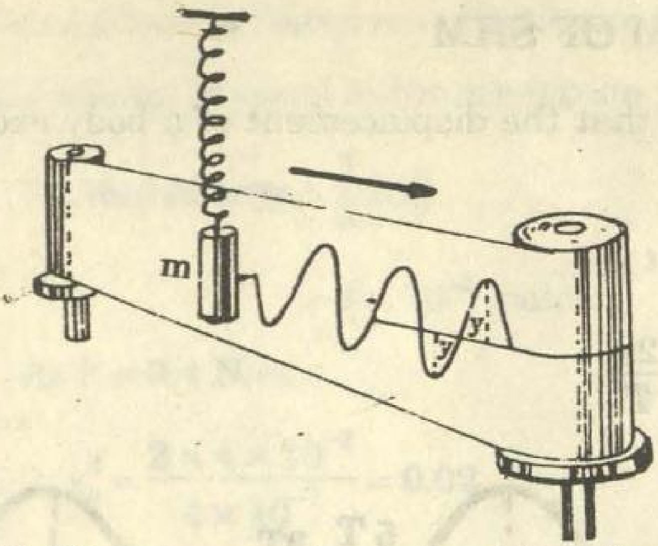


Figure 8.5 *When the mass m is raised, and then released, the pen attached to it, tracesout a wavy line on a paper which is moved at constant speed.*

8.5 THE SIMPLE PENDULUM

A simple pendulum consists of a single isolated particle suspended from a frictionless support by a light, inextensible string. When it is disturbed from its mean position, it perform a vibratory motion. We can identify its motion by calculating its acceleration. Suppose the bob is at the position B during its vibratory motion as shown in Fig. 8.6. Consider the forces acting on the bob at B. There are two forces acting upon it, one is the weight of the bob *mg* acting vertically downward and the other is the tension *T* acting along the string. The weight *mg* can be resolved into two rectangular components, one along the string and the other perpendicular to it.

Component of weight along the string $= mg\cos\theta$

Component of weight perpendicular to the string $= mg\sin\theta$

Since there is no motion along the string, the component $mg\cos\theta$ must balance the tension T.

$$T = mg\cos\theta$$

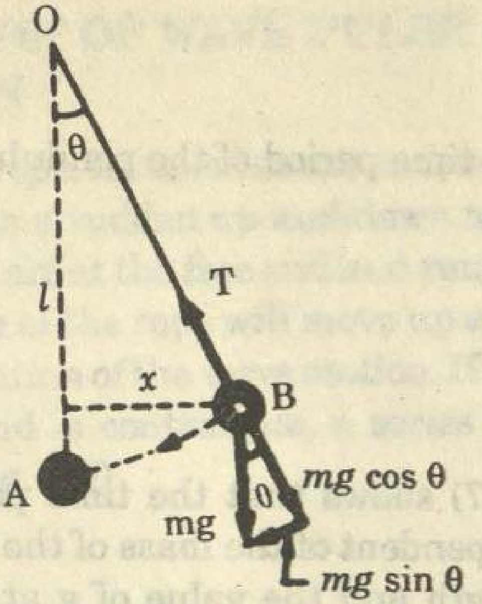


Figure 8.6 *The component mg sin θ of the weight mg of the bob tends to bring the bob back to its mean position.*

The component mg sin θ is responsible for the motion of the bob. As this component is directed towards the mean position, so we shall assign a negative sign to it. If a is the acceleration at B, then

$$ma = -\,mg\sin\theta$$

$$a = -\,g\sin\theta$$

$$= -\,g\theta$$

(since sin θ = θ when θ is small)

$$\therefore\ a = -\,g\frac{x}{l}\left(as\ \theta = \frac{x}{l}\ radians\right)$$

$$= -\,\frac{gx}{l}$$

As the acceleration of the bob is directed to the mean position and it is also proportional to displacement *x,* hence the motion of the bob is simple harmonic. In this case the proportionality constant

$$\omega^2 = \frac{g}{l}$$

Therefore the time period of the pendulum is

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

$$= 2\pi\sqrt{\frac{l}{g}} \qquad (8.17)$$

Equation (8.17) shows that the time period of a simple pendulum is independent of the mass of the bob and depends only upon its length and the value of g at that place. This enables us to measure g at a place with a simple pendulum since both T and l can be directly measured. If proper precautions are taken, this method for the determination of g is very accurate.

A Second's Pendulum is a pendulum which completes one vibration in two seconds. Thus the time period of such a pendulum is two seconds.

Example 8.3

A simple pendulum completes one vibration in one second. Calculate its length when g = 981 cm sec^{-2}.

We have g = 981 cm sec^{-2}, T = 1 sec, l = ?

$$\text{since } T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore l = \frac{T^2}{4\pi^2} \times g$$

$$= \frac{1}{4\pi^2} \times 981$$

$$= 24.8\ cm.$$

8.6 GENERATION OF WAVE PULSE & ITS PROPAGATION

If an ordinary rope, fastened at one end, is stretched across the room and given a sudden up-and-down motion at the free end, a wave will start at the free end and run the length of the rope. Each portion of the rope will move up and down perpendicular to the direction of the wave motion. If the up-and-down motion of the hand is continuous, a series of waves travels down the rope (Fig. 8.7).

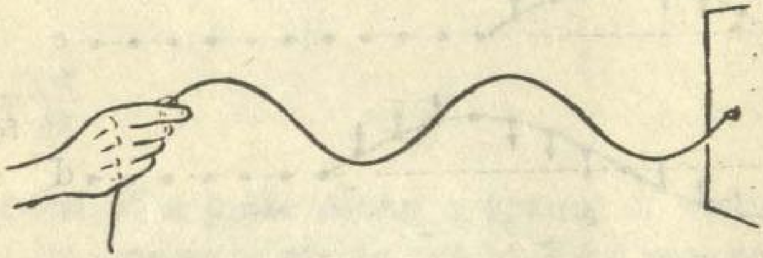


Figure 8.7 *Generation of a wave pulse*

In order to study the process by which a transverse wave advances, let us refer to the mechanical model of Fig.8.8. A long elastic cord, with markers, 1,2,3,4, etc. equally spaced along its length to represent the vibrating particles, is attached to a rigid support on one end and is free to oscillate at the other.

If particle 1 is given an upward motion at right angle to the cord, particle 2 will follow it because of the elasticity of the cord, but it will lag behind particle 1 because of its inertia. As particle 2 rises, particle 3 will follow but will lag behind particle 2. Particle 4 will lag behind 3, and so on. Let us assume that when 1 reaches its highest positon (+A), 5 is just begin to be raised, and at that instant 1 starts its downward motion. Particle 2 will continue in an upward motion, because of its inertia, until it reaches the height (+A); it will then begin to descend, while particles 3,4,5....follow the same pattern, moving upward because of elasticity and inertia until (+A) is reached and then beginning downward movement because of the elastic forces of the cord to the left.

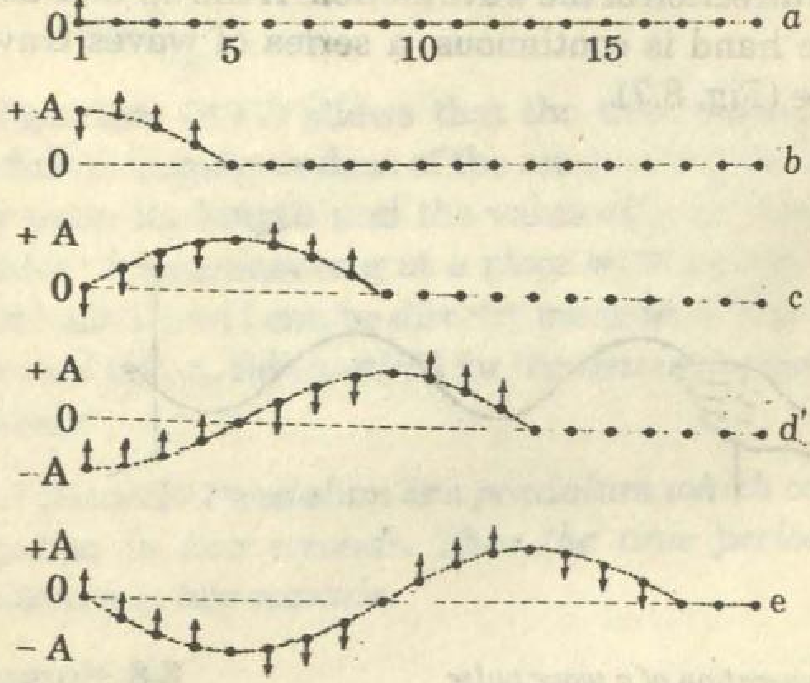


Fig.8.8

When particle 1 retuns to its original position (moving in a downward direction), the cord will have the form indicated in Fig. 8.8c. As 1 continues downward, the others follow, but always lag in motion a bit behind the particles to the left.

When particle 1 has gone through a complete vibration the particles are distributed as shown in Fig.8.8e. Particle 1 is now ready to start its second vibration. As can be observed from the figure, the wave has advanced to particle 17. As the particles continue their up-and-down motion, the wave advances to the right.

All types of mechanical wave pulses-whether on springs or strings, on water, or in the air-are characterized by the transfer of motion and energy from particle to particle in the medium, but in no case does any part of the medium move any appreciable distance. Waves transport energy, not matter.

8.7 WAVE SPEED

The speed of a wave pulse can be determined in the same way that we would determine the speed of a moving object. We can record the position of the wave pulse at several instants separated by equal intervals of time, Δt. In each such interval, the peak of the wave pulse moves forward a distance Δx. Therefore, according to the usual definition of speed, we can write

$$V = \frac{\Delta x}{\Delta t} \qquad (8.18)$$

The speed of a pulse along a spring or string will be constant if the spring or string has uniform properties along its length.

8.8 TRAVELING WAVES: PERIOD AND WAVELENGTH

A wave pulse is formed when a sharp displacement is given to the end of a spring or string. What will happen if we displace the end in simple harmonic motion? Fig. 8.9 shows that a repeating pattern of sinusidal wave pulses is formed. Each pulse is exactly the same as any other pulse and each joins smoothly onto the one in front of it and the one behind. The train of pulses moves uniformly along the string and the result is called a traveling wave and has the shape of a sine wave.

The driven end of the string in Fig. 8.9 moves up and down, and back to the starting position in a time interval indicated by T. The time T is the period of the driving force and is also

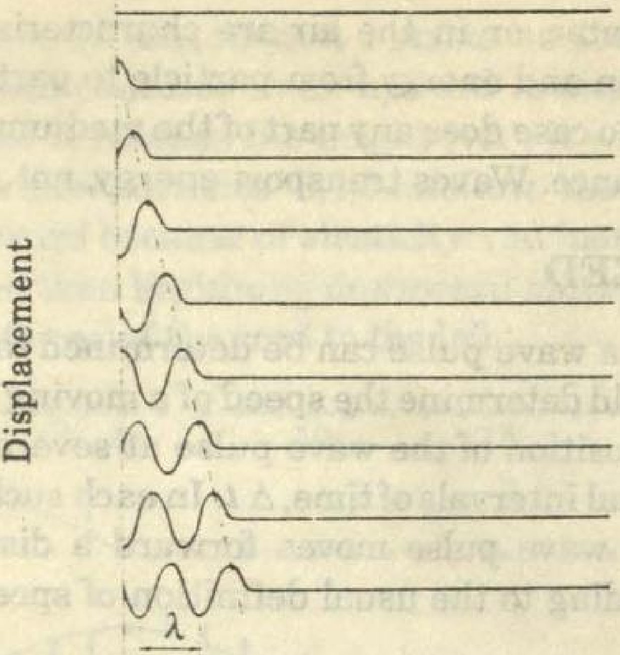


Fig.8.9 *Driving the end of a string up and down in simple harmonic motion forms a sinusoidal traveling wave in the string. The period of wave is T and the wavelength is λ. The dashed line shows the steady movement forward of the first wave maximum.*

the period of the traveling wave. That is, at any point along the string, the motion repeats itself in every time interval equal to T. During each interval T, the wave form moves forward by a certain distance, indicated by λ in Fig. 8.9. If We examine the shape of the string at any instant, we find that the pattern repeats itself in every distance interval λ. *This distance λ is called the wavelength of the traveling wave.* If the wave moves through a distance $\Delta x = \lambda$ in an interval of time T, we can use Eq.8.18 to express the relationship connecting wave speed, period, and wavelength.

$$\text{Wave speed} = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$$

$$= \frac{\textit{wave length}}{\textit{period}}$$

8.9 FREQUENCY

The period of a wave is the time period between the passage of two successive wave crests past a given point. The frequency of a wave is how frequently the wave crests pass a given point. If 100 wave crests pass a point in 1 second, the frequency is 100 per second (100 sec^{-1}). The time interval between the passage of successive wave crests past the point is $\frac{1}{100}$ of a second; therefore, the period is 0.01 sec. That is, the frequency ν is the reciprocal of the period T:

$$\nu = \frac{1}{T} \qquad (8.19)$$

Frequency is usually expressed in cycles per second. This unit of frequency is called the hertz (Hz). We can now combine Eqs. 8.19 and 8.20 and to write the wave speed as

$$V = \frac{\lambda}{T} = \nu\,\lambda \qquad (8.20)$$

This is a general expression and is valid for all types of wave motion-waves on spring, sound waves, water waves, and radio waves.

8.10 TRANSVERSE WAVES

When a pebble is dropped into a quiet pond, circular pattern spreads out from the point of impact. The pattern is that of water waves and we can see the waves moving on the surface of water. In order to investigate whether the water on the surface also moves when a wave is passing over it, we can drop a piece of cork on the surface of water and watch its motion. The motion of the cork as the wave passes across it, is shown in Fig. 8.10. It can be seen that the cork moves up and down only, executing a simple harmonic motion but it does not bodily move along with the wave. Such waves in which the particles of the medium vibrate along a line at right to the direction of propagation of waves, are known as *transverse*

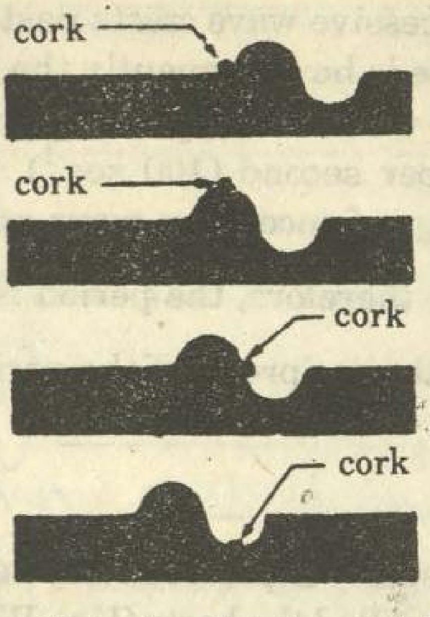


Fig.8.10 *The cork oscillates in a direction at right angles to the direction of propagation of waves as the wave passes over it.*

waves. Thus water waves are transverse in nature. The portion of the water above its mean level is said to form a *crest* and the portion below the mean level forms a *trough.*

Ripple Tank

The various features of wave phenomenon can be very conveniently studied by an apparatus known as *ripple tank* as shown in Fig. 8.11

It consists of a rectangular tray containing water, fitted with glass bottom. The transparent glass bottom makes it possible to project image of waves formed on the surface of water in the tray on a screen placed well below the glass bottom. The image is projected because the crests of the water act as converging lenses and tend to focus the light from the lamp while the troughs, acting as diverging lenses, tend to spread it with the result that the crests appear as bright bands on the screen while the troughs appear dark. A single circular pulse can be produced by simply dipping a finger into the water. The crest will appear as a bright circular band on the screen moving radially out. Similarly single straight pulse can be produced by taking a straight rod and dipping it into the water of the tray. However if water in the tray is disturbed periodically, we shall get waves one after the other passing through any point in the medium. Such waves are known as *transverse periodic waves.* Most of the waves that we usually come across in the actual physical world are periodic waves, so their detailed study is very essential.

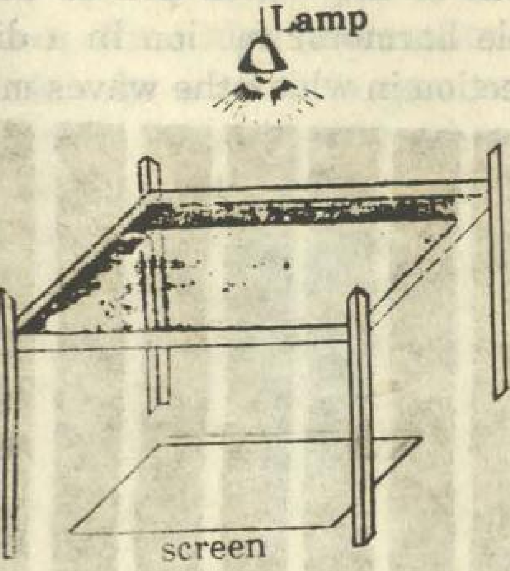


Fig.8.11 *The image of the wave is seen on the screen which is just a piece of white drawing paper placed under the ripple tank.*

Straight periodic waves can be generated in a ripple tank by dipping a straight rod periodically into the water. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rod can be altered by changing the speed of motor. The pattern of the wave that is obtained at any instant is as shown in Fig. 8.12.The distance between the centres of two successive bright bands which correspond to the tops of two successive crests is the wavelength λ. It should be noted that the pattern of waves as shown in Fig. 8.12 corresponds to a particular instant or in other words it is the photograph of the screen at an instant by a high speed camera. On the screen we can see the actual motion of the waves. They move in such a way that the distance between the successive bright bands or crests always remains equal to λ. To observe the motion of water (the medium), place a small piece of paper on the surface of water and also mark its shadow on the screen. Now generate waves one after the other by dipping the straight rod periodically into water. We shall notice that the piece of paper just moves up and down as each of the waves passes across it. Thus it performs a simple harmonic motion in a direction at right angles to the direction in which the waves move.

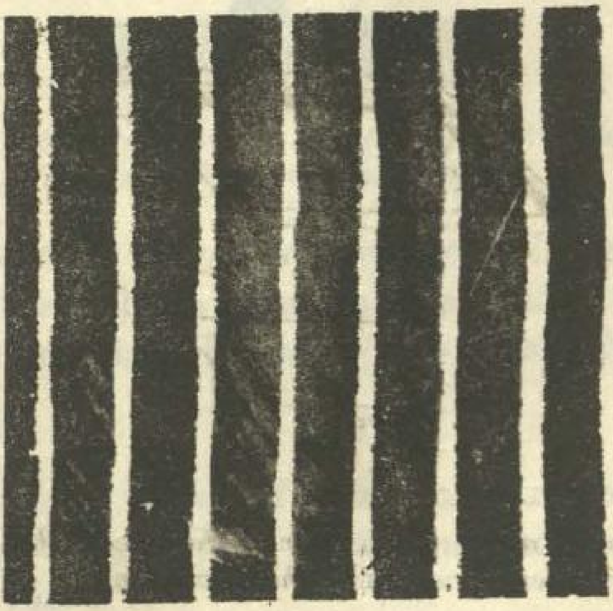


Fig.8.12 *Instantaneous photograph of moving periodic straight waves across a ripple tank.*

Now make the following observations:

(i) By means of a stop-watch measure the time period of the rod and that of the piece of paper. The two will be identical. Thus the particles of the medium execute *SHM* with the same period as that of the body generating the waves.

(ii) Now concentrate on the shadow of the piece of paper and count the number of waves that pass across it in a certain interval of time. It will be found that it is the same as the number of times the rod dips into the water in that interval of time. Hence, if we define the frequency of waves 'v' as the number of waves that pass across a certain point in the medium in one second, we can say that it is the same as the frequency of the body generating the waves. From this observation we can also find the time taken by a wave to pass across the piece of paper. This time comes out to be the same as the time period T of the rod. It may be noted that the distance between two consecutive waves is λ, therefore, when one wave passes across a certain point, the distance moved by it is λ. If V is the velocity of waves, then

$$VT = \lambda$$

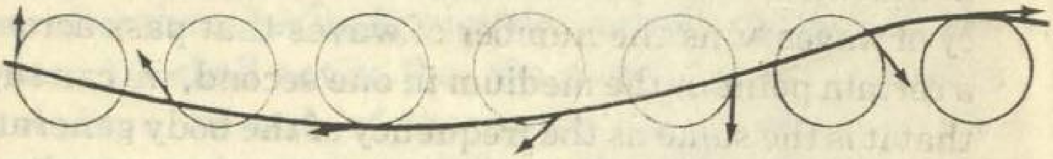
$$V = \frac{\lambda}{T} \qquad \nu = \frac{1}{T}$$

$$\therefore V = \nu\lambda \tag{8.21}$$

8.11 WATER WAVES

The surface waves that propagate across deep water are similar to the waves on springs, but water waves are not exactly transverse. Fig. 8.13 shows the motions of the water particles that take place when a wave moves on the surface of a body of water. The particles move around in small circular orbits as the wave advances. The particles at the top of the wave (the wave crest) move in the same direction as the wave, but the particles at the bottom of the wave (the wave trough) move in the opposite direction. As in the case of waves on springs, there is no net forward movement of the particles as a water wave propagates.

In an ideal water wave the particles do not undergo any net displacement and the wave cannot transport matter. But



circular orbits of water particles

Fig.8.13 *Water waves*

real waves are not perfect, and frictional drag effects combine with winds to permit the movement of matter by the waves. Driftwood is continually carried on to the beaches by the action of waves, and coastal areas undergo frequent changes as the beach sands are moved by waves.

8.12 OCEAN WAVES

Surface waves on open water build up due to the action of winds. But anyone who has ever seen the sea knows that the waves are not the simple ideal waves we have been discussing. The winds are variable in direction and strength, and ocean waves are therefore complex and irregular, ranging from ripples and chop to giant crashing waves. In describing these waves, we can only refer to average properties.

The size of wind-driven waves depends on the speed and the duration of the wind. In a strong, steady wind of 50km hr^{-1}, the waves will continue to grow for a day, eventually reaching an average height of about 4 *m*. For whole gale winds (90 km hr^{-1}), the average height of waves is about 13 metres or 45 feet.

8.13 WAVES AS CARRIERS OF ENERGY

It is easy to understand that a wave with a large height possesses more energy than a wave with a lesser height. How does wave energy depend on the height (or amplitude) of the wave? We can deduce the relationship by referring to Fig. 8.14. Here we have two wave patterns (drawn as rectangular pulses for simplicity), the second with twice the amplitude of the first. In order to form the wave pulse, a certain amount of work must be done. If we are considering a water wave, a volume of water must be raised from beneath the normal surface to a height equal to the amplitude of the wave. For the wave with an amplitude of I unit, this amounts to 1 unit of work (Fig. 8.14a). If the wave amplitude is increased to 2 units (Fig. 8.14b) an additional equal volume of water must be raised through a height 3 times greater; this requires 3 units of work. Therefore, the formation of a wave pulse with an amplitude of 2 units requires 4 times as much work as does a pulse with an amplitude of 1 unit. If we consider a pulse with an amplitude

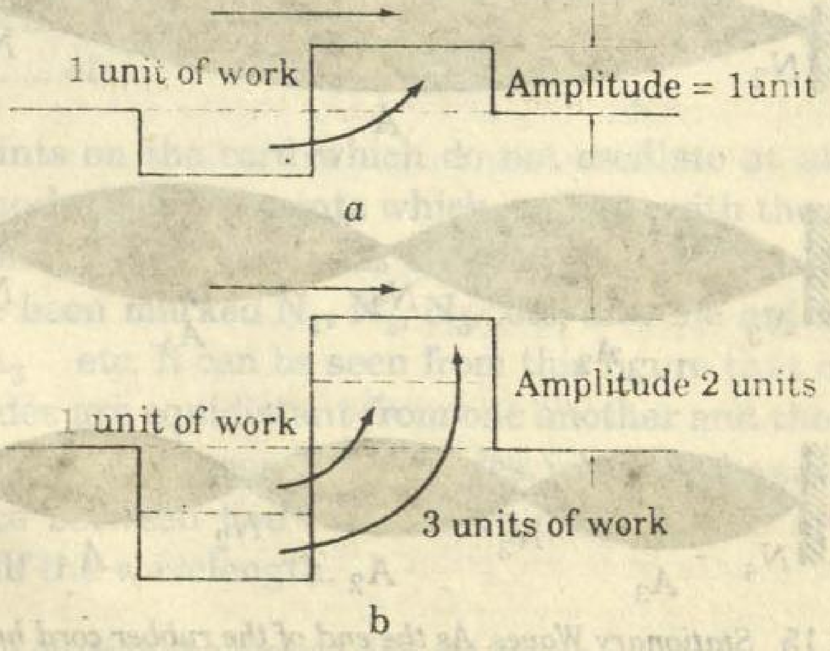


Fig.8.14 *The work required to form a wave with an amplitude of 2 units (b) is 4 times that required to form a wave with an amplitude of 1 unit (a). The energy content of a wave is proportional to the square of its amplitude.*

amplitude of 3 units, we would find that the work required is 9 times that for the first pulse. That is, the work required to form a wave pulse, and consequently the energy content of the wave, is proportional to the square of the amplitude. This relationship is valid for all types of of waves.

8.14 STANDING OR STATIONARY WAVES

Tie a rubber cord with a firm support and wiggle its other end by holding it in your hand. Now stop the motion of the hand. You will notice that the wave that was set up along the cord subsides. Again repeat the process by a different wiggling frequency. It will be seen that at a particular frequency, say v_1, the cord, even after the motion of the hand has been stopped, continues to oscillate as shown in Fig. 8.15a. Except for the friction losses, the motion persists, with the cord oscillating in the same fixed pattern.

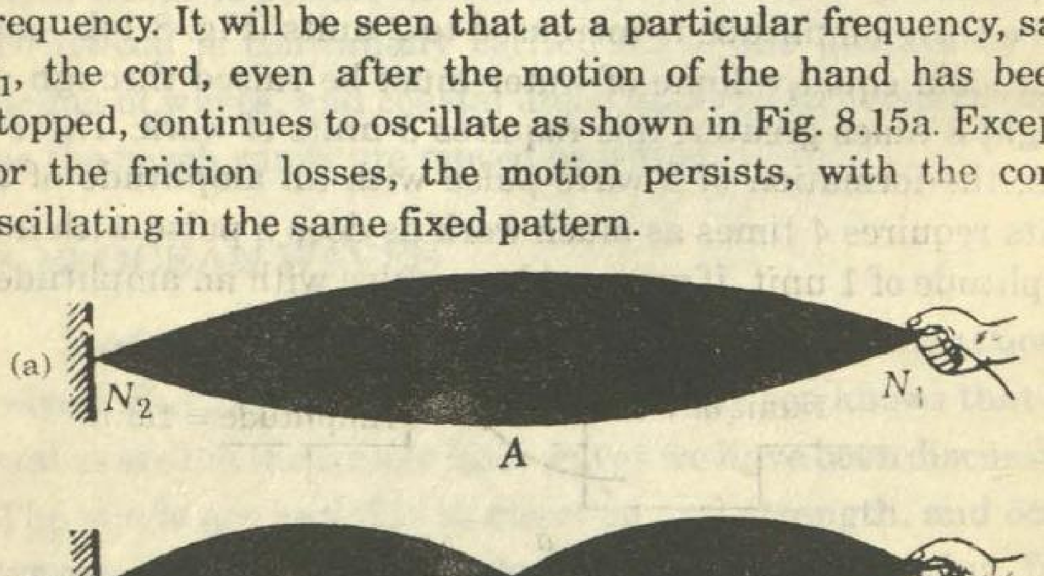


Fig.8.15 *Stationary Waves. As the end of the rubber cord held in hand, is wiggled with increasing frequency, the cord vibrates in more and more loops.*

Also in the spring under tension we have seen that whenever a spring under tension is plucked at one end, a transverse wave is set up which we can actually see moving along the spring. In the present case we do not see any wave moving along the cord, but instead, the whole of it continues to oscillate in one loop, with its two ends being at rest and the central portion vibrating with maximum amplitude. We can say that a wave has been set up on the cord that keeps it oscillating in this fashion. This wave is known as *stationary wave or standing wave* because no wave, moving on the cord, is visible.

If we increase the wiggling frequency beyond v_1, the stationary waves will not be set up, unless the frequency of the motion of the hand is 2 v_1. This time the cord will oscillate in two loops as shown in Fig. 8.15b. The two ends of the cord and its mid-point do not move at all. The points, A_1 and A_2 oscillate with the maximum amplitude. Similarly as the wiggling frequency is further increased, stationary waves are set up at the freqency 3 v_1. Now the cord oscillates in three loops as shown in Fig. 8.15c. The points N_1, N_2, N_3 and N_4 always remain at rest and the points A_1, A_2 and A_3 oscillate with the maximum amplitude. Generalising this experience we can say that if the wiggling frequency is n v_1, stationary waves will be set up with the cord oscillating in n loops where n is an integer equal to 1,2,3,4,5,........ etc.

The points on the cord which do not oscillate at all are known as *nodes* and the points which oscillate with the maximum amplitude are known as *antinodes*. In Fig. 8.15 the nodes have been marked N_1, N_2, N_3...etc, and the antinodes as A_1, A_2, A_3 etc. It can be seen from this figure that nodes and antinodes are equidistant from one another and that the distance between two consecutive nodes, which is the same as the distance between two consecutive antinodes, is always equal to half the wavelength.

Actually stationary waves are set up as a result of superposition of two exactly similar waves moving along the same line but in opposite directions. When we wiggle one end of the cord, transverse waves are set up which move towards the other end. When these waves reach the other end, they are reflected back.

These reflected waves along with the incoming waves from the end of cord which is held in hand, provide the two similar waves which while moving on the same cord but in opposite directions, superpose to give rise to stationary waves.

The phenomenon of stationary waves takes place in any medium wherever its particles are simultaneously agitated by two similar waves moving along the same line in opposite directions. The presence of stationary waves in a medium can be easily detected by the fact that the particles at the nodal points will be at rest and the particles at the antinodes will be vibrating quite strongly.

It should be noted carefully that in any medium stationary waves of all frequencies cannot be set up. The waves having a discrete set of frequencies only, can be set up in the medium. This fact is also known as *"quantization of frequencies."* Now we shall consider the stationary waves set up in different bodies and shall calculate the quantized frequencies of each.

8.15 TRANSVERSE STATIONARY WAVES IN A STRETCHED STRING

Consider a string of length l which is kept stretched by clamping its two ends so that the tension in the string is T. If the string is plucked at its middle point, two transverse waves will originate from the point. One of them will move towards the left end of the string and the other towards its right end. When these waves reach the two clamped ends, they are reflected back thus giving rise to stationary waves. The string will vibrate with such a frequency, say v_1, so that nodes are formed at the two clamped ends and the middle point of the string becomes an antinode. Thus the string vibrates in one loop as shown in Fig. 8.16. It should be noted that the string was plucked from the position of the antinode. As the distance between two consecutive nodes is half the wavelength, and λ_1 is the wavelength in this mode of vibration, so

$$l = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2\,l \tag{8.22}$$

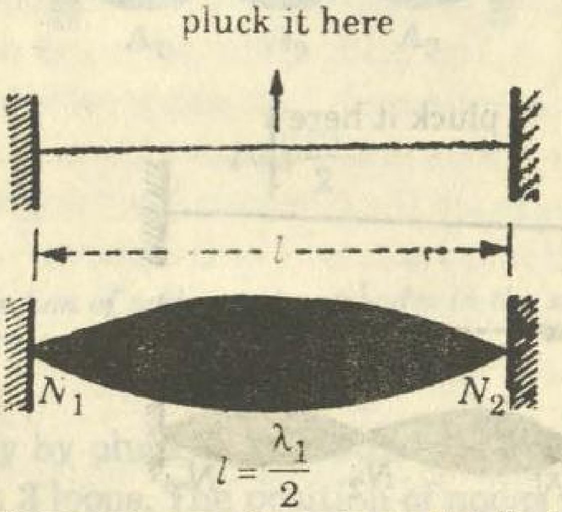


Fig.8.16 *Stationary waves set up in a stretched string with its two clamped ends as nodes and the centre as an antinode, are shown.*

If V is speed of either of the component waves, then from the equation

$$V = v_1 \lambda_1 = v_1 \times 2\,l$$

$$\text{or } v_1 = \frac{V}{2l} \tag{8.23}$$

If m' is the total mass of the string, it can be shown that the velocity V of the wave along the string is given by

$$V = \sqrt{\frac{T \times l}{m'}} \tag{8.24}$$

where T is the tension in the string.

Substituting the value of V in equation (8.23) we get

$$\nu_1 = \frac{1}{2l}\sqrt{\frac{T \times l}{m'}} \qquad (8.25)$$

If the same string is plucked from one quarter of its length then again stationary waves will be set up but now the string will vibrate with another frequency ν_2 and the position of nodes and antinodes will be as shown in Fig. 8.17. If λ_2 is the wavelength in this mode of vibration, then

$$\lambda_2 = l \qquad (8.26)$$

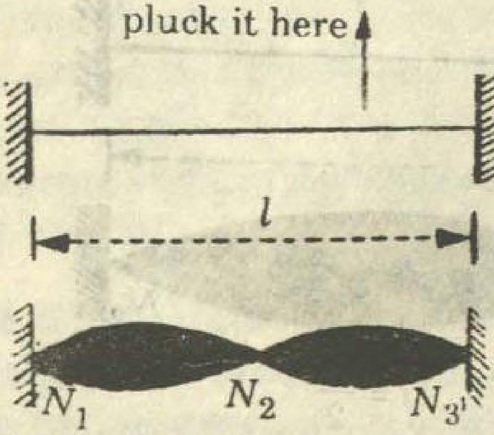


Fig.8.17 *When the string is plucked from one quarter of its length, stationary waves are set up with the string vibrating it two loops.*

A comparison of this equation with the equation (8.22), shows that the wavelength in this case becomes half that in the first case.

The equation (8.24) shows that the speed of wave depends upon the tension, length and total mass of the string and is independent of the point from where the string is plucked to generate the waves. So the speed of the wave in this case will be the same as that in the first case. Now by the equation (8.26) we have

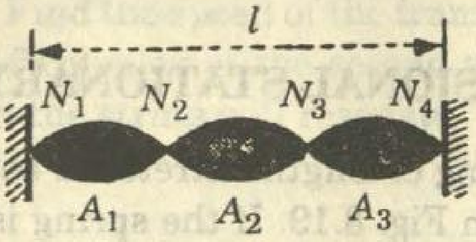
$$V = \lambda_2 \times \nu_2$$

$$= l \times \nu_2$$

$$\nu_2 = \frac{V}{l} \qquad (8.27)$$

Comparing it with equation (8.23) we get $\nu_2 = 2\nu_1$

Thus when the string vibrates in two loops, its frequency is double than when it vibrates in one loop.



$$l = 3\frac{\lambda_3}{2}$$

Fig.8.18 *Position of nodes and antinodes in the stationary waves when the string vibrates in 3 loops.*

Similarly by plucking the string properly, it can be made to vibrate in 3 loops. The position of nodes and antinodes will be as shown in Fig. 8.18. In this case the frequency of waves will be $\nu_3 = 3\nu_1$ and the wavelength will be equal to $\frac{2}{3}l$. Thus we can generalise that if the string is made to vibrate in n loops, then the frequency at which the stationary waves will be set up on the string will be $\nu_n = n\,\nu_1$, and the wavelength $\lambda_n = \frac{2}{n}l$. It is clear that as the string vibrates in more and more loops, its frequency goes on increasing and the wavelength gets correspondingly shorter. However the product of the frequency and wave-lengths is always equal to V, the velocity of waves.

Thus stationary waves can be set up on the string only with a discrete set of frequencies ν_1, $2\nu_1$, $3\nu_1$, $4\nu_1$ etc. The lowest of these is known as *fundamental* and the others which are the integral multiples of the fundamental are known as *overtones or harmonics.* If a stretched string is excited by a small periodic force having a frequency equal to any of the quantized frequencies of the string, the phenomenon of resonance will take place and stationary waves will be set up on the string.

8.16 COMPRESSIONAL STATIONARY WAVES

Consider a spring of length l stretched between two fixed clamps as shown in Fig. 8.19. If the spring is compressed at its centre and then released, two compressional waves will originate from the centre and will move towards the two ends. When these waves reach the two ends, they are reflected back, thus forming stationary waves. The frequency ν_1 of the waves is such that nodes are formed at the ends, that is the spring has no motion there and an antinode is formed at the centre. Now the central portion of the spring vibrates parallel to length of the spring with the maximum amplitude. If V is the speed of the waves, then as in case of stretched string, the

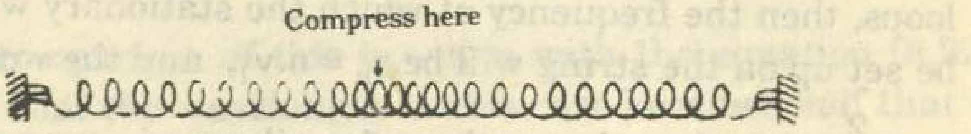


Fig.8.19 *The compressional stationary waves are set up in the spring by compressing it at its mid-point.*

frequency $\nu_1 = \frac{V}{2l}$. The other quantized frequencies will be $\nu_2 = 2\nu_1$, $\nu_3 = 3\nu_1$, and $\nu_n = n\nu_1$, where n is a positive integer. The various frequencies are generated depending upon the position of the initial compression.

Example 8.4

A string 2 *m* long and of mass 0.004 kg is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it. Find the speed of the transverse waves on the string and the frequency of the second and the fourth harmonics to which the string will resonate.

Solution

The speed of a transverse wave is given by equation

$$V = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Mg}{m/l}}$$

where T = Mg tension in the string
M = the total mass suspended from the string = 1 kg
g = the acceleration due to gravity = 9.8 m sec^{-2}

Substituting the various values, we get

$$V = \sqrt{\frac{1 \times 9.8}{0.004/2}}$$

$$V = 70 \text{ m sec}^{-1}$$

For resonance frequencies we know

$$\nu_n = n\,\frac{V}{2l}$$

For second harmonic (ν_2), $n = 2$

$$\therefore \nu_2 = \frac{2 \times 70}{2 \times 2} = 35 \text{ hertz}$$

For fourth harmonic (v_4), $n = 4$

$$\therefore v_4 = \frac{4 \times 70}{2 \times 2} = 70 \text{ hertz}$$

Example 8.5

A certain string resonates to several frequencies, the lowest of which is 50 hertz. What are the next four higher frequencies to which it resonates.

We have $v_n = n \frac{V}{2l}$, where $n = 1, 2, 3, 4, \ldots\ldots$

Therefore all the higher frequencies are integral multiples of the lowest frequency which corresponds to $n = 1$.

$v_1 = 50$ hertz

$v_2 = 2 \times v_1 = 2 \times 50 = 100$ hertz

$v_3 = 3 \times v_1 = 3 \times 50 = 150$ hertz

$v_4 = 4 \times v_1 = 4 \times 50 = 200$ hertz

$v_5 = 5 \times v_1 = 5 \times 50 = 250$ hertz

Example 8.6

The speed of a wave on a particular string is 24 m $\sec^{-1}$. If the string is 6 m long, to what frequencies will it resonate? Draw a picture of the string for each resonant frequency.

As $\lambda_n = 2l/n$, where $n = 1, 2, 3, 4, \ldots\ldots$

$$\text{therefore, } \lambda_1 = \frac{2 \times 6}{1} = 12\, m$$

$$\lambda_2 = \frac{2 \times 6}{2} = 6\, m$$

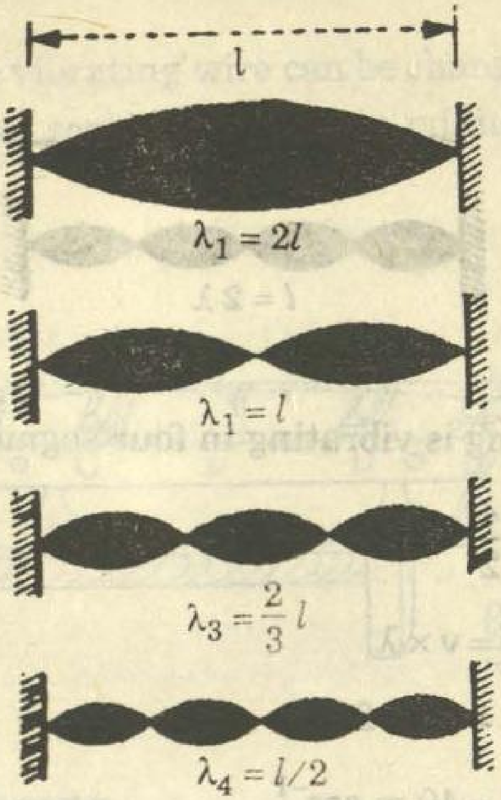

l
$\lambda_1 = 2l$
$\lambda_1 = l$
$\lambda_3 = \frac{2}{3}l$
$\lambda_4 = l/2$

Fig.8.20

$$\lambda_3 = \frac{2 \times 6}{3} = 4\, m$$

$$\lambda_4 = \frac{2 \times 6}{4} = 3\, m$$

As the frequency ν_n is given by

$$\nu_n = V/\lambda_n$$

$$\therefore\ \nu_1 = V/\lambda_1 = 24/12 = 2 \text{ hertz.}$$

$$\nu_2 = V/\lambda_2 = 24/6 = 4 \text{ hertz.}$$

$$\nu_3 = V/\lambda_3 = 24/4 = 6 \text{ hertz.}$$

$$\nu_4 = V/\lambda_4 = 24/3 = 8 \text{ hertz.}$$

Example 8.7

A spring 4 *m* long resonates in four segments (nodes at both ends). The frequency of driving system on the spring is 20 hertz. Calculate the speed of the wave in the spring.

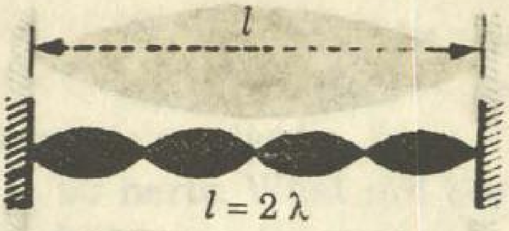


Fig.8.21

As the spring is vibrating in four segments, $l = 2\lambda$

$$\lambda = l/2 = \frac{4}{2} = 2\ m$$

again $V = \nu \times \lambda$

$$= 20 \times 2$$

$$= 40\ \text{m sec}^{-1}$$

We can obtain the same result from the equation

$$\nu = n\,\frac{V}{2l}$$

$$V = \frac{2\nu \times l}{n}$$

$$= \frac{2 \times 4 \times 20}{4} = 40\ \text{m sec}^{-1}$$

Thus speed of the wave in the spring = 40 m sec^{-1}.

8.17 SONOMETER

It consists of wooden box over which is stretched a steel wire, one end of which is fixed to a peg and the other end passes over a pulley and carries a hanger on which slotted weights can be slipped to vary the stretching force as shown in Fig. 8.22. Two sharp wedges C and D are placed below the wire. The distance between the wedges can be varied and thus the length of the vibrating wire can be changed and read from a fixed horizontal scale graduated in millimetres on the box below the wire.

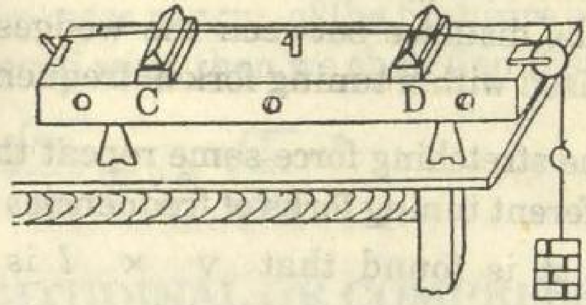


Fig.8.22 *The Sonometer*

Determination of Frequency

To find the frequency of a tuning fork a fine steel wire is stretched on the sonometer with a load of a few kilograms so that the wire remains taut. A V-shaped paper rider is placed in the middle of the wire between the two bridges. The tuning fork is struck gently with a rubber pad and its stem is pressed on the sonometer board. The distance between the wedges is adjusted till the rider flies off. The wire now vibrates in resonance with the tuning fork and hence produces a note of the same frequency. The length of the wire between the two knife edges is measured. A length 50 cms. of the wire is cut and weighed, from which the mass per unit length *m* is calculated. If T is the stretching force (tension), then the frequency ν is given by

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Verification of the laws of transverse vibration of strings

(a) To verify the law of length i.e., $\nu \propto \frac{1}{l}$

Place a suitable weight in the hanger so as to keep the wire taut. Adjust the distance between the wedges till the wire vibrates in unison with a tuning fork of frequency ν.

Keeping the stretching force same repeat the experiment with three different tuning forks of frequencies ν_1, ν_2, and ν_3. In each case it is found that $\nu \times l$ is constant or $\nu_1 l_1 = \nu_2 l_2 = \nu_3 l_3$

Hence $\nu \propto \frac{1}{l}$

(b) To verify the law of tension i.e., $\nu \propto \sqrt{T}$

Take two tuning forks A and B, frequency of B being double that of A e.g., 256. Place a suitable weight in the hanger so as to keep a wire taut. Adjust the distance between the wedges till the wire vibrates in unison with the tuning fork A of frequency 256. Now increase the load four times the previous value and find the length of the wire which vibrates in unison with the tuning fork B of frequency 512. It will be seen that the length of the wire in the second case is equal to that in the first within the experimental error. Thus when the load is increased to four times keeping the length of the wire constant the frequency is doubled. This shows that

$\nu \propto \sqrt{T}$

(c) To verify the law of mass i.e. $\nu \propto \sqrt{\frac{1}{m}}$

Adjust the distance between the wedges till the wire vibrates in resonance with a given tuning fork of frequency

ν_1. Change the wire and stretch it with the same force. Find the length l_2 of the wire which vibrates in resonance with the same tuning fork. Calculate, applying the law of length, the new frequency ν_2 with which this wire would have vibrated if the length had remained constant at l_1, then

$$\nu_2 = \frac{\nu_1 l_2}{l_1}$$

If m_1 is the mass per cm. of the first wire and m_2 the mass per cm. of second wire, then we shall find that

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{m_2}{m_1}} \text{ or } \nu \propto \sqrt{\frac{1}{m}}$$

8.18 LONGITUDINAL OR COMPRESSIONAL WAVES

Consider a long string of beads separated by light springs such as shown in Fig. 8.23a. If the bead I is pushed towards right a little and then clamped, the spring 1 is compressed (Fig. 8.23 b). Due to elasticity spring 1 exerts a force towards left on bead I which cannot be displaced as it is clamped. At the same time spring 1 exerts an equal force towards right on bead II which is displaced towards right thus compressing the spring 2. In this way the compression given to spring 1 is now transferred to spring 2. Now the compressed spring 2 will behave similarly and the compression is transferred to spring 3 and so on. We say that a pulse, which is a single unrepeated disturbance, is passing through the spring and it transfers energy from one spring to another. Similarly if the bead I is moved towards left, an elongation is produced in spring 1. This elongation also moves towards right from one spring to another.

If, however, bead I is caused to vibrate back and forth,

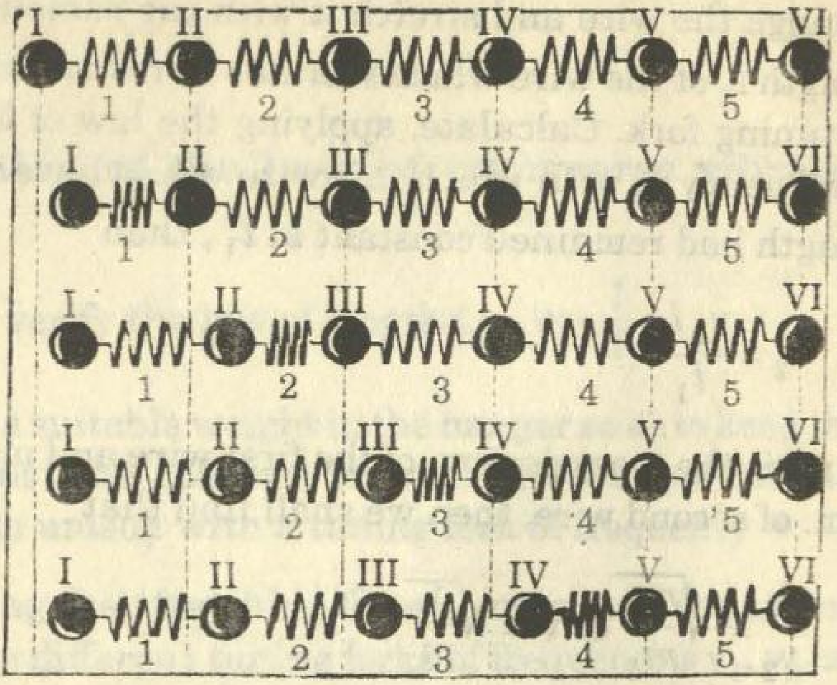


Fig.8.23 *The wave-motion in beaded springs is longitudinal wave-motion in which the various particles of the medium execute SHM about their respective mean positions along the direction of propagation of the wave.*

compressions and elongations will be sent along the spring one after the other. We say that waves are passing through the spring. It may be seen in this case that the various beads move to and fro about their mean positions along the line parallel to the direction of motion of the waves. Such waves in which the various particles of the medium vibrate about their mean positions along the direction of the waves, are known as *longitudinal waves. If, however, bead I vibrates with simple harmonic motion, the waves produced are known as periodic longitudinal waves.* The distance between two consecutive compressions or elongations is known as one *wavelength* which is denoted by Greek letter *lambda*(λ).

From the preceding discussion it is obvious that the properties of elasticity and inertia of the springs and beads are responsible for setting up a wave-motion. This is not only true for the waves along beaded springs but the wave-motion set up in any medium depends upon the elasticity and inertia of the medium. The elasticity of any substance is measured by its modulus E and its inertia can be estimated by its density. Thus the propagation of waves in a medium depends, in one way or the other on the modulus of elasticity and the density of the medium.

8.19 NEWTON'S FORMULA FOR THE VELOCITY OF SOUND IN FLUIDS

When one particle of medium starts vibrating, it produces a disturbances or wave motion in the medium. The disturbance travels in the medium in all directions. The velocity of the disturbance (or sound waves) depends upon the density of the medium. The lighter the medium, the more quickly will the disturbance move from point to point in the medium. Thus sound waves travel faster in hydrogen than in oxygen, as hydrogen is lighter than oxygen. Also, the velocity of disturbance depends upon the elasticity of the medium. The greater the elasticity, the more quickly will the disturbance be propagated from point to point in the medium.

The velocity of sound V thus depends upon two factors - the density ρ and the elasticity E of the medium. It was Newton who first proved the following formula for the velocity of sound in fluids (liquids & gases)

$$V = \sqrt{\frac{E}{\rho}} \qquad (8.28)$$

In gases sound waves travel in the form of compressions and rarefactions. In equation 8.28, the elasticity E is thus the bulk modulus which is given by the ratio stress/volumetric strain. Stress here means a change of pressure.

Newton assumed that sound travels through air and other gases under isothermal conditions, which means that when sound travels in air there is no change in temperature under such conditions, the coefficient of volume elasticity is equal to the pressure of the gas, which may be proved as:

Let V be the volume of the air at a pressure P. If we increase the pressure from P to P + p, temperature remaining constant, the volume will decrease from V to $V - v$, then according to Boyl's law

$$PV = (P + p)(V - v)$$

$$= PV - Pv + pV - pv$$

If the increase in pressure is very small, the decrease in volume will also be very small and hence the product pv is a negligible quantity. Neglecting pv in the above equation, we have

$$Pv = pV$$

$$\text{or } P = V\frac{p}{v} = \frac{p}{\left(\dfrac{v}{V}\right)}$$

$$= \frac{Stress}{Strain} = E_i \tag{8.29}$$

Hence for air and other gases, the velocity is equal to $\sqrt{\frac{P}{\rho}}$, where P is the pressure and ρ the density of the medium. For air at normal pressure

$P = 76$ cms of mercury, $\rho = 0.001293$ gm/cc at NTP.

$$\therefore \text{Velocity of sound } V = \sqrt{\frac{76 \times 13.6 \times 981}{0.001293}}$$

$$= 28100 \text{ cm sec}^{-1}.$$

The experimental determination of velocity of sound in air gives the result to be 33200 cm sec^{-1}. Thus the theoretical value is about 16% less than the experimental value. This cannot be an experimental error. The discrepancy in the theoretical and experimental values was explained by a French Mathematician Laplace.

Laplace's correction

Sound travels in air as a longitudinal wave motion. The wave motion is therefore, accompanied by compressions and rarefactions. At a compression the temperature of air rises and at a rarefaction a cooling effect is produced. The temperature does not, therefore, remains constant.

Air is a very poor conductor of heat and sound waves travel through it with a large velocity of about 33200 cm sec^{-1}. Therefore at a compression air cannot lose heat due to radiation and conduction, and at a rarefaction it cannot gain heat. The temperature throughout the medium does not remain constant and so Boyle's law is not applicable. On the other hand, the total heat of the medium is conserved, i.e. heat is neither lost by the air nor gained by it. *Such a change in which the total quantity of heat in a system remains constant is called an adiabatic change.* Hence in calculations of the velocity of sound we must use coefficient of adiabatic and not of isothermal elasticity.

Adiabatic Elasticity

Consider a cylinder having a volume V of air at a pressure P (Fig 8.24). Insulate the cylinder completely so that it can neither lose nor gain heat. Increase the pressure by a small amount p and let the volume decrease by a small amount v'. The temperature of air inside will rise due to compression. Hence

$$\text{Adiabatic Elasticity } E_a = \frac{p}{\frac{v'}{V}} = V\frac{p}{v'}$$

Now remove the insulation, the air in the cylinder being at a higher temperature will lose heat and the volume will further decrease. Let the total change in volume be v, then

Isothermal Elasticity $E_i = \dfrac{p}{\frac{v}{V}} = V\dfrac{p}{v}$

Since $v > v'$

$\therefore E_a > E_i$

It is found that $\dfrac{E_a}{E_i} = \gamma$. The value of γ is different for different gases. Its value is 3.1 for Carbon dioxide, 1.66 for Argon, 1.42 for air and 1.40 for all other diatomic gases. γ is not only the ratio of adiabatic elasticity to isotheremal elasticity but is also the ratio of specific heat of the gas at constant pressure to the specific heat at constant volume.

Since the value of γ is always greater than one, adiabatic elasticity is greater than isothermal elasticity.

In other words $E_a = \gamma E_i = \gamma P$

This relation can also be proved mathematically as follows:

For adiabatic changes of P and V, the relation is

$$P V^{\gamma} = \text{constant}$$

where γ is a constant depending upon the nature of the gas. If now the value of P is increased by a small amount p, the volume will decrease by a small amount v. Hence

$$P V^{\gamma} = (P+p)\,(V-v)^{\gamma}$$

$$= (P+p)\,V^{\gamma}\left(1-\frac{v}{V}\right)^{\gamma}$$

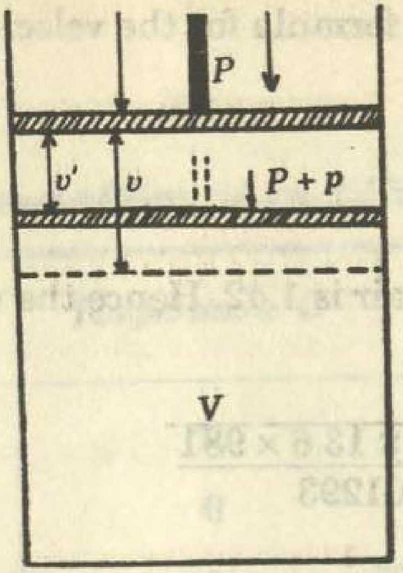


Fig.8.24 *Adiabatic expansion of a gas.*

$$\text{or } P = (P+p)\left(1-\frac{v}{V}\right)^{\gamma}$$

Applying Binomial theorem and neglecting squares and higher powers of $\left(\frac{v}{V}\right)$, as v is very small, we have

$$P = (P+p)\left(1-\gamma\frac{v}{V}\right)$$

$$\text{or } P = P - P\,\gamma\left(\frac{v}{V}\right) + p - p\,\gamma\left(\frac{v}{V}\right)$$

Neglecting the product $\left(\gamma p\,\frac{v}{V}\right)$ as it contains the product of two very small quantities, we have

$$\gamma P\left(\frac{v}{V}\right) = p$$

$$\text{or } \gamma P = \frac{p}{\left(\frac{v}{V}\right)} \qquad = \frac{Stress}{Strain} = E_a$$

Hence Newton's formula for the velocity of sound in a gas can be put as

$$V = \sqrt{\frac{\gamma P}{\rho}} \tag{8.30}$$

The value f γ for air is 1.42. Hence the velocity of sound in air at 0 ° C

$$= \sqrt{\frac{1.42 \times 76 \times 13.6 \times 981}{0.001293}}$$

$$= 33310 \text{ cm sec}^{-1}$$

which agrees with the experimental value of 33200 cm sec^{-1} within reasonable limits. Laplace's correction must, therefore, be correct.

The speed of sound varies with the temperature of the medium. The speed of sound in air increases approximately by 0.6 *m* sec^{-1} for each 1°C rise in temperature, if the temperature of air is not far removed from 0°C. An exact relation between the speed of sound and the temperature of air is given by

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \tag{8.31}$$

where V_1 and V_2 are the speeds of sound at absolute temperatures T_1 and T_2 respectively. Since both E and ρ are proportional to pressure, the speed of sound is independent of the pressure. We have seen that the speed of sound is different in different media. Table 8.1 gives the speed of the sound in some common substances.

Table 8.1 Speed of Sound in different media

Medium	*Temperature °C*	*Speed of Sound $m\ sec^{-1}$*
Air	0	331.3
Hydrogen	0	1286.0
Carbon dioxide	0	258.0
Water	15	1450.0
Iron	20	5130.0
Copper	20	3560.0
Glass	20	5500.0

8.20 INTENSITY AND LOUDNESS OF SOUND

We have already seen that compressional waves transmit energy. Since sound waves are compressional waves, they carry energy away from the source. The vibrating source does work on the surrounding media and this work appears as the energy of the waves. *The energy transmitted per second*

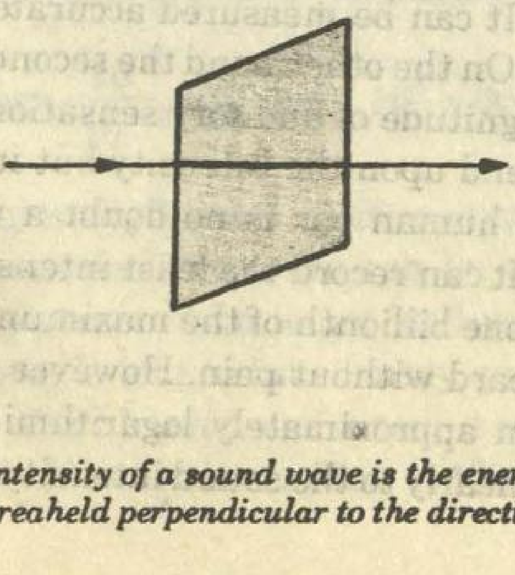


Fig.8.25 *The intensity of a sound wave is the energy flowing per second through a unit areaheld perpendicular to the direction of waves.*

through a unit area by the sound waves is called the intensity of the sound waves. The area is placed perpendicular to the direction of propagation of sound as shown in Fig. 8.25. Since energy per second is, by definition, power which is measured in MKS system in watts, so that unit of intensity of sound will be power unit divided by area unit i.e., watt/m^2. The intensities of several well known sounds are given in Table 8.2.

Table 8.2 Intensities of certain types of Sound

Type of Sound	*Intensity (watt m^{-2})*	*Intensity level (decibels)*
Threshold of pain	1	120
Riveter, elevated train	10^{-2}	100
Heavy street traffic	10^{-5}	70
Ordinary conversation	10^{-6}	60
Average whisper	10^{-10}	20
Rustle of leaves	10^{-11}	10
Threshold of hearing	10^{-12}	0

It may be pointed out that intensity of sound is purely a physical quantity. It can be measured accurately. It is independent of the ear. On the other hand the second quantity, *i.e. loudness* is the magnitude of auditory sensation produced by sound. It does depend upon the intensity but it also depends upon the ear. The human ear is no doubt a very sensitive detector of sound. It can record the least intense sound (10^{-12} watt/m^2) which is one billionth of the maximum sound intensity that can be heard without pain. However it is seen that ear operates on an approximately logarithmic scale rather than responding linearly to the sound intensity. It means that the loudness L is proportional to the logarithm of intensity I, i.e., $L \propto \log I$, this is called *Weber-Fechner law.* If I and I_0 represent the intensities of two sounds then the *difference in loudness,* known as the *intensity level* between them is given by the equation

$$\text{Intensity level} = K \log_{10}\left(\frac{I}{I_o}\right)$$

where K is a constant depending upon the units and I_0 is the intensity of the faintest audible sound taken as 10^{-12} watt m^{-2}. If I is taken as 10 times I_0 and intensity level is one unit, then K becomes equal to 1. This unit of intensity level is called a *bel* named after Alexander Graham Bell. This unit of loudness is too large. So *decibel* abbreviated as db, one tenth of *bel* is taken as unit. Thus intensity level in decibels

$$= 10 \log_{10}\left(\frac{I}{I_o}\right) \tag{8.32}$$

We can find the intensity level of various sounds using Eq. 8.32. Intensity level of faintest audible sound

$$= 10 \log_{10} \frac{10^{-12}}{10^{-12}} = 10 \log_{10} 1 = 0$$

Intensity level of ordinary conversation

$$= 10 \log_{10} \frac{10^{-6}}{10^{-12}} = 10 \log_{10} 10^6 = 60 \text{ db.}$$

Value of intensity level of various sounds are also given in Table 8.2. These decibel values can be verified by a simple experiment. A person, who is not well versed with the facts described above, is told to take barely audible sound as zero and the level of the loudness of a sound which begins to hurt his ear as 120. If he is then asked to place numbers against various sounds to indicate their loudness on this arbitrary scale, these numbers would turn out to be close to the number given in the Table 8.2.

The logarithmic response of the human ear has its merits as well as demerits. The advantage is that it can detect an extremely wide range of sound intensities. However the resolving power of the ear is limited.

8.21 AUDIBLE FREQUENCY RANGE

An average human ear can hear a sound if its frequency lies between certain limits. If the frequency of a sound is higher than 20,000 hertz, it cannot be heard. Sounds of frequency higher than this are called *ultrasonics* meaning 'beyond' or above sound. The sensitiveness of ear for high frequency sounds decreases with age. Children can generally hear sound of 20,000 frequency while elderly people cannot hear anything above 15,000. Moreover we are not able to hear sound below a certain frequency about 20 hertz. These do not blend into a note, but are heard separately.

The sensitivity of an average human ear is different in different frequency ranges even if the frequency lies within the lower and the upper limit of frequency. Fig. 8.26 shows a relationship between sensitivity of the ear and frequency. Intensities below the line indicating the threshold of hearing are unable to produce any sensation of hearing. The curve shows that the normal ear is most sensitive in the frequency range 2000 to 4000 hertzs. The intensity necessary for hearing in the regions near the high and low limits of audibility is many times as great as that necessary in the region of greatest sensitivity.

If the intensity becomes too great, the sensation becomes one of pain rather than of hearing. This is indicated by the

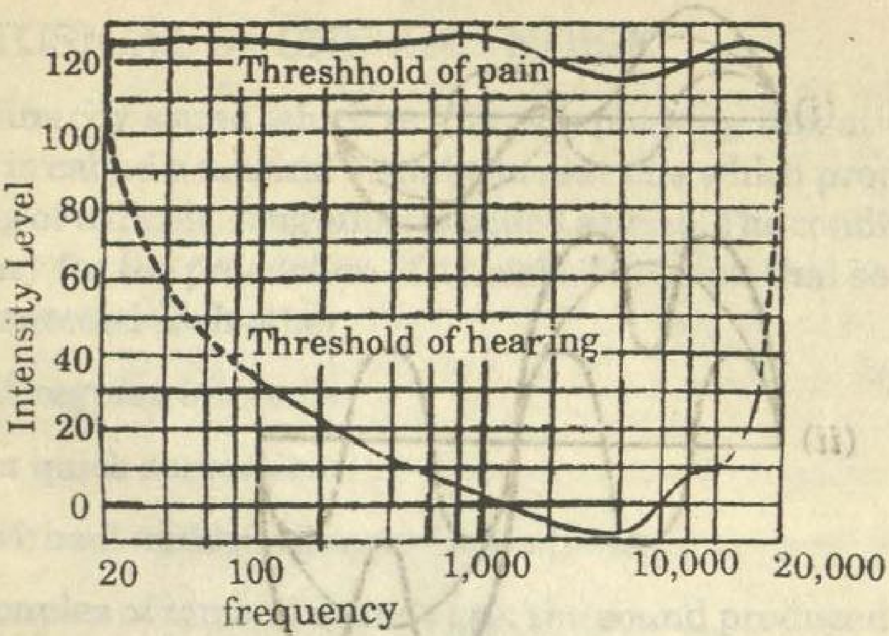


Fig.8.26 *An average human ear can hear a sound only if its frequency and intensity lie between the lower and upper curves.*

upper curve in Fig. 8.26 which shows the threshold of pain. Notice that the pain inducing intensity does not vary much with frequency. Fig 8.26 shows that there is sensation of hearing only in the region of intensity and frequency bounded by these two curves.

8.22 PITCH AND QUALITY OF SOUND

If a number of tuning forks of different frequencies are sounded one after the other, anyone with a normal ear can distinguish between sounds produced by these forks even if these are of equal loudness. Sound produced by a tuning fork of high frequency will be *shrill* while that by a fork of low frequency will be *grave. The characteristic of sound by which a shrill sound can be distinguihsed from a grave one is known as the pitch of the sound.* It depends upon frequency, the greater the frequency, the higher the pitch and lower the frequency, the lower the pitch. Sometimes pitch and frequency are considered as synonymous but in fact pitch is a sensation depending upon frequency.

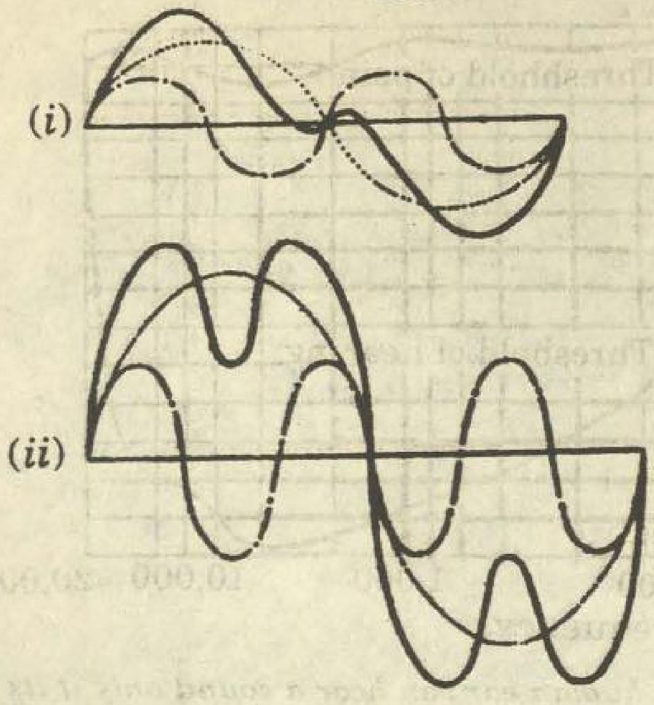


Fig.8.27 *Resultant waveform when two waves are combined.*

It is common experience that a note of given pitch and loudness sounded on piano is easily distinguished from one of exactly the same pitch and loudness played, for example, on violin. It is because the *quality* of the two notes is different.

A string under tension, when excited, vibrates not only with its fundamental frequency but several other harmonics are produced at the same time. The resultant vibration is found by adding together the waves of the various vibrations present. In Fig.8.27 the resultant vibration of (i) fundamental and second harmonic and (ii) fundamental and third harmonic are illustrated. It is seen that the presence of different harmonics affects the waveform of the note.

The quality of the sound depends upon the waveform of the resultant and is controlled by the number and relative intensities and phase of the harmonics that are present, the resultant waveforms shown in Fig. 8.27 will have different effects on the ear even though they have the same pitch and loudness. They will, however, give rise to notes of different qualities.

8.23 MUSICAL SOUND AND NOISE

Ordinarily sound which produces a pleasing sensation in the ear is called a musical sound whereas one which produces a jarring or a displeasing effect is called a noise. The conditions necessary for the production of a musical note are that sounds should succeed each other

(i) at regular intervals

(ii) in quick succession

(iii) without sudden changes in loudness

Examples of musical sounds are: the sound produced by a tuning fork, by plucking the string of sitar, the sound produced by blowing into an open or closed organ pipe.

Noise on the other hand are sounds of very short duration having no periodicity and their character is changing. Some familiar examples of noise are: the jingling of keys, clapping of hands, the report of a gun, the roar of street traffic, the rattling of wheels, and the sound of a hammer striking an anvil.

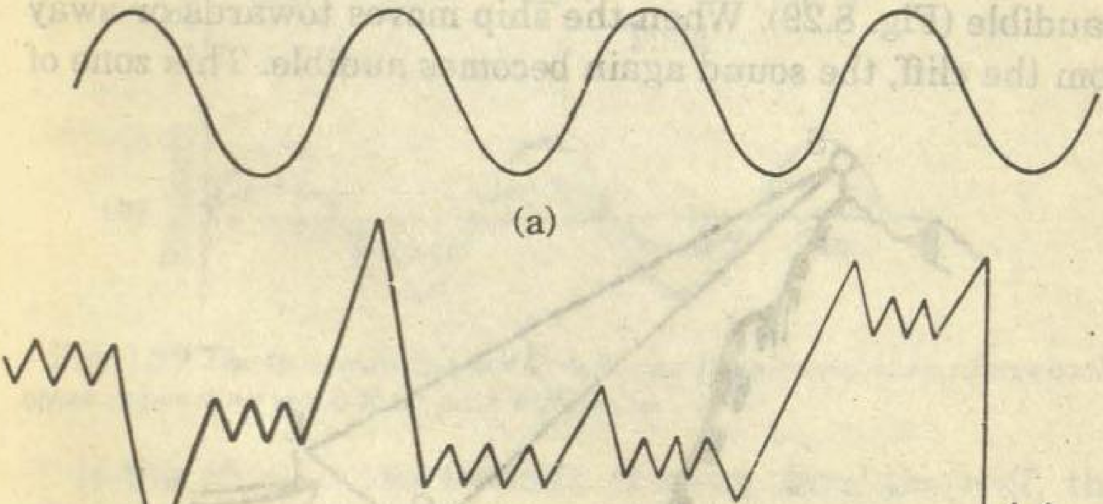


Fig.8.28 *(a) Musical sound, (b) Noise.*

The physical difference between a musical sound and a noise is clear from Fig. 8.28

The upper curve is uniform and regular and represents a musical sound. The lower curve is irregular and shows sudden changes in loudness and is a noise. Hence a musical sound is that which is produced by a series of similar impulses following each other regularly, at equal intervals, in quick succession, without any sudden changes in loudness.

8.24 INTERFERENCE OF SOUND WAVES

If two waves pass through the same region at the same time, the motion of the particles of the medium will be the result of the combined disturbance due to the two sets of waves. One of the effect due to the combined action of the two sets of waves is known as *interference*

Let us consider some examples of interference of sound waves. If a fog-siren is situated on a high cliff, a ship approaching it may find itself in a zone of silence where the siren is inaudible (Fig. 8.29). When the ship moves towards or away from the cliff, the sound again becomes audible. This zone of

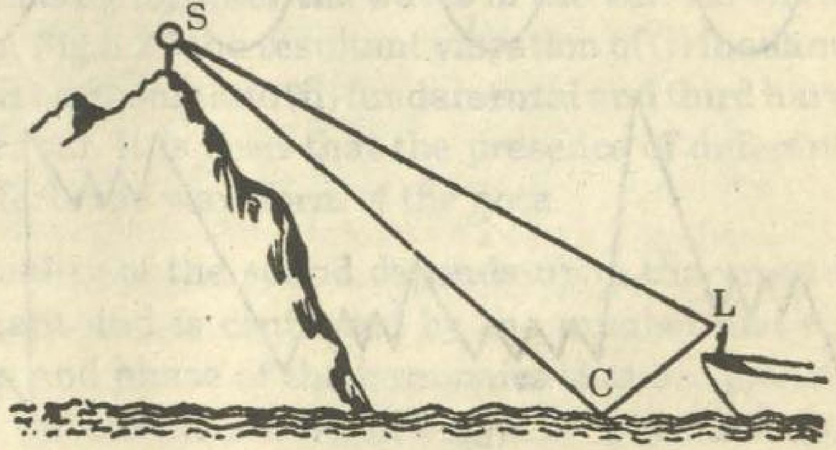


Fig.8.29 *The sound of the siren is not heard at L when the waves interfere destructively.*

silence can be explained as being due to interference.

The sound of a siren S can reach the listener L on the ship through two paths, one by the direct path SL and the other by the path SCL after suffering reflection from the surface of the sea. If these two paths differ by half a wave-length or any odd integral multiple of half the wavelength, then the compression due to one wave will find a rarefaction due to the other wave at the point L. Since the compression tends to increase the pressure while the rarefaction tends to decrease the pressure at L, the two will cancel the effect of each other. As a result no sound will be heard at L. This situation is represented graphically in Fig. 8.29' where the change in pressure at L due to one wave is plotted as function of time and it is cancelled, by the change in pressure at L due to the other wave consequently there is no resultant change at L. This type of situation is known as *destructive interference.*

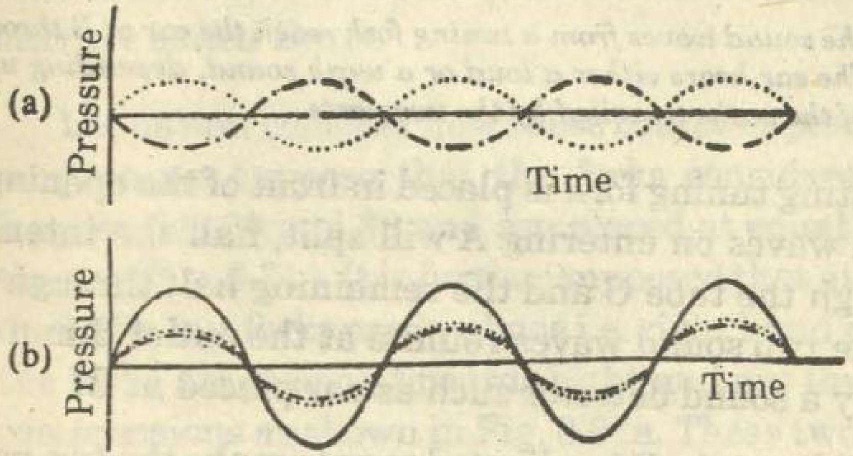


Fig. 8.29' *The two waves shown by separate lines cancel or reinforce each other depending upon their path difference.*

If the ship moves towards or away from the cliff, the difference between the two paths changes and the sound is again heard. when the difference of these paths is exactly one wavelength or any integral multiple of a wavelength, sound will be loudest. It is because the compression due to two waves arrive at L simultaneously and so also the rarefactions. The two waves thus reinforce each other and the resultant sound is loudest. This sort of interference is known as *constructive interference* which is illustrated in Fig. 8.29'b.

We shall now consider a simple experiment which illustrates the phenomenon of interference in a very striking manner. The apparatus used is shown in Fig. 8.30.

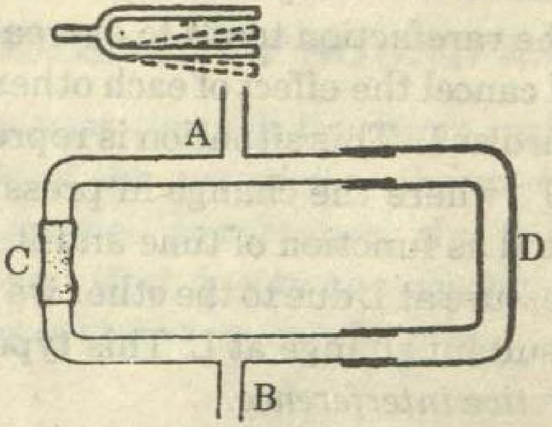


Fig.8.30 *The sound waves from a tuning fork reach the ear at B through two paths. The ear hears either a loud or a weak sound, depending upon the lengths of the paths travelled by the two parts.*

A vibrating tuning fork is placed in front of the opening A. The sound waves on entering A will split, half the intensity goes through the tube C and the remaining half through the tube D. The two sound waves reunite at the outlet B and can be heard by a sound detector such as ear placed at B.

If the sliding tube D is adjusted so as to make the two paths ACB and ADB equal, the waves arrive at B in phase (without any path difference). Then these waves interfere at B constructively and a loud sound is heard at B. If the sliding tube D is now drawn out, path ADB becomes longer than the path ACB. The sound waves arriving at B via D will fall more and more behind those coming via C. When the difference of path between the waves is half a wavelength, they interfere destructively and consequently no sound is heard at B. If the rubber portion of the tube C, is pinched so as to stop the sound waves coming through C, the ear will again hear the sound. This proves that the silence is due to the destructive interference of the two sound waves.

Beats

We have so far considered only the interference of two sound waves of the same frequency. We shall now study the interference of two sound waves of two slightly different frequencies sent through air at the same time.

Take two tuning forks having the same frequency. slightly load a prong of one fork with a little wax. Thus the two forks now have slightly different frequencies. Set the two forks vibrating together. We will not hear two separate notes, but a single note which rises and falls in intensity. *These periodic alternations of sound between maximum and minimum loudness are called Beats.*

Let us now consider how these beats are produced. For this purpose we suppose that the forks considered above have frequencies 30 and 32 and are placed at equal distance from the ear (Fig. 8.31). It is further supposed that at a certain time $t = 0$, the two forks are in phase i.e. right hand prongs, of both the forks are moving towards right and are thus sending out compressions as shown in Fig. 8.31a. These two compressions will arrive at the ear together and thus a loud sound is heard.

As time goes on, the fork B, vibrating at a slightly lower frequency than A, will begin to fall behind. After ¼ second the fork A will complete 8 vibrations and will just be sending out a compression. On the other hand the fork B will complete 7½ vibrations and will be sending out a rarefaction as shown in Fig. 8.31b. The compression from A and rarefaction from B will reach the ear at the same time. They will cancel each other. Hence at this instant no sound will be heard.

As time passes the fork B will fall still behind A. After half a second the fork A will complete 16 vibrations while the fork B will complete 15 vibrations (Fig. 8.31c). Both the forks will be sending out compressions together and thus again a loud sound will be heard.

After ¾ second fork A will complete 24 vibrations and fork B will complete 22½ vibrations (Fig. 8.31d). At this instant fork A will besending a compression while fork B will be sending a rarefaction. Thus no sound will be heard.

After 1 second fork A will complete 32 vibrations and fork B will cmplete 30 vibrations. Both these forks will be sending compressions and again a loud sound will be heard.

We have seen that in one second two beats are produced. The difference between the frequencies of the two forks is also two.Thus we can conclude that *the number of beats per second is equal to the difference between the frequencies of the forks.*

We can understand the

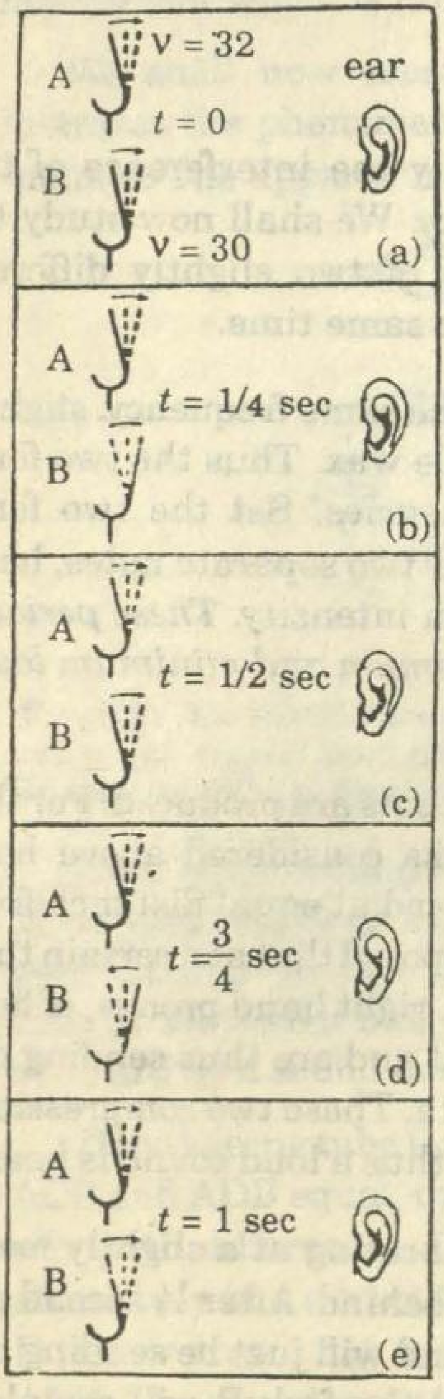


Fig.8.31 *Beats are produced when two similar tuning forks vibrate with slightly different frequencies. One prong of each tuning fork is shown for simplicity.*

phenomenon of beats by considering the displacement curves of sound waves produced by the forks. The displacements of the particles of medium due to two waves are plotted separately as a function of time as shown in Fig. 8.32. The total extent of the time axis is 2 seconds. If both waves travel simultaneously along the same line then according to the principle of superposition, the resultant displacement of any particle will be the sum of the displacements due to each of the two waves. Thus the resultant wave which is produced, is shown in Fig. 8.32c. It is seen that the amplitude varies with time. The variations of amplitude give rise to variations of loudness which we call beats.

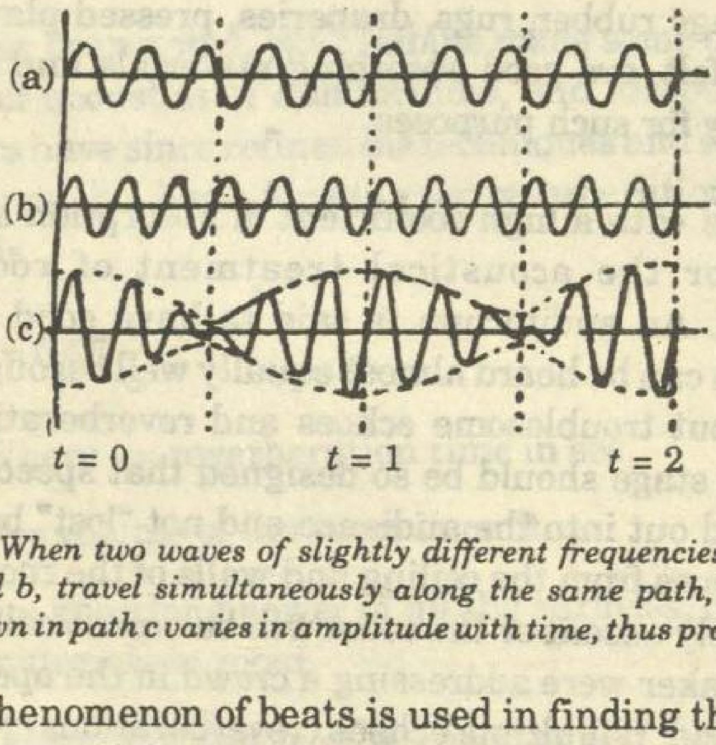


Fig.8.32 *When two waves of slightly different frequencies as shown in part a and b, travel simultaneously along the same path, the resultant waves shown in path c varies in amplitude with time, thus producing beats.*

The phenomenon of beats is used in finding the unknown frequencies and also in tuning the musical instruments.

8.25 ACOUSTICS

A sound wave will continue to recede from its source until it is converted into some other form of energy. When a sound wave passes through a given material, some of the sound-wave energy is absorbed and converted into heat energy. That is, as the soundwave energy strikes the absorbing material, it increases the motion of its molecules. This increase in molecular motion appears as added heat energy. Porous materials are effective sound absorbers because they contains, many pockets of air whose molecules can readily be set into increased motion.

The greater the conversion to heat, the greater the absorption coefficient. The absorption coefficient of a given material is the fraction of the sound energy that it will absorb at each reflection or transmission. Some materials have low absorption coefficients, and sound waves pass through them are reflected from them with little loss of energy. Other materials, such as sponge rubber, rugs, draperies, pressed plant fibers, and porous felt, are good absorbing materials and are used commercially for such purposes.

Materials with a high coefficient of absorption are of importance for the acoustical treatment of rooms and auditoriums. An auditorium is said to have good acoustics when speech can be heard almost equally well throughout the space, without troublesome echoes and reverberations. The podium and stage should be so designed that speech sounds are projected out into the audience and not "lost" backstage. Multiple echoes from the ceiling and walls of the room should not be entirely absent or the room will be acoustically "dead," as if the speaker were addressing a crowd in the open air. On the other hand, if multiple echoes (reverberations) persist for too long a time, the echoes from previous syllables uttered by the speaker will arrive at the listener's ear just in time to interfere with the hearing of the next syllable. Music, too, can be adversely affected by excessive reverberation.

Interference is another factor that must be considered in designing auditoriums. Interference will cause variations in intensity-loud spots in some places, and dead spots in other locations. Interference may be minimised by a proper choice of the dimensions and shape of the auditorium and by having "clean" lines, free from pillars, overhangs, and unnecessary architectural embellishments.

The time it takes for sound to diminish until it is no longer audible is called the reverberation time. For speech, the optimum reverberation time for small auditoriums is from 0.5 to 1.0 sec. For large auditoriums a reverberation time up to 2.0 sec may be allowed. Values somewhat larger than these are satisfactory for listening to music.

As has been noted, W.C. Sabine made some of the earliest studies of acoustics of auditoriums, and although many researchers have since refined his techniques and supplemented his findings, his basic formula for reverberation time is still used. It is

$$t = 0.05 \frac{V}{a} \tag{8.33}$$

Where t = reverberation time in sec.

V = volume of the room in cu ft.

a = absorbing power of all the surfaces, objects, and materials in room.

The absorbing power of a 1 square foot open window is taken as unit and is called 1 sabin. The absorption coefficient of a material is a number, always less than 1.00, which compares the absorbing power of 1 sq ft of that material or surface to the absorbing power of 1 sq ft of open window. The symbol β is used for absorption coefficient. Total absorbing power of all the surfaces and materials in a room is equal to the sum of separate areas multiplied by their respective absorption coefficients.

$$a = \beta_1 s_1 + \beta_2 s_2 + \beta_3 s_3 \tag{8.34}$$

TABLE 8.3 Sound Absorption

Absorption Coefficient β	
Open window	1.00
Acoustic ltex	0.82
Acoustic plaster	0.25
Carpet, heavy	0.40
Chalkboard	0.06
Draperies, cotton	0.50
Felt, heavy.	0.70
Glass	0.025
Linoleum	0.03
Marble	0.01
Plaster wall (smooth)	0.03
Wood wall (painted)	0.04
Wood wall, floor, or paneling (unpainted)	0.08

Absorbing power (sabins)	
Audience, each person	4.0
Auditorium chairs, wood, each	0.3
Auditorium chairs upholstered, each (empty)	2.0
Auditorium chairs upholstered, each (occupied)	1.5

Table 8.3 gives the absorption coefficients of certain com-

mon surfaces and acoustical materials.

Example 8.8

Compute the reverberation time of a small auditorium 70 ft. × 110 ft × 25 ft. The following information is given with regard to the areas, in square feet, of the building surfaces: plaster, 10,500; wood planel, 6,000; linoleum, 6,700; heavy carpet, 1,000; heavy drapes, 500; Acoustic-celotex, 1,000. There are 500 people (seated) and 300 empty upholstered chairs in the auditorium.

Solution

The calculation of absorbing power, *a*, is carried out in tabular form.

Plaster, 10,500 × 0.03 = 315 sabins
wood panel, 6,000× 0.08 = 480
linoleum, 6,700× 0.03 = 200
heavy carpet, 1,000× 0.40 = 400
heavy draperies, 500× 0.70 = 350
Acoustic-celotex. 1,000× 0.82 = 820
500 people, 500× 4.0 = 2,000
500 chairs, occupied, 500× 1.5 = 750
300 chairs, empty, 300× 2.0 = 600

a = 5,915 sabins

$$\text{From Eq. 8.34 } t = 0.05 \times \frac{70 \times 110 \times 25}{5915} = 1.63 \textit{ sec.}$$

In addition to acoustical problems in auditoriums, the science of acoustics is intensively applied to public-address and hi-fi equipment, to the design of recording studios, and to the problems of suppressing unwanted sound in this "noisy" age of ours.

8.26 DOPPLER'S EFFECT

If a train while whistling passes near you, a considerable change in the pitch of the note, produced by the whistle, will occur. When the train is approaching, the pitch of the note increases and when the train is receding, the pitch decreases. A similar change in pitch also occurs when a moving listener passes a stationary horn or siren. *This change in the pitch of sound caused by the relative motion of either the source of sound or the listener is called the Doppler effect,* after an Austrian Physicist, Christian John Doppler.

Let us consider a simple case of a source moving towards the listener. Suppose that the frequency of sound emitted by source is ν the speed of the source towards the listener is a and the speed of sound is V. If the source were at rest, the events which would have occurred during one second are illustrated in Fig 8.33a. The first wave emitted by the whistle has reached the listener at point A, at a distance V from the source. The number of the waves emitted by the source during one second is ν and these occupy a length V. The wavelength of the waves is given by

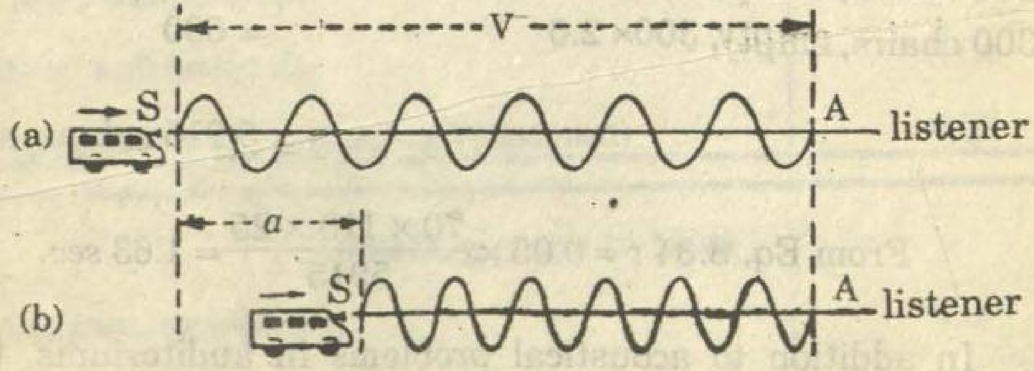


Fig.8.33

$$\lambda = \frac{\text{Distance which waves occupied}}{\text{Number of waves}}$$

$$= \frac{V}{\nu}$$

If the source were moving towards the listener with a velocity a, the situation during the first second is shown in Fig. 8.33b. The first wave emitted by the whistle reaches the point A distant V from the initial position of the source after one second. As time goes on, the source moves towards the listener. At the end of the first second the source reaches the position S after travelling a distance a. The ν waves emitted by the source in one second are now contained in the length $(V-a)$. It is clear that V waves are now compressed into a smaller distance. Therefore the wavelength decreases and is given by

$$\lambda' = \frac{\text{Distance in which } \nu \text{ waves are contained}}{\text{Number of waves}}$$

$$= \frac{V-a}{\nu}$$

The changed frequency ν' of the sound wave is given by

$$\nu' = \frac{V}{\lambda'}$$

$$\text{or } \nu' = \frac{V}{V-a}\nu \qquad (8.35)$$

Thus the pitch of sound emitted by a source, approaching the listener, increases. If the train is moving away from the listener, the sign of a should be reversed with the result that

$$\nu' = \frac{V}{V+a}\nu$$

In this case the pitch of the sound decreases.

However if the source is at rest and the listener is moving with velocity b towards the source, a little reasoning shows that the frequency of the sound heard is given by

$$\nu'' = \frac{V+b}{V}\,\nu \qquad (8.36)$$

So, the apparent frequency of the sound when the listener is moving towards the source, increases.

If the listener is moving away from the stationary source, the result will be

$$\nu'' = \frac{V-b}{V}\,\nu$$

In this case the frequency appears to decrease.

The Doppler effect is not, of course, confined to sound waves, but it has been successfully applied to light. The frequency of light from certain stars is found to be slightly more and from other stars slightly less than the frequency of the same light emitted from the source of the same light on earth. The reason is that these stars are moving towards or away from the earth and their velocities can be obtained from this observed frequency difference.

A bat determines the location and nature of objects by sending ultrasonic waves. It is likely that bats have also developed sensory devices to find the speed of the objects in their surrounding by sensing the Doppler frequency shift.

Another interesting application of Doppler effect is in the reflection of radar waves from the aeroplanes. The frequency of the reflected waves is decreased if the plane is moving away from the source and increased if it is moving towards the source. From the observed frequency difference the speed and the direction of the plane can be calculated. The same phenomenon occurs when under-water sound waves (sonar) are reflected from a moving submarine. The velocities of earth satellites are also determined from the Doppler shift in the frequency of radio waves which they transmit.

SUMMARY

If a system vibrates such that its acceleration (or force) is directly proportional to the displacement and is directed toward the center, the system is said to execute simple harmonic motion. The potential energy of a spring executing *SHM* is given by $\frac{1}{2}\text{k}x^2$, and total energy $= \frac{1}{2}mV^2 + \frac{1}{2}\text{k}x^2 = \text{K} + \text{U}$ remains constant in *SHM*. Wave motion is a form of disturbance which traverse through a medium due to the repeated periodical vibrations of the particles of the medium about their mean positions, the motion being handed on from one particle to the next. It provides a mechanism for transfer of energy from one point to another without physical transfer of any material medium between the points. In transverse waves the particles of the medium vibrate at right angles to the direction of propagation, while in longitudinal waves, the particles of the medium vibrate back and forth about their equilibrium position along the direction of propagation of the wave motion. wavelength λ is the distance between any two successive points that are in the same phase.

$$V = \frac{\lambda}{T} = \nu\lambda$$

The velocity of the transverse waves is $V = \sqrt{\frac{T}{m}}$ in a string and that of the longitudinal waves is $V = \sqrt{\frac{E}{\rho}}$ in fluids. For velocity of sound in air, $V = \sqrt{\frac{\gamma P}{\rho}}$ and

$$\frac{V}{V_o} = \sqrt{\frac{T}{T_o}} \text{ or } V = (331 + 0.62\,T)\text{ m sec}^{-1}$$

Stationary or standing waves are formed when two waves of the same frequency and speed travel in opposite directions.

The points where the amplitudes are minimum in a standing wave pattern are called nodes; the distance between adjacent nodes is $\lambda/2$. Antinodes are the locations of maximum amplitude, located midway between the nodes. For a stretched string, the fixed ends are nodal points.

In strings the fundamental frequency is $\nu_1 = \frac{v}{2l}$ while for the $(n - 1)$ th overtone, $\nu_n = n\left(\frac{v}{2l}\right)$, $n = 2, 3, 4, \ldots$..

The frequency of the nth harmonic in the string is

$$\nu_n = \left(\frac{n}{2l}\right)\sqrt{\frac{T}{\rho}}$$

Waves can interfere with one another. When two waves are present at a point, the resulting wave is found by algebraically adding the displacements of the individual waves. Two waves in phase add constructively, while two waves a half wavelength out of phase interfere destructively.

When two waves of slightly different frequencies traveling through the same region of space are observed at a given point, the result is a sound with pulsating intensity called beats (interference in time). The frequency of the beats is $(\nu_1 - \nu_2)$.

The loudness is intimately connected with the intensity of sound and usually increases with the frequency. The difference in the intensive levels of two sounds is $L_1 - L_2 = 10 \log (I_1/I_2)$ db. The pitch refers to that characteristic of sound sensation that enables one to classify a note as a high note or a low note. For a pure note of constant intensity, the pitch increases with increasing frequency. The tone quality of any musical sound is determined by the presence of the number of overtones and their relative intensities.

The observed change in the pitch due to the relative motion between the source and the observer is called the Doppler effect.

QUESTIONS

8.1. State the basic conditions for frictionless system to execute simple harmonic motion.

8.2. Give some examples of motions that are simple harmonic.

8.3. A certain simple pendulum has an iron bob. Would its behaviour change if we replace the iron bob with a lead bob of the same size?

8.4. A wire hangs from a dark high tower so that its upper end is not visible. How can we determine the length of the wire?

8.5. Suppose we have block of unknown mass and a spring of unknown stiffness. How can we predict the period of oscillation of block-spring system by measuring the extension of the spring by attaching the block to it?

8.6. How can you compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring?

8.7. Will a pendulum that keeps correct time at Karachi, be accurate at Murree or at the top of Mount Everest?

8.8. Will the period of a vibrating spring increase, decrease or remain constant by addition of more weight?

8.9. What is the frequency of a second's pendulum?

8.10. Would you keep the amplitude of a simple pendulum small or large? Why?

8.11. Clearly explain the difference between transverse and longitudinal waves.

8.12. Is it possible for two identical waves travelling in the same direction along a string to give rise to a standing wave?

8.13. Explain the terms:
trough, crest, compression, node, and antinode.

8.14. Explain how the speed of transverse wave in a string will change if its tension is made four times.

8.15. Keeping tension constant, how the speed of wave will change if the string is changed by another string of the same material but having a diameter four times larger?

8.16. What are the conditions of constructive and destructive interference?

8.17 What is meant by phase coherence? How does it affect the interference pattern?

8.18 Describe why the sound produced by explosions going on in the sun cannot be heard on the earth?

8.19 Why does sound travel faster in solids than in gases?

8.20 Why does the speed of a sound wave in gas change with temperature?

8.21 It has been claimed that a certain singer could shatter a glass goblet by singing a particular note. Is it possible?

8.22 Mention some illustrations of importance of resonance in musical instruments.

8.23 How are beats useful in tuning a musical instrument?

8.24. From where does the energy in a sound wave come in the following cases? *(a)* sitar *(b)* fire-cracker *(c)* drum and *(d)* loudspeaker.

8.25: Persons speaking with the same loudness can be recognised from their voice. How is that?

PROBLEMS

8.1 A stone swinging in a horizontal circle of 50 cm diameter makes 15 revolutions/min. A distant light causes a shadow of the stone to be formed on nearby wall. What is the frequency, period and the amplitude of the motion of the shadow?

8.2 The buffer of a railway wagon is compressed by 5 cm when a force of 200 N is applied to it. Calculate the spring constant of the buffer and the work done in compressing it.

8.3 A mass of 0.5 kg is suspended from a spring. The spring is stretched by 0.098 metre. Calculate the period of oscillation of the mass when it is given a small displacement.

8.4 A ball of mass 0.5 kg is attached to an elastic spring. The spring is compressed by 10 cm by a force of 45 N. Calculate the energy stored in the spring. What will be the maximum velocity of the ball if the compressing force is suddenly removed?

8.5 Calculate the length of a second's pendulum at a place where $g = 9.8\ m\ \text{sec}^{-2}$. (A second's pendulum has a time period equal to 2 seconds).

8.6 A ball of mass 0.5 kg vibrates with an amplitude of 0.04 *m* when attached to a spring of spring constnt 0.5 N/*m*. Calculate (a) the potential energy when the displacement is maximum (b) the speed of the ball at the middle point.

8.7 A mass at the end of a spring describes simple harmonic motion with a period of 0.40 sec. Find the acceleration when the displacement is 4 cm.

8.8 A body of mass *m*, suspended from a spring with a spring constant k, vibrates with a frequency v_1. When the spring is cut into half and the same body is suspended from one of the halves, the frequency is v_2.

What is the ratio $\frac{v_1}{v_2}$?

8.9 If a wave generator produces 20 pulses in 4 seconds what is (*a*) its period (*b*) its frequency?

8.10 (*a*) In a ripple tank, ten waves pass across a certain point in one second. If the wavelength is 4 cm, what is the speed of propagation?

(*b*) In the same medium if we send two waves, the second one ¼ second after the first, how far apart are they?

8.11 A wire of length 1.4 *m* and mass 0.052 kg is stretched by means of a load of 15 kilograms. Calculate the frequency of the fundamental vibration.($g = 9.8$ m sec^{-2}).

8.12 A certain string 1 m long and of mass 0.004 kg is stretched with a certain force. Calculate the force acting on the string, given that the velocity of a transverse wave in the string is 140 m sec^{-1}.

8.13 A wire 100 cm long plucked at its centre, vibrates with a frequency of 260 hertz. Calculate the wavelength and the speed of the wave in the wire.

8.14 A string can resonate to several frequencies, the lowest of which is 200 hertz. What are the next two higher frequencies?

8.15 A coil spring 200 cm long lies on a smooth table, with one end fixed and the other free. The speed of the compressional wave in the spring is 1600 cm sec^{-1}. Calculate five possible frequencies of the driving (hitting) system at the free end for sending resonating compressional waves along the spring.

8.16 The frequency of a certain string is 125 hertz, when it vibrates in five segments. Calculate the frequencies when the string is vibrating in three and four segments. What is the speed of the wave in it, if its length is 0.80 m?

8.17 A wire 0.80 *m* long vibrates in three loops with nodes at the two ends. Find the wavelength and speed of the wave in the wire if it vibrates with the frequency of 300 hertz.

8.18 Flash of lightning occurs 1.5 seconds earlier than the thunder. How far away is the cloud in which the flash has occurred? (Speed of sound = 332 m sec^{-1}).

8.19 Calculate the velocity of sound in steel rails of a railroad track. The weight density of steel is 490 lb/ft^3 and Young's modulus for steel is 29×10^6 lb/in^2.

8.20 The speed of sound in air at 0 °C is 332 *m* sec^{-1}. What will be its speed at (*i*) 22 ° C (*ii*) 273 ° C (*iii*) 819 °C?

8.21 Find the velocity of sound in a copper cable if the density of copper is 8.9 gm cm^{-3} and its Young's modulus is 12×10^{11} dynes cm^{-2}.

8.22 A copper rod projects 50 cm from a concrete block in which it is firmly set. The rod is tapped on the end with a hammer to set up compressional waves in it. What is the fundamental frequency of the sound it emits? (Velocity of sound in copper = 3560 *m* sec^{-1}).

8.23 A closed pipe (i.e. closed at one end, open at the other) produces a sound of 240 hertz. How long must the pipe be? (Velocity of sound in air = 330 m sec^{-1}).

8.24 A car has a siren sounding a 2000 cps tone. What frequency will be heard by a stationary observer as the car approaches him at 50 mph?
(Velocity of sound = 750 mph).

8.25 Compare frequencies heard by a listener under the following circumstances (*a*) A listener moves with a velocity of 100 ft/sec^{-1} directly towards a stationary source emitting waves of frequency 1000 hertzs (*b*) The listener remains at rest while the source moves directly towards him at 100 ft/sec^{-1}.
(Velocity of sound = 1100ft/sec^{-1}).

8.26 Two tuning forks A and B give 4 beats per second. On loading A slightly we get 3 beats per second. What is the frequency of A before and after loading if that of B is 256?

8.27 A tuning fork A of frequency 256 hertz produces 4 beats per second with another tuning fork B. When the prongs of the fork A are loaded with one gm ring, the number of beats is one per second and when loaded with 2 gm ring, the number of beats becomes two per second. What is the frequency of the fork B?.

9

PHYSICAL OPTICS

9.1 DUAL NATURE OF LIGHT

The study of properties and nature of light had been an active field of research since Newton's time and even before. Especially, the nature of light remained in dispute even until the first decade of the present century. In this regard the two "school of thoughts" were active in the beginning of the seventeenth century. One was that of Newton who proposed the corpuscular nature of light and the other was that of Christian Huygens who believed that light was a wave of some sort.

Most scientists accepted Newton's corpuscular nature of light as it successfully explained many experimental facts concerning the properties of light. However Huygens showed that the laws of reflection and refraction could be understood assuming the wave nature of light. The theory did not receive immediate attention and acceptance for several reasons. All the waves known at that time (sound, water etc.) travelled through some sort of medium. On the other hand, light could travel to us from the Sun through the vacuum. Furthermore, it was argued that if light were some form of wave motion, the wave could bend around obstacles, hence, we should be able to see around corners. It is now known that the bending of light around the edges of an object does indeed occur. Although experimental evidence for the occurence of the diffraction of light was discovered by Francesco Grimaldi (1618-1663) around 1660, most scientists rejected the wave theory and adhered to the corpuscular theory for more than a century. This was, for the most part, due to the Newton's great reputation as a scientist.

The first clear demonstration of the wave nature of light was provided in 1801 by experiments of Thomas Young (1773-1829). In his experiment, Young showed that under appropriate conditions, light exhibits an interference behaviour. Several years later, a French physicist, Augustin Fresnel (1788-1829), performed a number of detailed experiments dealing with interference and diffraction phenomena. In 1850, Jean Focault proved further inadequacy of the corpuscular theory by showing that the speed of light in liquids is less than that in air, (according to the corpuscular theory of light, the speed of light would be higher in glasses or in liquids than in air). Additional developments during the nineteenth century led to the general acceptance of the wave nature of light.

The most important development concerning the nature of light was the work of Maxwell, who, in 1873, showed that the light was a form of high frequency electromagnetic waves. The theory predicts that these waves should have a velocity of about 3×10^8 m $\sec^{-1}$. Within experimental error, this value turned out to be equal to the velocity of light. Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves. Furthermore, he and other investigators showed that these waves could be reflected and refracted and so on. In other words, electromagnetic waves exhibited all the characteristic effects of light waves.

An electromagnetic wave travels with the velocity of light in free space and is accompanied by vibrating electric and magnetic fields which are mutually perpendicular and also perpendicular to the direction of the propagation of the wave as shown in Fig. 9.1, where the electric field $\overrightarrow{E}$ is shown in the vertical plane. The magnetic field $\overrightarrow{H}$ is shown in the horizontal plane, the electric field and magnetic field are at right angles to each other as they travel through space.

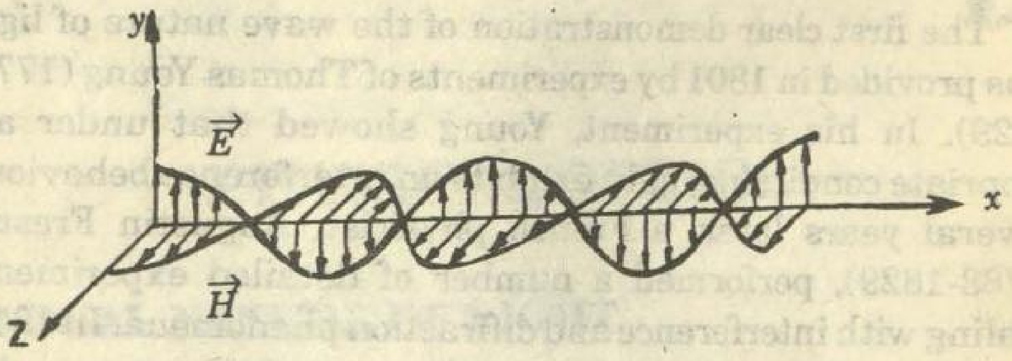


Fig. 9.1

Although the classical theory of electromagnetism was able to explain most of the known properties of light, subsequent discoveries demonstrated its shortcomings. In photoelectric effect, which is the ejection of electrons from a metal whose surface is exposed to light, it was experienced that the kinetic energy of an individual electron is independent of the light intensity. An explanation of the phenomenon could only be given by Einstein in 1905 on the basis of quantum theory. According to this theory the light waves exist in discrete bundles called photon. The energy E of a photon is proportional to the frequency 'ν' of the electromagnetic waves, so that

$$E = h\nu \qquad (9.1)$$

where, $h = 6.63 \times 10^{-34}$ J sec and is called Planck's constant. It is important to note that this theory retains some features of both the wave theory and the corpuscular theory of light. The photoelectric effect is the result of energy transfer from a single photon to an electron in the metal.

The energy of the ejected electrons does not change with intensity, but depends only on the frequency of incident light in a manner consistent with Eq. 9.1. Einstein's theory suggested that if the light wave had a frequency below some threshold value (which depends on the type of metal), no photoelectric effect would occur regardless of the fact how intense is the source of light.

One must regard light as having dual nature, depending on the phenomena under consideration. On one hand, the classical electromagnetic wave theory provides an adequate explanation of light propagation and the effect of interference and diffraction. On the other hand, the photoelectric effect and other experiments involving the interaction of light with matter are best explained by particle model.

9.2 WAVEFRONTS AND HUYGENS' PRINCIPLE

When the surface of water in a tank is disturbed periodically by dipping a ball point into it, circular ripples are produced which travel outward. These ripples are said to form a circular wavefront.

The circle 'a' represents a crest, as does the circle 'b'. Between these circles there is another circle '*t*', which is a trough. In general, *a wave front is defined as the locus of points having the same phase of vibration*. All the points on 'a' have the same phase of vibration, since all the molecules on this particular wavefront are at their maximum displacement above the normal level of water. Likewise, '*t*' is a wavefront on which all the particles are at their maximum displacement

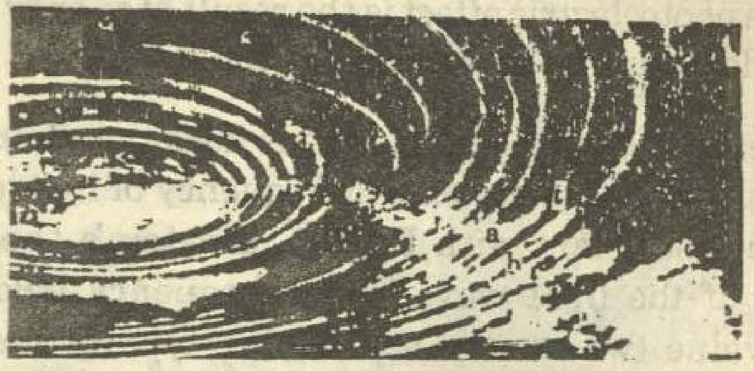


Fig. 9.2 *Water Waves*

below the normal position of water. Halfway between 'b' and '*t*' is another wavefront where the particles are momentarily at the normal level.

Geometrically, concentric circles are drawn to represent the spherical wavefronts in such a way that adjacent circles are one wavelength, λ, apart. The radial lines pointing outward from the centre (source) are called rays (Fig. 9.3a).

If we consider a small portion of a wavefront at large distance from the source (Fig. 9.3b), the rays are nearly

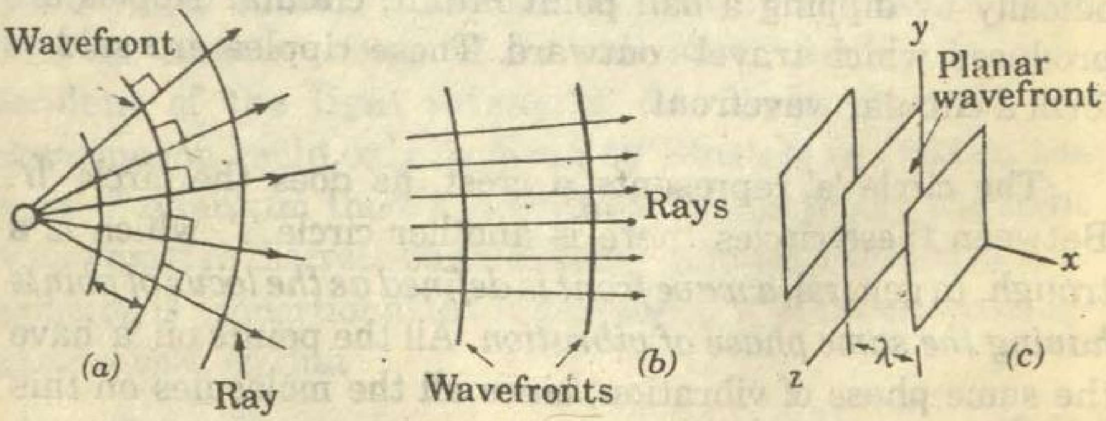

Wavefront
(a)
Ray
Rays
Wavefronts
(b)
y
Planar wavefront
x
z
λ
(c)

Fig. 9.3 *Different types of Wave-fronts.*

parallel and wavefronts are very close to being plane. Therefore, at large distances from the source spherical wavefronts reduce to plane wave-fronts. Fig. 9.3c illustrates a set of plane wavefronts propagating along the x-axis. If x is taken to be the direction of the wave motion then the wavefronts are parallel to the yz-plane.

In Huygens' construction, *all points on a given wavefront are taken as point sources for the production of spherical secondary waves, called wavelets,* which propagate outward with the same speed.

Fig. 9.4 illustrates two simple examples of Huygen s construction. First, consider the plane wavefront moving through medium as in Fig. 9.4a. At $t = 0$, the wavefront is indicated by the plane labelled AA′. According to Huygens' principle, each point on this wavefront is considered as a point source. Only a few points on AA′ are shown for clarity. Using these points as sources for the wavelets, we draw circles of radius 'ct', where 'c' is the speed of light and 't' is the time of propagation from one wavefront to the next. The plane tangent to these wavelets is BB′, which is parallel to AA′. In a similar manner Fig. 9.4b, shows Huygen s construction for spherical wavefront.

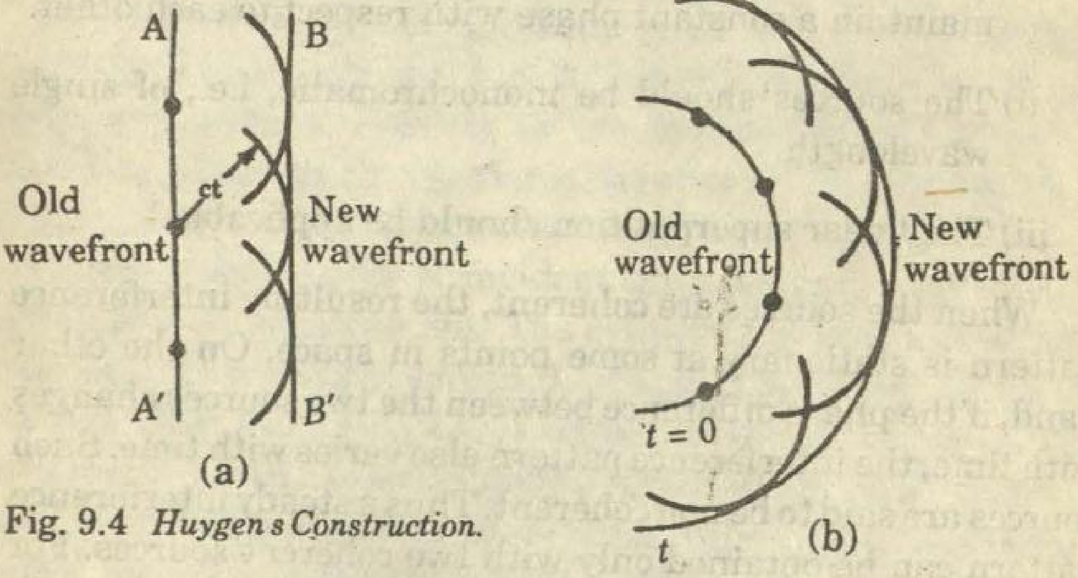


Fig. 9.4 *Huygen s Construction.*

9.3. INTERFERENCE - YOUNG'S DOUBLE SLIT EXPERIMENT

In chapter 8 we found that interference results from the superposition of two waves. *If the two waves superimpose so as to produce a resultant wave of amplitude greater than the amplitudes of original waves the interference is called constructive one. On the other hand, if the two waves combine so as to produce a resultant wave of amplitude less than the amplitude of the original waves the interference is called destructive one.* Interference effects associated with light are not easy to observe because of its short wavelength (about 5×10^{-7}m). In order to obtain interference of light waves, the following conditions must be met.

i) The sources must be phase coherent, i.e., they must maintain a constant phase with respect to each other.

ii) The sources should be monochromatic, i.e., of single wavelength.

iii) The linear superposition should be applicable.

When the sources are coherent, the resulting interference pattern is stationary at some points in space. On the other hand, if the phase difference between the two sources changes with time, the interference pattern also varies with time. Such sources are said to be non coherent. Thus a steady interference pattern can be obtained only with two coherent sources. For example, two radio antennas driven by the same oscillator will maintain a constant relative phase. In this case, the phase of the source is effectively "locked in" by a common driving force. On the other hand, two independent ordinary light sources are incoherent, since the sources of the light waves are randomly vibrating atom.

It is possible to overcome this difficulty by using one

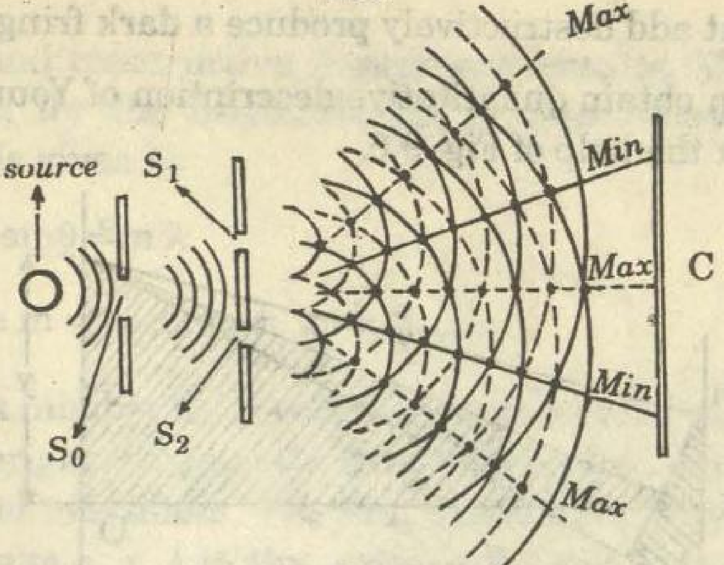


Fig. 9.5 *Huygen's construction of Interference pattern.*

chromatic source to illuminate a screen containing two small holes. The light emerging from the two holes is then coherent, since it originates from a common source.

The phenomenon of interference of light waves arising from two sources was first demonstrated by Thomas Young in 1801. A schematic diagram of the apparatus used in this experiment and the Huygen's construction of the interference pattern observed in this experiment are shown in Figs 9.5 and 9.6 respectively. Light is incident on a screen A, which is provided with a narrow slit. The cylindrical waves emerging from this slit arrives at screen B, which contains two narrow, parallel slits, S_1 and S_2. Light emerges from the two slits as cylindrical waves. (In his original experiment, Young actually used sunlight as source, and screens contained pinholes rather than slits.). In fact slits S_1 and S_2 act as individual light sources that are in phase since they originate from the same cylindrical wavefront. The light from the two slits produces a visible pattern on screen C; the pattern consists of a series of light and dark parallel bands called fringes. The overall light amplitude at a given point on the screen is the result of superposition of two wave amplitudes from S_1 and S_2. Two waves that add constructively give a bright fringe and any two waves that add destructively produce a dark fringe.

We can obtain quantitative description of Young's experiment with the help of Fig. 9.6.

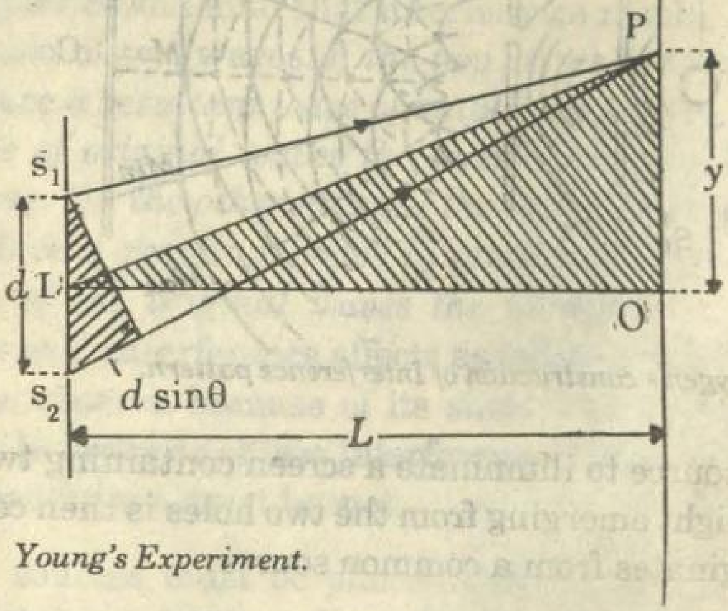


Fig. 9.6 *Young's Experiment.*

Consider a point P on the screen; this point is located at a perpendicular distance 'L' from the two identical slits S_1 and S_2, which are separated by a distance '*d*'. Let us assume that the source is equidistant from the two slits and is monochromatic, i.e., emitting light of single wavelength. Under these assumptions, the waves emerging from S_1 and S_2 have the same frequency and amplitude and are in phase. The light amplitude on the screen at P is the resultant of light amplitudes coming from both slits. Note that a wave from the lower slit S_2 ; travels farther than a wave from the upper slit, S_1, by an amount equal to $d \sin\theta$. This distance is called the path difference S, where

$$S = r_2 - r_1 = d \sin\theta \tag{9.2}$$

As shown in Fig. 9.6. the value of this path difference will determine whether or not the two waves are in phase when they arrive at P. If the path difference is either zero or some integral multiple of the wavelength, the two waves are in phase at P and constructive interference results. Therefore the condition for the bright fringes or constructive interference at P is given by

$$S = d\sin\theta = m\lambda \tag{9.3}$$

where m = 0, ± 1, ± 2, ± 3, ...

The index number 'm' is called order number of the fringe. The central bright fringe at $\theta = 0$ (m = 0) is called the zeroeth order fringe or maximum. The first maximum on either side is given when m = ± 1 in the equation 9.3 and is called first order maximum. Similarly, when the path difference is an odd multiple of $\lambda/2$ the two waves arriving at P will be out of phase and will give rise to destructive interference (dark fringe). Therefore the condition for the dark fringes at P is given by

$$S = d\sin\theta = (m + \tfrac{1}{2})\lambda \tag{9.4}$$

where m = ± 1, ± 2, ± 3, ...

It is useful to obtain expressions for the positions of bright and dark fringes measured vertically from 'O' to 'P'. We shall assume that L > d (Fig. 9.6) and consider only points P that are close to O. In this case, θ is small, and so we can use the approximation $\sin\theta \approx \tan\theta \approx \theta$ (is in radians). From the triangle OPQ, we see that

$$\sin\theta \approx \tan\theta = \frac{y}{L} \tag{9.5}$$

Using this result together with equation 9.3. we see that positions of the bright fringes measured from O are given by

$$y_{bright} = \frac{\lambda L}{d} m \tag{9.6}$$

From this expression we find that the separation between any two adjacent bright fringes is equal to $\frac{\lambda L}{d}$ called fringe spacing, i.e

$$y_{m+1} - y_m = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d}m = \frac{\lambda L}{d} \qquad (9.7)$$

Similarly, using Eq. 9.5 together with Eq. 9.4 we find that dark fringes are located at

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \qquad (9.8)$$

This result shows that the separation between adjacent dark fringes is also equal to $\frac{\lambda L}{d}$. Since the quantities L and *d* are both measureable, we see that the double slit interference pattern, together with Eq. 9.7, provides a direct determination of the wavelength λ. Young used this technique to make the first measurement of the wavelength of light.

Example 9.1

A screen is separated from a double slit source by 1.2 m. The distance between the two slits is 0.03 mm. The second order bright fringe is measured to be 4.5 cm from the centre line. Determine (a) the wavelength of the light, (b) the fringe spacing.

Solution

(a). We can use Eq. 9.6 with m = 2, $y = 4.5 \times 10^{-2}$ m, $d = 3 \times 10^{-5}$m and L = 1.2 m

$$y = \frac{\lambda L}{d} m$$

$$4.5 \times 10^{-2} = \frac{\lambda \times 1.2}{3 \times 10^{-5}} \times 2$$

$$\lambda = 5.625 \times 10^{-7}\, m$$

(b) Fringe width $= \dfrac{\lambda L}{d}$

$$= \frac{5.625 \times 10^{-7}\,(1.2)}{3 \times 10^{-5}}$$

$$= 2.25 \times 10^{-2}\, m$$

9.4 INTERFERENCE IN THIN FILMS

Interference effects are commonly observed in thin films, such as thin layers of oil on water and soap bubbles. The various colours that are observed with ordinary white light result from the interference of waves reflected from the opposite surfaces of the film.

Consider a thin film of uniform thickness '*t*' and of index of refraction 'n' as in Fig. 9.7. Let us assume that the light rays travelling in air are nearly normal to the surface. To determine whether the reflected rays interfere constructively or destructively, we must first note the following facts:

i) A wave that travels from a medium of lower refractive

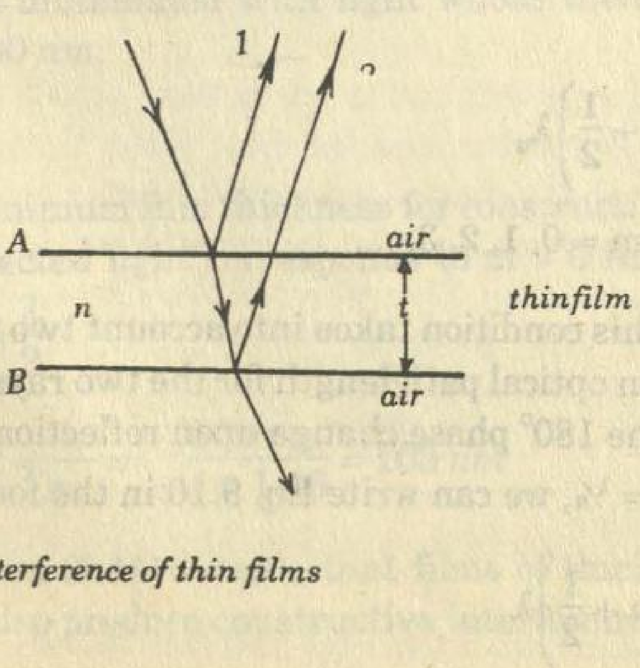


Fig. 9.7 *Interference of thin films*

index to one of higher refractive index undergoes a 180° phase change relative to the incident wave on reflection. There is no phase change in the reflected wave if it travels from a medium of higher refractive index to one of lower refractive index.

ii) The wavelength of light λ_n in a medium whose refractive index is 'n' is given by

$$\lambda_n = \frac{\lambda}{n} \tag{9.9}$$

where λ is the wavelength of light in free space.

Let us apply these rules to the thin films described above. According to the first rule, ray 1, which is reflected from the upper surface A, undergoes a phase change of 180° with respect to the incident wave. On the other hand, ray 2, which is reflected from the lower surface B, undergoes no phase change with respect to the incident wave. Therefore ray 1 is 180° out of phase with respect to ray 2, which is equivalent to a path difference of $\lambda_{n/2}$, since the path difference between the two rays for normal incidence is twice the film thickness or 2t, we can write the condition for constructive interference in the form

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \tag{9.10}$$

where, m = 0, 1, 2, 3, ...

Note that this condition takes into account two factors: (a) the difference in optical path length for the two rays (the term $m\,\lambda_n$) and (b) the 180° phase change upon reflection (the term $\lambda_{n/2}$). Since $\lambda_n = \lambda/n$, we can write Eq. 9.10 in the form,

$$2nt = \left(m + \frac{1}{2}\right)\lambda \tag{9.11}$$

In a similar manner, the condition for destructive interference is

$$2nt = m\lambda \qquad (9.12)$$

Note that these conditions for constructive and destructive interference are valid only when the film is surrounded by a common medium. The surrounding medium may have a refractive index less than or greater than that of the film. In either case, the rays reflected from the two surfaces will be out of phase by 180°. On the other hand, if the film is located between two different media, one of lower refractive index and one of higher refractive index, the conditions for constructive and destructive interference are reversed. In this case, either there is a phase change of 180° for both ray 1 reflecting from surface A and ray 2 reflecting from surface B or there is no phase change for either ray; hence, the net change in relative phase due to the reflection is zero.

Example 9.2

Calculate the thickness of a soap bubble film, $n = 1.46$, that will result in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is 600 nm.

Solution

The minimum film thickness for constructive interference in the reflected light corresponds to m = 0 in Eq. 9.11. This gives $2nt = \frac{\lambda}{2}$

$$\text{or } t = \frac{\lambda}{4n} = \frac{600\,nm}{4 \times 1.46} = 103\,nm$$

Equation (9.11) reveals that films of thickness 3t, 5t, 7t etc., will also produce constructive interference.

Example 9.3

Solar cells are often coated with transparent thin film such as silicon monoxide (n = 1.45), in order to minimize reflective losses from the surface (Fig. 9.8). A silicon solar cell (n = 3.5) is coated with a thin film of SiO for this purpose. Determine the minimum thickness of the SiO that will produce the least reflection at a wavelength of 550 nm.

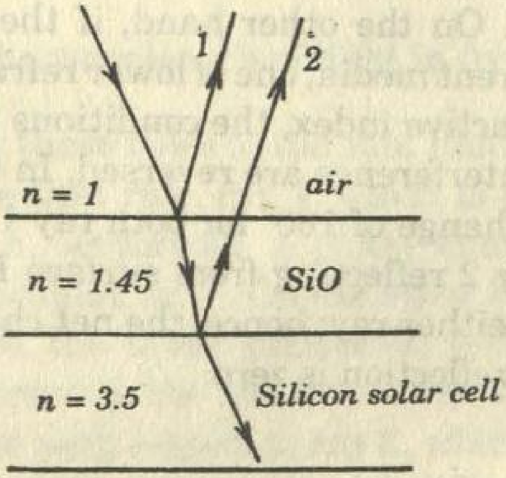


Fig. 9.8 *Reflective losses are minimised in a silicon solar cell by coating it with a thin film of SiO.*

Solution

The reflection is minimum when rays 1 and 2 meet the condition of destructive interference. Note that both rays undergo a 180° phase change upon reflection in this case, one from the upper and one from the lower surface. Hence, the net change in phase is zero. Therefore, the condition for a reflection minimum is given by Eq. 9.11. Since the minimum thickness corresponds to m = 0, we get $2nt = \frac{\lambda}{2}$, or

$$t = \frac{\lambda}{4n} = \frac{550\ nm}{4 \times 1.45} = 94.8\ \text{nm} = 948\ \text{A}^\circ.$$

Typically, such anti reflecting coatings reduce the reflective loss from 30% to 10%. Such coatings will therefore increase the cell's efficiency, since more light will be available to create charge carriers in the silicon cell. Note that in reality, the coating will never be perfectly non reflecting because the required thickness is wavelength - dependent and the incident light covers a wide range of wavelengths.

Glass lenses used in cameras and other optical instruments are often coated with a transparent thin film, such as Magnesium Fluoride (Mg F_2), to reduce or eliminate unwanted reflections.

Example 9.4

A thin wedge-shaped film of refractive index n is illuminated with monochromatic light of wavelength as illustrated in Fig. 9.9. Describe the interference pattern observed in this case.

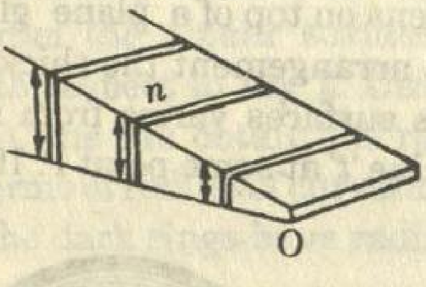


Fig. 9.9 *A thin wedge shaped film.*

Solution

The interference pattern will be equivalent to that of a thin film of variable thickness surrounded by air. Hence, the pattern will be an alternating series of bright and dark parallel bands. A dark band corresponding to destructive interference appears at the apex O since the upper reflected ray undergoes a 180° phase change while the lower wave does not. According to Eq. 9.12, other dark bands appear when

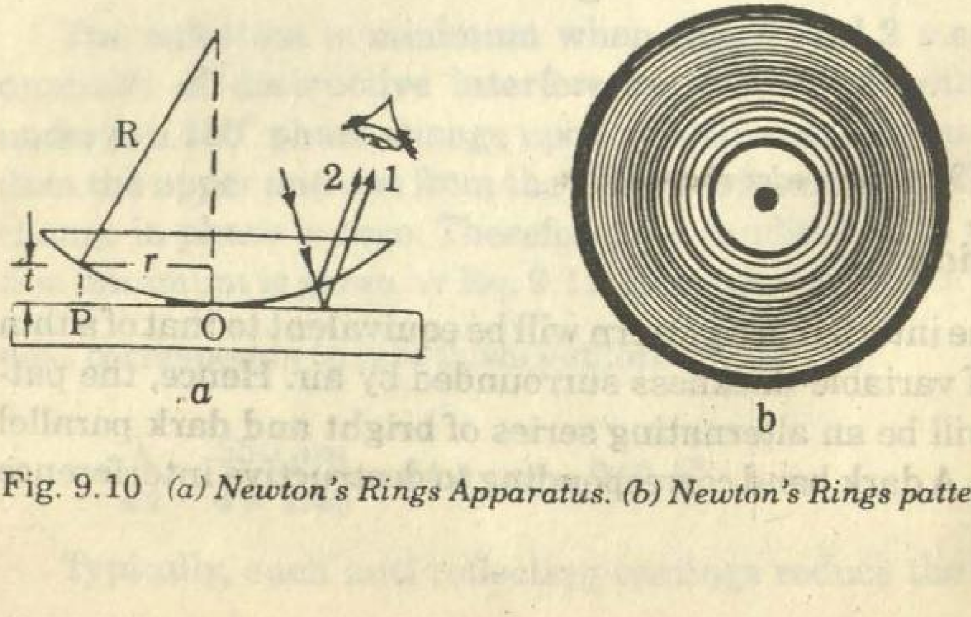
$$2nt = m\,\lambda,$$

$$\text{so that } t_1 = \frac{\lambda}{2n},\ t_2 = \frac{\lambda}{n},\ t_3 = \frac{3\,\lambda}{2n}, \text{ etc.}$$

Similarly, bright bands will be observed when the thickness satisfies the condition $2nt = \left(m + \frac{1}{2}\right)\lambda$, corresponding to the thickness of $\frac{\lambda}{4n}, \frac{3\,\lambda}{4n}, \frac{5\,\lambda}{4n}$, etc. If white light is used to observe the pattern, bands of different colours will be observed at different points, corresponding to the different wavelength of light.

9.5 NEWTON'S RINGS

Another method for observing interference of light waves is to place a plano-convex lens on top of a plane glass surface as in Fig. 9.10a. With this arrangement the thickness of the air film between the glass surfaces varies from zero at the point of contact to some value '*t*' at some point P. If the radius



a

b

Fig. 9.10 *(a) Newton's Rings Apparatus. (b) Newton's Rings pattern.*

of curvature of lens R is very large as compared with the distance '*r*' and if the system is viewed from above using light of wavelength λ, a series of dark rings is observed. A photograph of such a pattern is shown is Fig. 9.10b. These circular fringes, discovered by Newton, are called Newton's Rings. Newton's particle theory of light could not explain the origin of these rings.

In Fig. 9.10(a), the interference effect is due to the combination of ray 1, reflected from the glass plate, with ray 2 reflected from the lower part of the lens. Ray 1 undergoes a phase change of 180° upon reflection, since it is reflected from a medium of higher index of refraction, whereas ray 2 undergoes no phase change. Hence, the conditions for constructive and destructive interference are given by Eq. 9.11 and 9.12 respectively, with n = 1 since the film is that of air. Here again one might guess that the contact point O would be bright, corresponding to constructive interference. Instead, the contact point is dark, as seen in Fig. 9.10 (b), because ray 1, reflected from the planar surface, undergoes a 180° phase change with respect to ray 2. Using the geometry shown in Fig. 9.10(a), one can obtain the radii of the bright and dark bands in terms of radius of curvature R and wavelength λ. For example, the dark rings have radii given by

$$r = \sqrt{m \lambda R}$$

The details of the proof are left as those are out of the scope of this book. By measuring the radii of the rings, one can obtain the wavelength λ, provided R is known.

9.6 THE MICHELSON INTERFEROMETER

The interferometer, invented by American physicist A.A Michelson (1852 - 1931), is an ingenious device which splits a light beam into two parts and then recombines them to form an interference pattern after they have travelled over different paths. The device can be used for obtaining accurate measurements of wavelength and for precise length measurements.

A schematic diagram of the interferometer is shown in Fig. 9.11. A beam of light provided by a monochromatic source is split into two rays by a semi silvered mirror M inclined at 45° relative to the incident light beam. One ray is reflected vertically upward toward mirror M_1 while the second ray is transmitted horizontally through M toward mirror M_2. Hence, the two rays travel separate paths l_1 and l_2. After reflecting from mirrors M_1 and M_2 the two rays eventually recombine to produce an interference pattern, which can be viewed through a telescope. The glass plate P, equal in thickness to M, is placed in the path of the horizontal ray in order to equalize the path length of the two rays. With this arrangement each ray will then pass through the same thickness of glass.

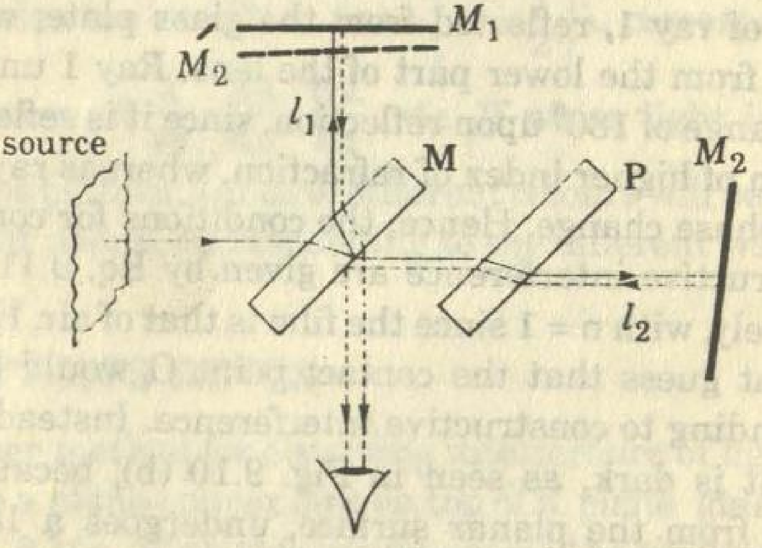


Fig. 9.11 *Michelson Interferometer.*

The interference condition for the two rays is determined by the difference in the optical path lengths. When the two rays are viewed as shown, the image of M_2 is at M_2' parallel to M_1. Hence M_1 and M_2' form the equivalent of a parallel air film.

The effective thickness of the film is varied by moving mirror M_1 parallel to itself with a finely threaded screw. Under these conditions, the interference pattern is series of bright and dark circular rings which resemble Newton's rings. If a dark circle appears at the centre of the pattern, the two rays interfere destructively. If the mirror M_1 is moved a distance of $\frac{\lambda}{4}$, the path difference changes by $\frac{\lambda}{2}$ (twice the separation between M_1 and M_2'). The two rays will now interfere constructively, giving a bright circle in the middle. As M_1 is moved, an additional distance of $\frac{\lambda}{4}$ (total distance of $\frac{\lambda}{2}$), a dark circle will appear once again. Thus we see that successive dark and bright circles are formed each time M_1 is moved a distance $\frac{\lambda}{4}$. The wave length of light is then measured by counting the number of fringe shifts for a given displacement of M_1. Conversely, if the wavelength is accurately known, mirror displacements can be measured to within a fraction of the wavelength. With this discussion, we infer that if we have a dark fringe, say, in the beginning, the next dark fringe will appear by moving the mirror M_1 to a distance of $\frac{\lambda}{2}$ every time. If number of fringes are passed through a reference point when the mirror M_1, is moved a distance p, then

$$p = \frac{1}{2} m \lambda, \qquad (9.13)$$

Since the Interferometer can accurately measure displacement, it is often used to make highly precise measurements. The fundamental difinition of the meter is based on a certain wavelength of light from krypton-86. The interferometer makes such precise measurements of wavelength possible.

9.7 DIFFRACTION

In the previous sections we have found that light passing through two slits does not produce two distinct bright regions on a screen. Instead, an interference pattern is observed which shows that the light has deviated from a straight-line path and has entered the otherwise shadowed region. This deviation of light from a straight line path is called *DIFFRACTION*. Diffraction result from the interference of light from many coherent sources. In principle, the intensity of diffraction pattern at a given point in space can be computed using Huygens principle.

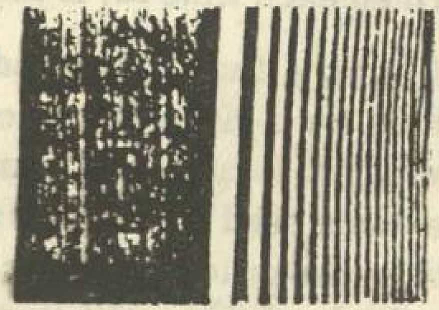


Fig. 9.12 *Diffraction pattern.*

In general, diffraction occurs when waves pass through holes or around obstacles. For example, if an opaque object is placed between a point source and screen, the boundry between the shadowed and illuminated region is not very sharp. A careful inspection of the boundry shows a series of bright and dark bands within the geometric shadow (Fig. 9.12) which is due to the bending of light around the edge of the obstacle.

Diffraction phenomena are usually classified as being one of two types, which are named after the men who first explained them. The first type, called Fraunhofer Diffraction, occurs when the rays reaching a point are approximately parallel. This can be achieved experimentally either by placing the screen at a large distance from the hole or by using a converging lens to focus parallel rays on the screen, as shown in Fig. 9.13a. Note that a bright fringe is observed along the axis at $\theta = 0$, with alternating bright and dark fringes on either side of the central bright fringe.

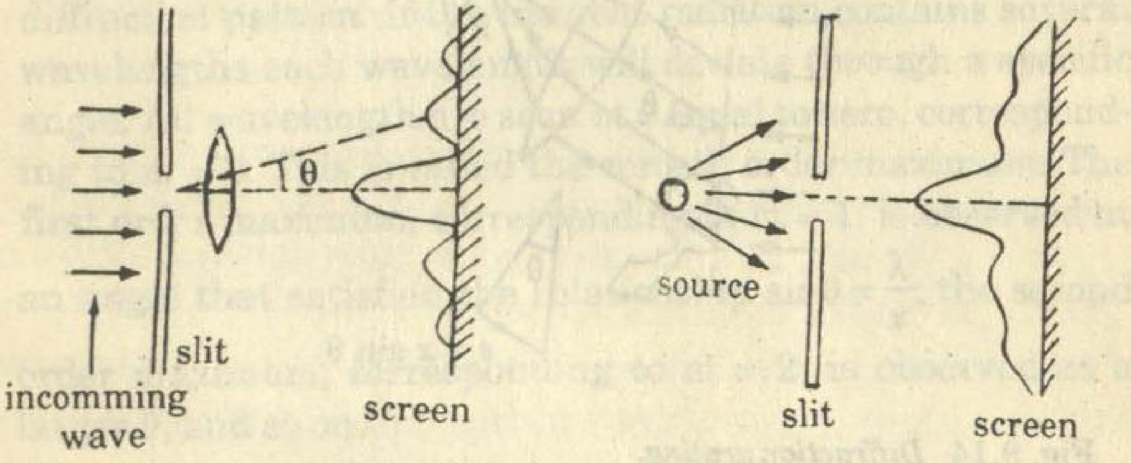


Fig. 9.13 *(a) Fraunhofer Diffraction Pattern.*
(b) Fresnel Diffraction Pattern.

When the screen is placed at a finite distance from the slit and no lens is used to focus the parallel rays, the observed pattern is called a *Fresnel Diffraction Pattern* Fig. 9.13b.

9.8 THE DIFFRACTION GRATING

The diffraction grating, a very useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A grating can be made by scratching parallel lines on a glass plate with a precision machining technique. The spaces between each scratch are transparent to the light and hence act as separate slits. A typical grating contains several thousand lines per centimetre. For example, grating ruled with 5000 lines cm^{-1} has a slit spacing x equal to the inverse of this number,

$$x = \left(\frac{1}{5000}\right) cm = 2 \times 10^{-4}\ cm.$$

A schematic diagram of a section of plane diffraction grating is illustrated in Fig. 9. 14. A plane wave is incident from the left, normal to the plane of the grating. A converging lens can be used to bring the rays together at the point P. The intensity of the observed pattern on the screen is the result of the combined effects of interference and diffraction. Each slit produces diffraction, as was described in the previous section. The diffracted beams in turn interfere with each other to produce the final pattern. Each slit acts as a source of waves, where all waves start at the slits in phase. However, for some arbitrary direction θ measured from the horizontal, the waves must travel different path lengths before reaching a particular point P on the screen. From Fig. 9.14 note that the path difference between waves from two adjacent slits is equal to x sinθ. If this path difference equals one wavelength or some

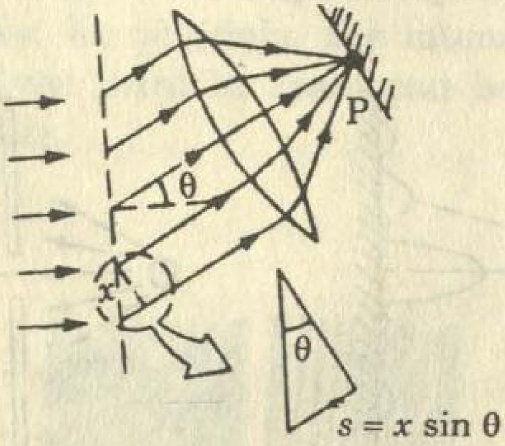


Fig. 9.14 *Diffraction grating.*

integral multiple of a wavelength, waves from all slits will be in phase at P and a bright line will be observed. Therefore, the condition for maxima in the interference pattern at the angle θ is,

$$x \sin\theta = m\lambda \tag{9.14}$$

where, m = 0, 1, 2, 3, ...

This expression can be used to calculate the wavelength from a knowledge of the grating spacing and the angle of deviation θ. The integer m represents the order number of the diffraction pattern. If the incident radiation contains several wavelengths each wavelength will deviate through a specific angle. All wavelength are seen at θ equal to zero, corresponding to m = 0. This is called the zeroeth order maximum. The first order maximum, corresponding to m = 1, is observed at an angle that satisfied the relationship $\sin\theta = \frac{\lambda}{x}$; the second order maximum, corresponding to m = 2, is observed at a larger θ, and so on.

A simple arrangement that can be used to measure the various orders of the diffraction pattern is shown in Fig. 9.15. This is a form of diffraction grating spectrometer (discussed in Chapter 10). The light to be analyzed passes through a slit, and a parallel beam of light's incident from the collimator, which is perpendicular to the grating. The diffracted light leaves the grating at angles that satisfies the grating equation. A telescope is used to view the image of the slit. By measuring the precise angles at which the images of the slit appear for the various orders, the wavelength can be determined.

The diffraction grating is most useful for taking accurate wavelength measurement. Like the prism, the diffraction grating can be used to disperse a spectrum into its com-

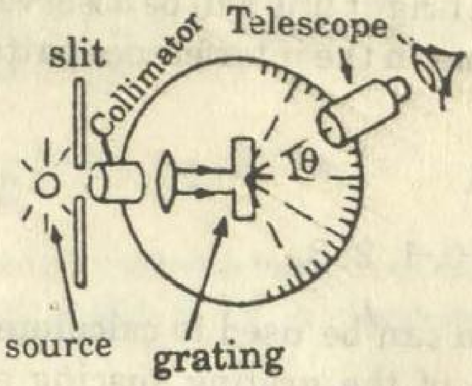


Fig. 9.15 *Arrangement to measure the various orders of the Diffraction pattern.*

ponents. Of the two devices, the grating is more precise if one wants to distinguish between two closely spaced wavelengths. We say that the grating spectrometer has a greater or higher resolution than a prism spectrometer. If λ_1 and λ_2 are the two nearly equal wavelengths between which the spectrometer can just barely distinguish, the resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta \lambda} \qquad (9.15)$$

where $\lambda \approx \lambda_1 \approx \lambda_2$ and $\Delta \lambda = \lambda_2 - \lambda_1$. Thus we see that a grating with a high resolving power can distinguish small wavelength difference. Furthermore, if N lines of the grating are illuminated, it can be shown that the resolving power in the mth order diffraction equals the product Nm

$$R = N \cdot m \qquad (9.16)$$

Example 9.5

Monochromatic light from a helium-neon source (Laser with $\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6000 lines cm^{-1}. Find the angle at which one would observe the first order maximum, the second order maximum, etc.

Solution

First, we must calculate the slit separation, which is equal to the inverse of number of lines per cm:

$$x = \left(\frac{1}{6000}\right) cm = 1.667 \times 10^{-4}\ cm = 1667\ nm$$

For the first order maximum (m = 1) we get

$$\sin\theta_1 = \frac{\lambda}{x} = \frac{632.8\ nm}{1667\ nm} = 0.3796$$

or $\theta_1 = 22.31^{\circ}$

Likewise, for m = 2, we find

$\theta_2 = 49.39^{\circ}$

However for m = 3, $\sin\theta_3 = 1.139$. Since $\sin\theta$ cannot exceed unity, so third order diffraction is not possible.

9.9 DIFFRACTION OF X-RAYS BY CRYSTALS

We have seen that the wavelength of light can be measured with a diffraction grating having a known number of rulings per unit length. In principle, the wavelength of any electromagnetic wave can be determined if a grating of proper spacing (of the order of λ) is available. X-rays, discovered by W. Roentgen (1845-1923) in 1895, are electromagnetic waves with very short wavelengths (of the order of 1 °A). Obviously, it would be impossible to construct a grating with such a small spacing. However, the atomic spacing in a solid is known to be about 10^{-10} m. In 1913, Max von Laue (1879-1960) suggested that the regular array of atoms in a crystal could act as a three dimensional diffraction grating for X-rays. Subsequent ex-

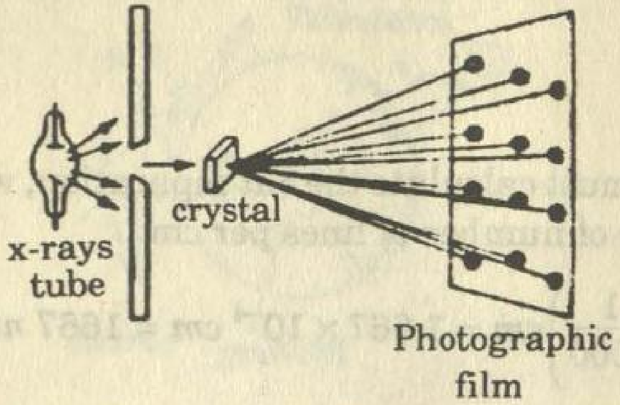


Fig. 9.16 *Diffraction of X-rays by crystals.*

periment confirmed this prediction. The diffraction patterns that one observes are rather complicated because of the three dimensional nature of the crystal. Nevertheless, X-ray diffraction has proved to be an invaluable technique for determining crystalline structures and understanding structure of matter. Fig. 9.16 is an experimental arrangement for observing X-ray diffraction from a crystal, such as one of sodium chloride. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams can be detected by a photographic film where they form an array of spots known as a 'Laue Pattern'. The crystalline structure is determined by analyzing the positions and intensities of various spots in the pattern.

The arrangement of atoms in a crystal of NaCl is shown in Fig. 9.17. the smaller, dark spheres represent Na^+ ions and larger hollow spheres represent Cl^- ions.

A careful examination of the NaC1 structure shows that the ions appear to lie in various planes. The shaded areas in Fig. 9.17 represents one example in which the atoms lie in equally spaced planes. When an X-ray beam is incident at an

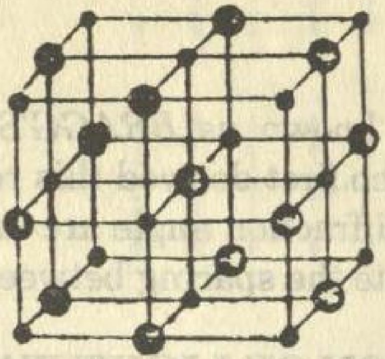


Fig. 9.17 *Sodium chloride crystal structure.*

angle θ on one of the planes, as in Fig 9.18., the beam can be reflected from both the upper and lower plane of atoms. However the geometric construction in Fig. 9.18 shows that the beam reflected from the lower surface travels farther than the beam reflected from the upper surface. The effective path difference between the two beams is 2*y* sinθ as shown in the Fig. 9.18. The two beams will reinforce each other (constructive interference) when this path difference equals some integral multiple of the wavelength λ. The same is true for the reflection from the entire family of parallel planes. Hence, the

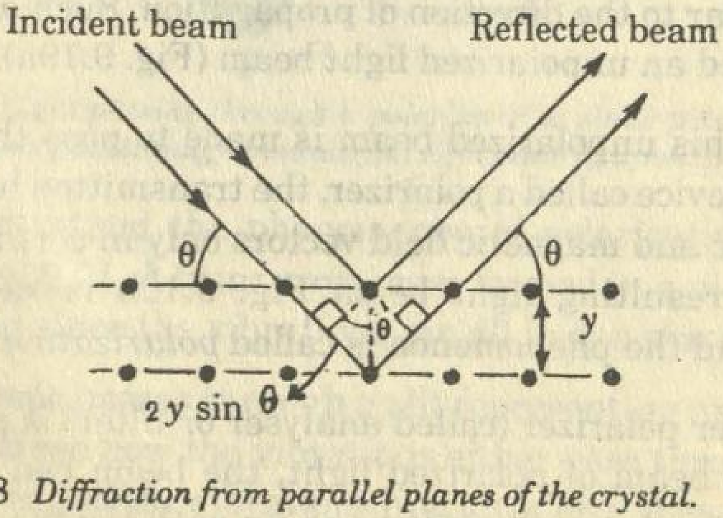


Fig. 9.18 *Diffraction from parallel planes of the crystal.*

condition for constructive interference (maximum in diffracted waves) is given by

$$2y \sin\theta = m\,\lambda. \qquad (9.17)$$

This condition is known as *BRAGG'S LAW* after W.L. Bragg (1890-1971), who first derived this relationship. If the wavelength and the diffraction angle are measured, Eq. 9.17 can be used to calculate the spacing between atomic planes.

9.10 POLARIZATION OF LIGHT WAVES

In previous sections we have established the wave nature of light by discussing its application to explain the phenomena of interference and diffraction. However, this does not establish that light waves are transverse in nature. That is, the electric and magnetic vectors, which are at right angles to each other, are also perpendicular to the direction of propagation of the wave, the phenomenon of polarization establishes transverse nature of light and electromagnetic waves in general.

A beam of ordinary light consists of a large number of waves, each in its own plane of vibration. In this case all directions of vibration are equally probable and are always perpendicular to the direction of propagation. Such a beam of light is called an unpolarized light beam (Fig. 9.19a).

Now if this unpolarized beam is made to pass through a polarizing device called a polarizer, the transmitted beam will have electric and magnetic field vectors only in certain directions. The resulting light beam Fig. 9.19b is said to be polarized and the phenomenon is called *polarization*.

If another polarizer (called analyser or filter) is placed in front of the beam of polarized light, the beam can only get through when the analyser is lined up with the plane of the

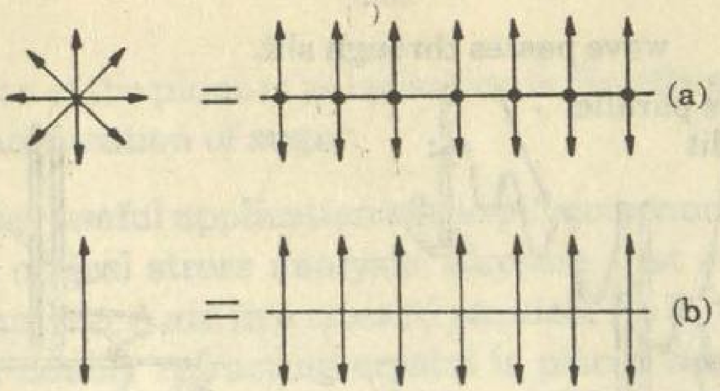


Fig. 9.19 *(a) Unpolarized ordinary light. (b) Plane polarized light.*

polarized light. If the two are not aligned, the polarized light is absorbed and no light at all passes. This is shown in Fig. 9.20.

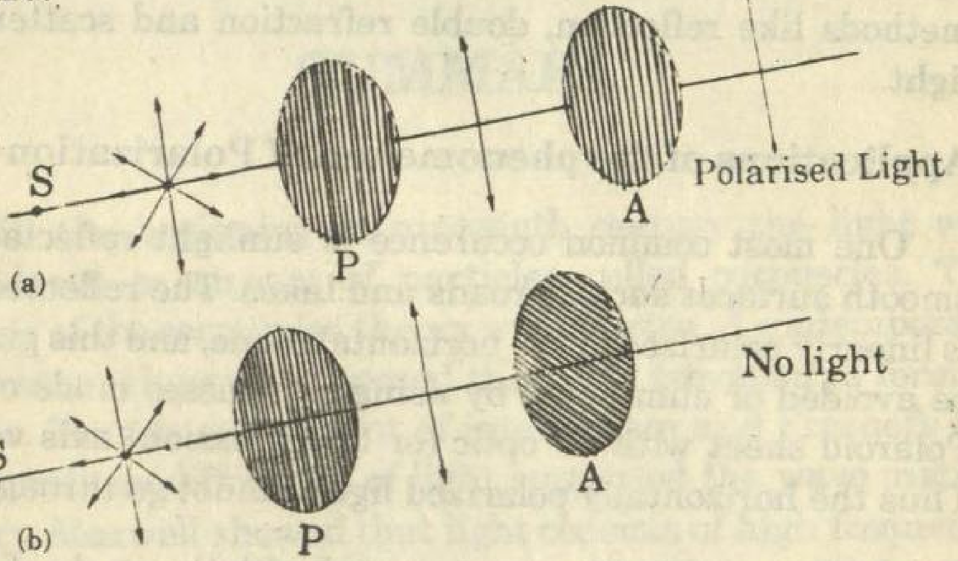


Fig. 9.20 ***Light passing through a polarizer P is plane polarized. Light may or may not pass through the analyser A, depending upon its orientation.***

To understand the phenomenon of polarization of light refer to Fig. 9.21. A transverse wave formed in a piece of rope is polarized since the vibrations are all in one plane.

If the rope passes through a slit representing an analyser, it is easy to see how the vibrations either pass through when the analyser and the vibrations line up or how they are cut off

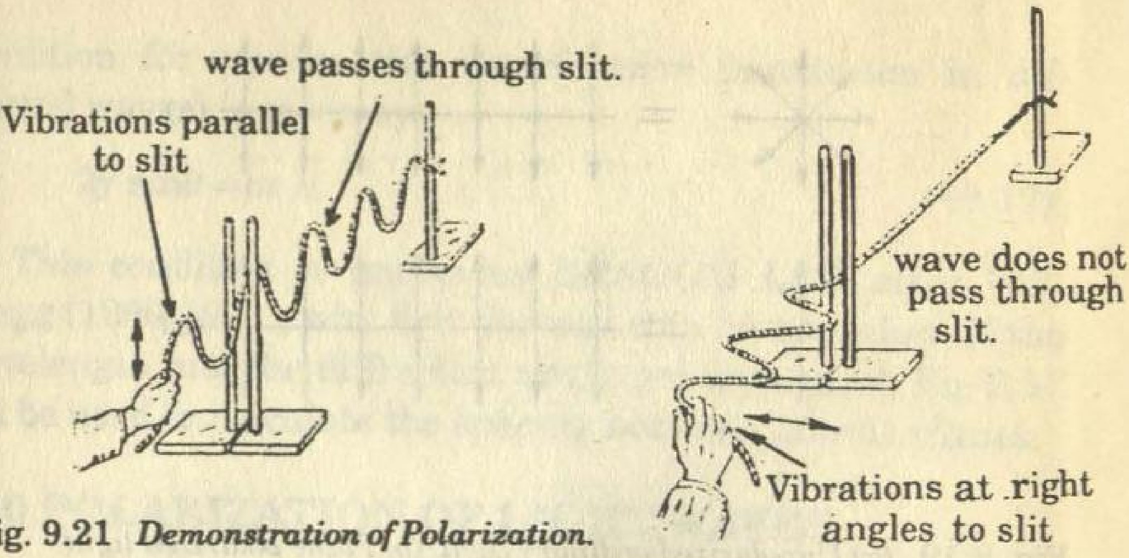

wave passes through slit.
Vibrations parallel to slit
wave does not pass through slit.
Vibrations at right angles to slit

Fig. 9.21 *Demonstration of Polarization.*

and absorbed when they are not aligned. The method of polarizing the light discussed above is called polarization by selective absorption. However, light can be polarized by other methods like reflection, double refraction and scattering of light.

Applications of the phenomenon of Polarization

One most common occurence is sunlight reflected from smooth surfaces such as roads and lakes. The reflected light, is linearly polarized in the horizontal plane, and this glare can be avoided or eliminated by using sunglasses made out of a Polaroid sheet with its optic (or transmission) axis vertical. Thus the horizontally polarized light cannot go through.

Another application is connected with the study of optical activity. When a beam of light is made to pass through certain crystals or liquids, the direction of vibration of the transmitted polarized light is found to be rotated. The phenomenon of the rotation of the plane of polarization is called *optical activity*. The amount of rotation of the plane of polarization by certain liquids, such as sugar solutions, depends upon the density of the molecules in the liquid. This method is used commercially for measuring the amount of sugar present in a given sample;

the rotation of the plane of polarization is directly proportional to the concentration of sugar.

Another useful application of the phenomenon of polarization is in optical stress analysis. Suppose that a polarizer P and an analyzer A are in a crossed position. No light comes out of A. If a doubly refracting crystal is placed between them, some light will come out of A. Some substances, such as glass and plastics, under great stress become doubly refracting. Hence, when such an object is placed between P and A, light will come out of A. The pattern of the interference fringes obtained reveals the strong and weak points in the structure. Non-transparent objects may be analyzed by reflected light.

SUMMARY

Till the beginning of ninteenth century the light was considered as stream of particles called corpuscles. The deviser of the corpuscles theory was Newton. A contemporary of Newton, Huygens proposed that light travelled in form of waves. Young's experiment of interference and Fresnel's experiments on diffraction of light supported the wave motion theory. Maxwell showed that light consists of high frequency electromagnetic waves which travel with velocity of light. The latest view about light is that it behaves like particles as well as like wave.

According to Huygens' principle all points on the wavefront are taken as point sources for the production of spherical secondary waves which propagate outward with speed characteristics of wave in that medium.

In Young's double slit experiment the bright and dark fringes are located at.

$$y_{bright} = \frac{\lambda L}{d} m$$

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)$$

In a thin film, the condition of constructive interference is

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

and that for destructive interference is

$$2nt = m\,\lambda$$

The Michelson interferometer is used for precise measurement of wavelength. Wavelength is also measured by Diffraction grating using the formula $x\sin\theta = m\,\lambda$

Bragg's law $2y\sin\theta = m\,\lambda$ is used for the diffraction of X-rays from the crystal. Polarisation shows that light travels in the form of transverse electromegnetic waves.

QUESTIONS

9.1. What is the necessary condition on the path difference between two waves that interfere

a) Constructively

b) Destructively

9.2. Explain why two distant flash lights will not produce an interference pattern.

9.3. If a soap film on a wire loop is held in air, it appears black in the thinnest regions when observed by reflected light and shows a variety of colours in thicker regions. Explain.

9.4. Observe the shadow of your book when it is held a few centimetre above a table with a lamp serveral metre above the book. Why is the shadow of the book somewhat fuzzy at the edges.

9.5. What is the difference between Fraunhofer and Fresnel diffraction?

9.6. Although we can hear around corners, we cannot see around corners. How can you explain this in view of the fact that sound and light are both waves?

9.7. As a soap bubble evaporates, just before it breaks its surface, appears black when viewed by reflected light. Explain in detail why it is dark and not bright?

9.8. What is the difference between interference and diffraction?

9.9. What aspect of the nature of light is proved by the phenomenon of polarization?

9.10. Describe the use of a Polaroid as an analyzer and as a polarizer.

9.11. Explain whether a path difference of $\frac{\lambda}{4}$ is associated with a constructive interference or destructive interference or neither of them.

9.12. Describe and account for the interference phenomenon which can be seen when a convex lens is pressed against a glass plate.

9.13. Under what conditions can interference of light takes place? Describe some simple method based on interference for measurement of wavelength of light.

9.14. What are the main facts which support the following statements:

a) That light is wave-motion and not cospuscular.

b) The light travels in the form of transverse waves.

c) Light waves are much shorter than sound waves.

9.15. Two spectral lines of different colour are observed by means of a grating and it is found that the third order image of one line coincides with the fourth order image of second line. What is the ratio of the wavelength of the two colours.

PROBLEMS

9.1. A light source emits light of two wavelengths in the visible region, given by 430 nm and 510nm. The source is used in double slit interference experiment in which L = 1.5 m and $d = 0.025$ m. Find the separation between the third order bright fringe corresponding to these wavelengths.

9.2. In a double slit experiment yellow light from a sodium discharge tube is used in place of sunlight. The geometry is such that L = 100 cms $y = 0.50$ cms, $d = 0.023$ cms. Where y is the distance from the central light point to the second order maximum. Find the wavelength of the sodium light.

9.3. A pair of narrow, parallel slits separated by 0.25 mm are illuminated by the green component from a mercury vapour lamp of wavelength 5461 °A. The interference pattern is observed on a screen located 1.2 m from the plane of the parallel slits. Calculate (a) distance from the central maximum to the first bright fringe on either side of central maximum and (b) the distance between the first and second dark fringe.

9.4. The yellow component of light from a helium discharge tube with wavelength 587.5 nm is allowed to fall on a plane containing parallel slits that are 0.2 mm apart. A screen is located so that the second bright band in the interference pattern is at a distance equal to 10 slit spacings from the central maximum. What is the distance between the source plane and screen?

9.5. Light of wavelength 546 nm produces a Young's interference pattern in which the second order minimum (dark fringe) is along a direction that makes an angle of 18 minutes of arc relative to the direction to the central maximum. What is the distance between the parallel slits?

9.6. Let the film shown in Fig. 9.7 have an index of refraction of 1.36 and a thickness of 7×10^{-5} cm. A beam of sunlight is incident in air on the top surface of the film. Determine the wavelengths (within 4000-7000 °A) that will be strongly reflected by the film.

9.7. A material having an index of refraction of 1.30 is used to coat a piece of glass of refractive index 1.50. What should be the minimum thickness of this film in order to minimize reflected light at a wavelength of 500 nm.

9.8. In Michelson Interferometer, the movable mirror is shifted by 0.0589 mm and shift of 200 fringes is observed. What is the wavelength of the light used?

9.9. The mirror on one arm of a Michelson interferometer is displaced by '*p*'. During this displacement, 250 fringe shifts are counted, the light being used has a wavelength of 632.8 nm. Calculate the displacement '*p*'.

9.10. Light of wavelength 550.5nm is used to calibrate a Michelson interferometer. By use of a micrometer screw, the plateform on which one mirror is mounted is moved through 0.18 mm. How many dark fringes shifts are counted?

9.11. A collimated light from a Hydrogen discharge tube is incident normally on a diffraction grating. The incident light includes four wavelength components: 410.1 nm, 434 nm, 486.1 nm and 656.3 nm. There are 410 lines/mm in the grating. Find the angle between (a) first and last wavelength in the first order diffraction and (b) second and third wavelength in the third order diffraction.

9.12. The 546.1nm wavelength in mercury is observed at an angle of 81° in the third order spectrum of a diffraction grating. Calculate the number of lines per mm for the grating.

9.13. Two wavelengths of Hydrogen, 6563 °A and 6564.8 °A are observed with the help of a grating. Determine the minimum number of lines the grating must have in order to, resolve these two wavelengths in the first order.

9.14. A monochromatic X-rays beam is incident on a NaCl crystal surface. The second order maximum in the reflected beam is found when the angle between the incident beam and the surface is 20.5°. Determine the wavelength of X-rays.

10

GEOMETRICAL OPTICS

10.1 LENSES

A portion of a transparent medium such as glass bounded by two faces, one of which is at least curved, *is called a lens*. The surfaces are generally spherical, but one of them may be plane, which is then considered to be a small part of a spherical surface of infinite radius.

A lens of which one of the surfaces is plane but the other a part of a cylinder is also often used for special purposes. Such lenses are called cylinderical and are used by opticians for correcting the defect of the eye, known as *astigmatism*. For this reason a cylinderical lens is also called an astigmatic one. We shall confine our attention only to spherical lenses which are of two types. *Those which converge a beam of parallel rays are called converging lenses or convex lenses. Those which diverge a beam of parallel rays are called diverging lenses or concave lenses.* Fig. 10.1 and 10.2 show different shapes of converging and diverging lenses. Observe that the converging lenses are thicker in the centre and thinner at the edges, while diverging lenses are thinner at the centre and thicker at the edges.

A straight line joining the centres of curvature of the two surfaces of the lens is called the principal axis of the lens. The

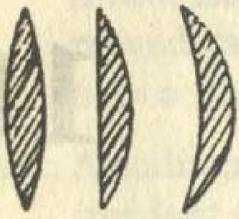


Fig. 10.1 *Converging lenses*

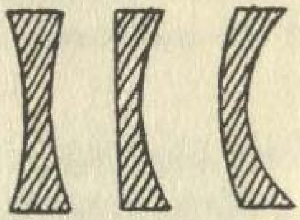
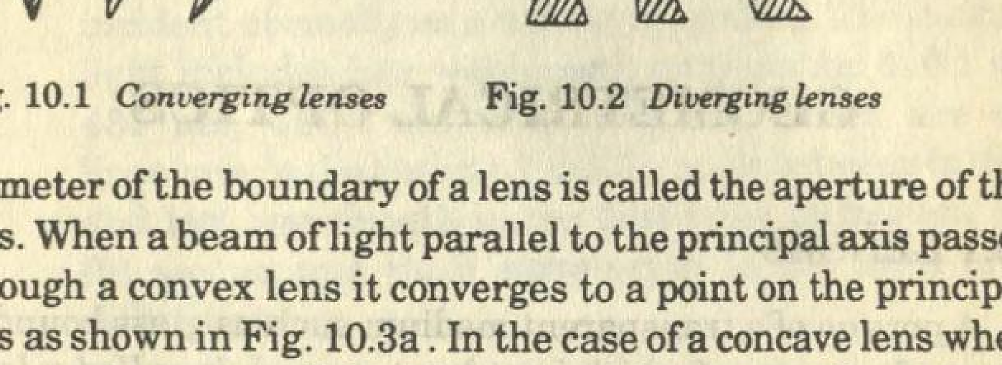


Fig. 10.2 *Diverging lenses*

diameter of the boundary of a lens is called the aperture of the lens. When a beam of light parallel to the principal axis passes through a convex lens it converges to a point on the principal axis as shown in Fig. 10.3a . In the case of a concave lens when a beam of rays parallel to the principal axis pass through the lens it appears to diverge from a point on the principal axis (Fig. 10.3b). This point is called the principal focus of the lens.

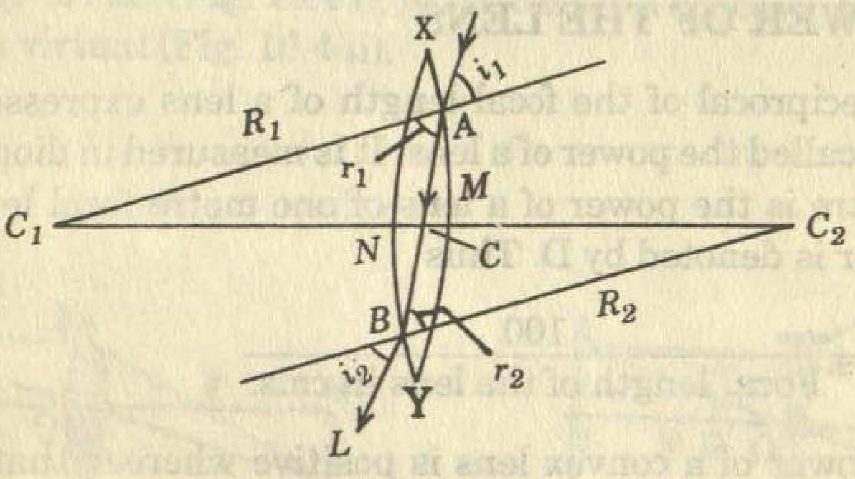


(a) (b)

Fig. 10.3 *(a) Refraction by a convex lens. (b) Refraction by a concave lens*

Since light can pass through a lens in either direction, a lens has two principal focii, one on either side of the lens and the distance of each from the lens is the same. *The distance of either of the principal focii from a point known as the optical centre of the lens is called its focal length.* The focal length of a convex lens is positive, as the rays actually pass through its principal focus. The focal length of a concave lens is negative as the rays do not actually pass through its principal focus.

It may be mentioned that a convex lens behaves as a converging lens only when it is placed in a medium whose refractive index is less than that of the material of the lens. In such a case, the velocity of light in the lens is less than the velocity of light in the surrounding medium and the lens behaves as a converging lens. A fixed point of a lens lying inside or outside on its axis such that a ray of light passing through it suffers no deviation is called *optical centre*. If the lens is thick the ray on passing through it suffers lateral displacement although there is no deviation.



Let C_1 and C_2 be the centres of curvature of the two surfaces of a double convex lens. Draw a radius C_1A of the surface XMY and a radius C_2B of the surface XNY parallel to each other. Join A B cutting the principal axis $C_1 C_2$ at C. Let K A be the incident ray for which A B is the refracted ray and B L the corresponding emergent ray, then $\angle r_1 = \angle r_2$, being alternate angles. As

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2} = \mu$$

$$\therefore i_1 = i_2$$

Hence K A is parallel to B L. In other words the incident ray K A for which the refracted ray A B passes through C, suffers no deviation. Hence C is the optical centre. Since triangles ACC_1 and BCC_2 are similar,

$$\therefore \frac{C_1 C}{C_2 C} = \frac{C_1 A}{C_2 B} = \frac{R_1}{R_2}$$

Hence the optical centre of the lens divides the lens joining the centres of curvature of the two surfaces in the ratio of their radii of curvatures. Hence the optical centre is a fixed point for a given lens. It may be noted that for a ray passing through the optical centre, the lens acts as a thin sheet or a plate.

10.2 POWER OF THE LENS

The reciprocal of the focal length of a lens expressed in metres is called the power of a lens. It is measured in dioptres. One dioptre is the power of a lens of one metre focal length. The power is denoted by D. Thus

$$D = \frac{100}{\text{Focal length of the lens in cms.}}$$

The power of a convex lens is positive whereas that of a concave lens is negative. If D_1 and D_2 are the powers of the two lenses of focal lengths f_1 and f_2 respectively placed in contact, then the power D of the combination is given by

$$D = D_1 + D_2$$

10.3 GRAPHICAL REPRESENTATION OF IMAGES

To get the position of the image of an object two rays are taken starting from any point of the object (i) a ray parallel to the axis, which on emergence from the lens either converges to (convex lens) or appears to diverge from (concave lens) the principal focus and (ii) a ray passing through the optical centre of the lens which follows its course undeviated. The intersection of these two rays gives the position of an image of the corresponding point on the object, the image being real or imaginary (virtual) depending as the rays actually intersect or only appear to do so. The image of the whole object can be obtained in this manner. For this purpose we take the object A B at right angles to the axis of the lens of which F is the principal focus and O the optical centre (Fig. 10.4 i, ii). *A′* is the image of A obtained in the above mentioned manner. From A ′ draw a perpendicular *A′ B′* on the principal axis, then *A′ B′* is evidently the image of A B. In the case of a convex lens, the image is real (Fig. 10.4 i), in the case of a concave lens, the image is virtual (Fig. 10.4 ii).

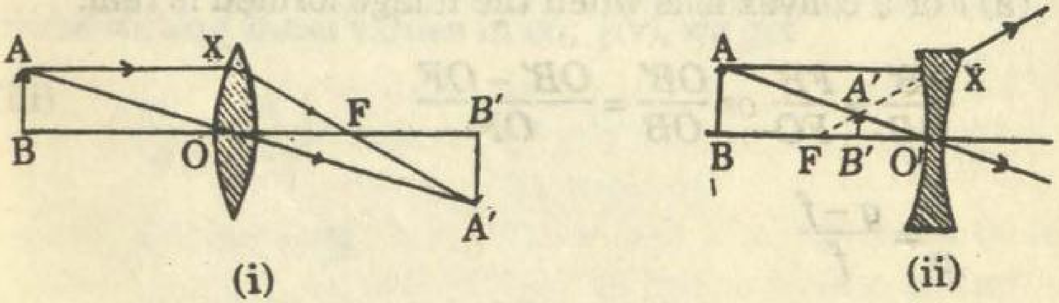


Fig. 10.4 *(i) Image produced by a convex lens. (ii) Image produced by a concave lens.*

The triangles O A′ B′ and O A B are similar, then

$$\frac{A'B'}{AB} = \frac{OB'}{OB} \qquad \text{(i)}$$

The triangle F A′ B′ and F X O are similar

$$\frac{A'B'}{XO}=\frac{FB'}{OF}$$

But XO = AB

$$\therefore \frac{A'B'}{AB}=\frac{FB'}{OF} \qquad \text{(ii)}$$

From the equations (i) and (ii), it is clear that

$$\frac{OB'}{OB}=\frac{FB'}{OF}$$

Conventionally, all distances are measured from the optical centre of lens, distances of the real objects and real images are taken as positive, whereas, those of virtual objects and virtual images are taken as negative. Accordingly the focal length of a convex lens is positive and that of a concave lens is negative. For thin lenses of small apertures. We have

(a) For a convex lens when the image formed is real.

$$\frac{OB'}{OB}=\frac{FB'}{FO} \text{ or } \frac{OB'}{OB}=\frac{OB'-OF}{OF} \qquad \text{(iii)}$$

$$\frac{q}{p}=\frac{q-f}{f}$$

Where q, p and f are the distances of the image, the object and the principal focus from the optical centre of the lens.

$$qf=pq-pf$$

$$\therefore \quad pq=qf+pf$$

Dividing by pqf, we have

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

(b) For a convex lens when the image formed is virtual

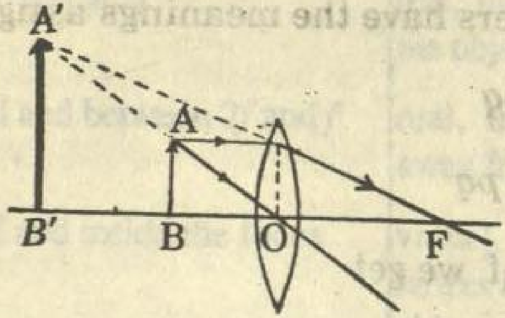


Fig. 10.5

Referring to the Fig. 10.5

$$\frac{BO}{B'O}=\frac{OF}{B'F}=\frac{OF}{OF+B'O} \qquad \text{(iv)}$$

In this case, p is positive, q is negative and f is positive. Substituting these values in eq; (iv), we get

$$\frac{p}{-q}=\frac{f}{f-q}$$

$$pf-pq=-qf$$

$$\text{or } qf+pf=pq$$

Dividing by pqf, we get

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

(ii) For a concave lens

Since the focal length and the image are both virtual and have negative sign we have

$$\frac{-OB'}{OB}=\frac{FB'}{FO} \text{ or } \frac{OB'}{OB}=\frac{FO-FB}{FO}$$

$$-\frac{q}{p}=\frac{-f-(-q)}{-f}=\frac{-f+q}{-f}$$

where the letters have the meanings assigned above.

$$qf=-fp+pq$$

$$\text{or } qf+pf=pq$$

Dividing by pqf, we get

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

This is the same relation as in the case of a convex lens. The points like B and B′ which are connected by the above relation are called conjugate focii.

10.4 THE SIZE, NATURE AND POSITION OF THE IMAGES FORMED BY LENSES

(i) In concave lenses, the image of a real object is always virtual, erect, smaller and on the same side of the object.

(ii) In a convex lens, the nature, position and size of the image is determined by the distance of the object from the lens and also whether the object is real or virtual. They can be calculated by giving different values to p varying from infinity to 0 and finding the corresponding values of q. They can also be obtained by drawing the course of rays. They are summarized in the following table

Nature of lens	*Nature and position of object*	*Nature and position of image*
Converging	real and away from $2f$	real, inverted diminished and between f and 2f

Nature of lens	*Nature and position of object*	*Nature and position of image*
-do-	real and at 2*f*	real, inverted, same size as the object and at 2*f*
-do-	real and between 2*f* and *f*	real, inverted, magnified, away from 2*f*
-do-	real and inside the focus	virtual, erect, magnified and farther from the lens than the object
-do-	virtual in any position	real, erect, diminished and inside the focus
Diverging	real in any position	virtual, erect, diminished and inside the focus
-do-	virtual outside the focus	virtual, inverted and outside the focus
-do-	virtual inside the focus	real, erect, magnified and farther from the lens than the object

10.5 LINEAR OR TRANSVERSE MAGNIFICATION

It is defined as the ratio of the linear dimensions of the image to those of the object and is given by

$$\text{magnification} = \frac{\text{size of the image}}{\text{size of the object}} = \frac{I}{O}$$

Referring to Figs 10.4 (a,b) and 10.5 triangles ABO and *A′ B′ O* are similar

Therefore magnification =

$$\frac{\text{distance of the image from the lens}}{\text{distance of the object from the lens}}$$

$$\therefore \frac{A'B'}{AB} = \frac{B'O}{BO} = \frac{I}{O}$$

Where I is the size of the image and O that of the object. The distance BO = p is always positive for a real object while the distance B ′ O = q may be positive or negative.

(a) For a convex lens producing a real image, q is positive.

$$\therefore \frac{I}{O} = \frac{q}{p}$$

(b) For a convex lens producing a virtual image q is negative.

$$\therefore \frac{I}{O} = \frac{-q}{p}$$

(c) For a concave lens the image formed is virtual, so q is negative

$$\therefore \frac{I}{O} = \frac{-q}{p}$$

It is clear that the magnification is positive when the image is real and inverted and is negative when the image is virtual and erect.

Magnifying Power (Angular Magnification)

The linear size of the image produced by a mirror or a lens is given by $I = \frac{q}{p} O$, but this is divided by O *i.e.* simply $\frac{q}{p}$ does not in a general case give the magnifying power of the instrument so far as the eye is concerned. For the eye judges the size of the object not only by its actual size but by the angle which it subtends at the eye. An object as it is brought nearer and nearer to the eye, keeps on appearing bigger and bigger although the size of the object remains the same, because the angle subtended by the object at the eye becomes larger and larger. In the case of optical instruments where the final image is seen by the eye and is virtual, the magnifying power is given by the ratio of the angle θ_i subtended by the image formed at the least distance of distinct vision to the angle θ_o subtended by the object placed at the least distance of distinct vision. The nearest point to the eye at which a small object can be clearly seen is called the near point. Its distance from the eye is called the least distance of distinct vision. For adults, it is between 25 and 30 cm. When the image is formed at the least distance of distinct vision, (D) the value of angular magnification or magnifying power is equal to

$$\frac{\theta_i}{\theta_o} = \frac{\frac{I}{D}}{\frac{O}{D}} = \frac{I}{O} = \text{Linear magnification}$$

10.6 LENS ABERRATIONS: (DEFECTS IN LENSES)

Lenses suffer from two important defects which are known as chromatic aberration and spherical aberration. We shall discuss these defects one by one.

Chromatic Aberration

A lens may be regard as made up of prisms placed one above the other. It is evident that when a ray of white light passes through it, it will be dispersed into its constituent colours. All the red rays are brought to focus at F_r, whereas all the blue rays are brought to focus at F_b. A complete image will consist of a small linear spectrum lying along the axis which can be projected on a white screen. A screen will be coloured and image will not be well defined. This defect in the image is called *chromatic aberration*. If the screen is placed at F_b the blue part will be in focus and if the screen is placed at

F_r the red part will be in focus, but the image at AB will practically be white. This point AB is known as the circle of *least confusion*.

Defect in the lens can be removed by using a combination of a convex lens and a concave lens made of two different materials having unequal dispersive powers. The lenses are given such suitable shapes that the dispersion produced by one lens is exactly equal and opposite to that produced by the other. The focal lengths of the lenses are, of course, unequal in numerical values; so that the focal length of the combination has a finite value. Such a combination is called an *achromatic combination of lenses*.

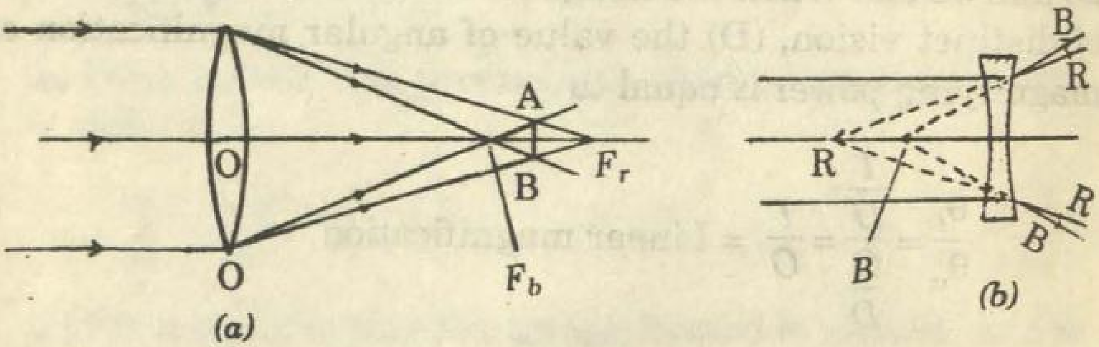


Fig. 10.6 *Chromatic aberration of (a) Convex lens. (b) Concave lens.*

Spherical Aberration

A beam of perallel rays is focused at a point by a lens only if the aperture of the lens is small, otherwise the lens will refract outer rays slightly more than the inner rays. The image produced will not be well defined and sharp. This defect in a lens is called *spherical aberration*. To reduce this defect, optical instruments using lenses are provided with a stop which allows only the central rays to pass through the lens. In this way, the effective aperture of the lens remains small and so the spherical aberration is almost removed. The spherical aberration can also be reduced by taking suitable values of the radii of curvature of the surfaces of a lens or by using two lenses kept at a suitable distance apart. Now we shall discuss how the lenses are used in optical instruments.

10.7. OPTICAL INSTRUMENTS

(a) Telescopes

When an object is at a great distance, it subtends a small angle at the eye. A telescope serves to magnify the angular separation of distant objects or to increase the amount of light received by the eye from a distant object. One of the primary functions of a telescope is to collect the light from a weak source and to concentrate the rays so that the eye can register an image of the object. Telescopes are of two types, called *refracting telescopes* (Fig. 10.7a) and *reflecting telescopes* (Fig. 10.7b). In a refracting telescope, a real image of a distant object is formed at the focus of the objective. The image is then seen through a manifying glass, called the eye-piece or eye-lens.

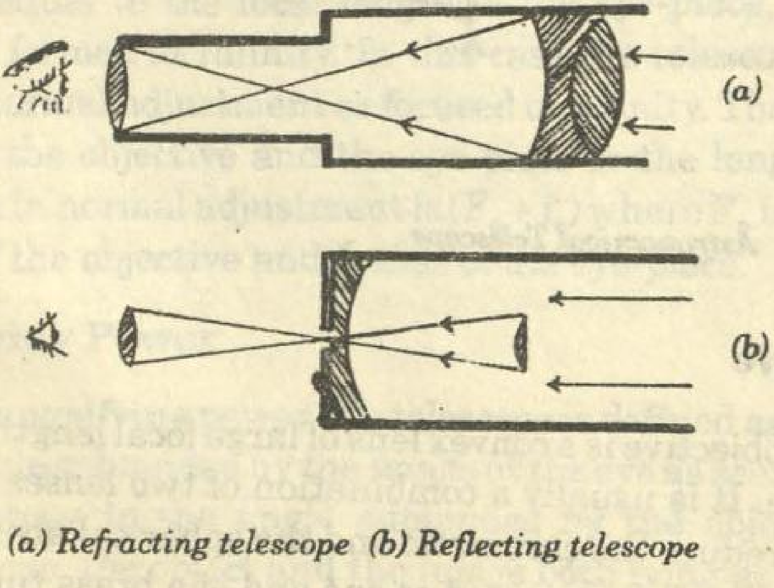


Fig. 10.7 *(a) Refracting telescope (b) Reflecting telescope*

The brightness of the image depends upon the amount of light entering the objective. Hence the objectives of powerful telescopes have large diameters.

In a reflecting telescope, a real image of a distant object is formed at the focus of a concave mirror. this image is then seen through an eye-piece. We shall consider different types of telescopes one by one.

(i) Astronomical Telescope

A simple astronomical telescope essentially consists of two convex lenses one of which serves the purpose of an objective and the other that of an eye-piece.

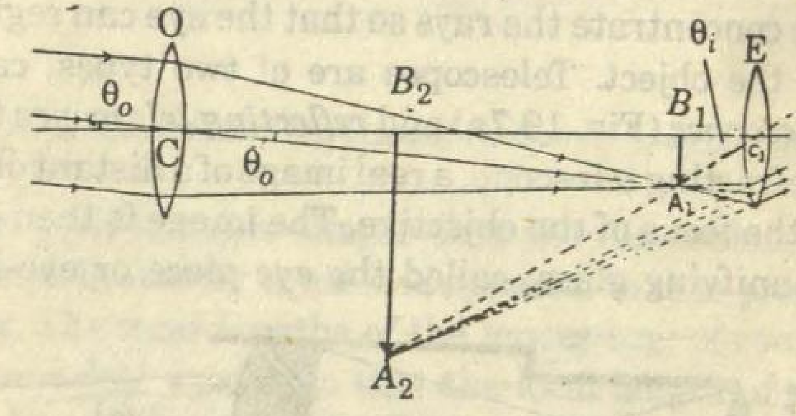


Fig. 10.8 *Astronomical Telescope*

Objective

The objective is a convex lens of large focal length and large aperture. It is usually a combination of two lenses in contact so as to reduce the chromatic and the spherical aberrations to a minimum. It is mounted at one end of a brass tube.

Eye-Piece

The eye-piece is also a convex lens but is of short focal length and small aperture. In order to increase the field of view and also to minimise the aberrations, the eye-piece consists of two lenses separated by a suitable distance. It is mounted in a small tube which can slide inside the bigger tube carrying the objective.

Working

The objective O form a real, inverted and diminished image A_1B_1 of a far off object at its focal plane, B_1 in front of the eye piece E as shown in the Fig. 10.8.

The rays incident on the objective are taken in the form of a parallel beam as they are coming from a far off object. The distance between the eye lens and this image formed is adjusted so that it is less than the focal length of the eye-piece which, therefore, forms a magnified virtual image A_2B_2 at the least distance of distant vision. An eye placed behind the eye-piece can see this magnified image, which is inverted with respect to the object.

When the distance between the image A_1B_1 and the eye-piece is equal to the focal length of the eye-piece, the final image is formed at infinity. In this case the telescope is said to be in normal adjustment or focused to infinity. The distance between the objective and the eye-piece or the length of the telescope in normal adjustment is $(F_o + f_e)$ where F_o is the focal length of the objective and f_e that of the eye-piece.

Magnifying Power

The magnifying power of a telescope is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object at the unaided eye, the object and the image both lying at infinity.

As the object lies at a very great distance, the angle subtended by it at the eye is the same as subtended at the optical centre of the objective C and is equal to $\angle A_1 CB_1 = \theta_o$. The angle subtended by the image at the eye is the same as that subtended at C_1 as the eye lies very close to the eye-piece. This angle is equal to $\angle B_2C_1A_2 = \angle B_1C_1A_1 = \theta_i$ Thus magnifying power, $M = \frac{\theta_i}{\theta_o}$

Now as the angles are small, therefore, they can be replaced by their tangents. Hence

$$M = \frac{\tan\theta_i}{\tan\theta_o} = \frac{A_1 B_1}{B_1 C_1} \times \frac{B_1 C}{A_1 B_1} = \frac{B_1 C}{B_1 C_1}$$

When the rays which emerge from the eye-piece are parallel, the image A_2B_2 lies at infinity and the distance B_1C_1 is equal to the focal length of the eye-piece. Therefore

$$M = \frac{F_o}{f_e} = \frac{\textit{focal length of the objective}}{\textit{focal length of the eye piece}}$$

(ii) Galileo's Telescope

It essentially consists of two lenses, placed coaxially at a distance. One of the lenses is called the objective and the other an eye-piece. The objective is a convex lens of large focal length and large aperture whereas the eye-piece is a concave lens of small focal length and small aperture. The eye-piece is fitted in a small tube which can slide inside a bigger tube so that the distance between the two lenses can be altered.

Working

Parallel rays from one extremity of a distant object fall on the objective O and after refraction tend to converge to a point A_1 as shown in Fig. 10.9. A real, diminished image A_1B_1 of the

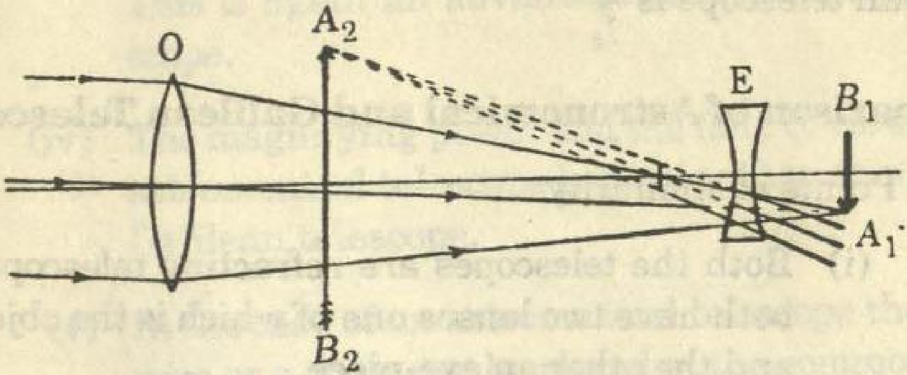


Fig. 10.9 *Galileo's telescope*

far off object is formed in this way. Before these rays actually come to focus, they are intercepted by the eye-piece E which is a concave lens of short focal length. The rays diverge and appear to proceed from a point A_2 which, therefore, is the final, virtual and erect image of the extermity of the far off object. Generally, the final image A_2B_2 is formed at the least distance of distinct vision from the eye. This can be achieved by making the distance of the eye lens from the image A_1B_1 slightly greater than the focal length of the lens.

When the final image A_2B_2 is formed at infinity the distance of the eye-piece from the image A_1B_1 is equal to its focal length. In such a case the distance between the two lenses is equal to the difference between their focal lengths. If F_o is the focal length of the objective and f_e that of the eye-pieces then the distance between the two lenses is $F_o - f_e$. The field of view of this instrument is very limited because the eye can receive only a limited number of rays. But its advantage is that the final image formed is erect. The length is also small as compared to the terrestrial telescope and hence this can be used in the construction of binoculars. The magnifying power of a

Galilean telescope is $\frac{F_o}{f_e}$

Comparison of Astronomical and Galilean Telescopes

a) Points of similarity

(i) Both the telescopes are refracting telescopes *i.e.* both have two lenses one of which is the objective and the other an eye-piece.

(ii) The focal length of the objective in both of them is greater than that of the eye-piece.

(iii) Both the telescopes are used to see distant objects.

(iv) In both the telescopes, the objective is a convex lens of large focal length and large aperture.

(v) When focussed for parallel rays the magnifying power in both is $\frac{F_o}{f_e}$

b) Points of dissimilarity

(i) In an astronomical telescope, the eye-piece is a convex lens while in the Galilean telescope it is concave lens.

(ii) The astronomical telescope forms an inverted image and is usually used to view heavenly bodies whereas the Galilean telescope form an erect image and is used for viewing terrestrial objects, which is an advantage in favour of the Galilean telescope.

(iii) The astronomical telescope is longer in length than the Galilean telescope. For normal adjustment the length of the astronomical telescope is $F_o + f_e$ while in the latter case the length is $F_o - f_e$. This is again an advantage of the Galilean telescope.

(iv) The magnifying power and the field of view of the astronomical telescope is greater than that of the Galilean telescope.

(v) In the case of an astronomical telescope the cross wire or a scale can be placed at the common focal plane of the objective and the eye-piece for measurements whereas no such arrangement is possible in the case of Galilean telescope.

(b) Microscopes

(i) Simple Microscope or magnifying glass

An ordinary convex lens held close to the eye can be used as a magnifying glass or a simple microscope. If an object AB is placed at a distance *p*, less than the focal length of the lens L, then a virtual, erect and magnified image of this object is obtained on the same side. Generally, this image is made to lie at the least distance of distinct vision from the eye. The course of rays showing the formation of the image *A′ B′* is shown in Fig. 10.10.

Magnification of a Simple Microscope

If the image A′ B′ is formed at the least distance of distinct vision, then the angle that it subtends at the centre of an unaided eye is ∠ B′ O A′ which is the same as ∠ BOA, subtended by the object AB. (Fig. 10.10). To see the object clearly by a naked eye the object must be displaced to the position A′ C at the least distance of distinct vision. The angle subtended by the object in this position is COA′.

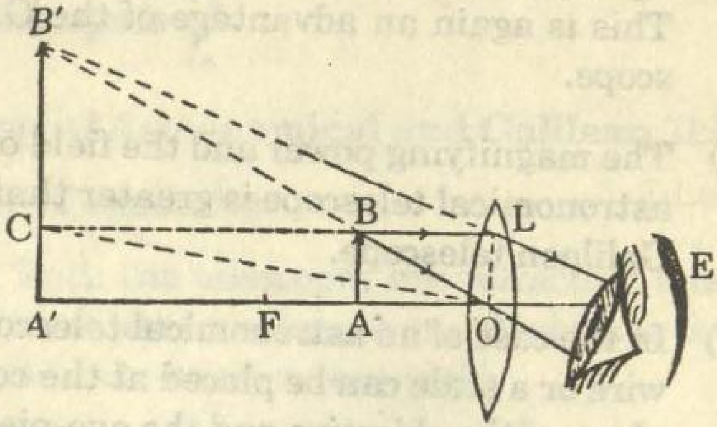


Fig. 10.10 *Simple Microscope*

$$\text{Magnifying Power } M = \frac{\angle B'OA'}{\angle COA'} = \frac{\angle BOA}{\angle COA'}$$

If the angles are small, then these can be replaced by their tangents.

$$M = \frac{\tan BOA}{\tan COA'} = \frac{BA}{OA} \times \frac{OA'}{CA'}$$

But $BA = CA'$

$$\therefore M = \frac{OA'}{OA} = \frac{q}{p} = \frac{D}{p}$$

Let f be the focal length of the lens and q = D. Where D is the least distance of distinct vision (As image is virtual D is negative) Substituting these values in the relation for the lens formula, we get

$$-\frac{1}{D} + \frac{1}{p} = \frac{1}{f}$$

$$\text{or } \frac{D}{p} = \frac{D}{f} + 1$$

Magnifying Power $M = 1 + \frac{D}{f}$

Since D is constant, therefore, smaller the value of f greater is the magnifying power of microscope. *A microscope is a device by which very minute objects can be seen considerably magnified.* A single convex lens may be used as a simple microscope but the magnification produced is small. For large magnification a compound microscope is used.

(ii) Compound Microscope

It is used to produce a very large magnification of very small objects. It essentially consists of two lenses O and E as shown in Fig. 10.11.

The lens O called the objective is a convex lens of short focal length f and small aperture. The lens E through which the final image is seen is called the eye-piece. It is a convex lens of large focal length and large aperture as compared to those of the objective. The objective is mounted in a brass tube. The eye-piece is also mounted in a small tube which can slide

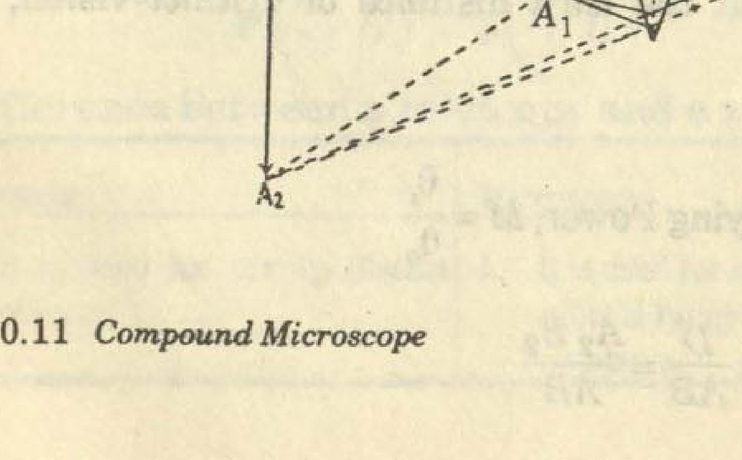


Fig.10.11 *Compound Microscope*

inside the bigger tube.

Working

The object AB is placed at a point whose distance p is slightly greater than the focal length of the objective. A real inverted image of this object is formed after refraction through the objective at A_1B_1 at a distance q from it.

The eye-piece is then placed in such a manner that the image A_1B_1 fall within its focal length. The final image which is virtual, magnified and inverted with respect to the object is thus formed at A_2B_2 at a distance equal to the least distance of distinct vision D, when eye is placed close to the eye-piece. The path of ray is shown in Fig. 10.11.

Magnifying power

The magnifying power of a microscope is defined *as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object when both are placed at the least distance of distinct vision from the eye.* As the angles subtended are small, these are replaced by their tangents. The angle subtended by the final image A_2B_2 at the eye,

$$\theta_i = \frac{A_2 B_2}{D}$$

Let θ_o be the angle subtended by the object AB at the eye when placed at the least distance of distinct vision, then $\frac{AB}{D} = \theta_o$

$$\text{Magnifying Power, } M = \frac{\theta_i}{\theta_o}$$

$$\frac{A_2 B_2}{D} \times \frac{D}{AB} = \frac{A_2 B_2}{AB}$$

$$= \frac{A_2 B_2}{A_1 B_1} \times \frac{A_1 B_1}{AB}$$

$$= M_1 \times M_2$$

Where M_1 is the magnification produced by the eye-piece and M_2 that produced by the objective.

Now magnification produced by the objective $= \frac{A_1 B_1}{AB} = \frac{q}{p}$

Magnification produced by the eye-piece

$$= \frac{A_2 B_2}{A_1 B_1} = \left(1 + \frac{D}{f_e}\right)$$

Where f_e is the focal length of the eye-piece.

Hence magnifying power, $M = \frac{q}{p}\left(1 + \frac{D}{f_e}\right)$

Generally, the object AB lies very close to the focus of the objective of focal length F_o.

$$\therefore\ p = F_o$$

The image A_1B_1 lies very close to the eye-piece.

q = L (the length of the microscope tube)

$$\text{Hence } M = \frac{q}{p}\left(1 + \frac{D}{f_e}\right) = \frac{L}{F_o}\left(1 + \frac{D}{f_e}\right)$$

Difference between a telescope and a microscope

Telescope	*Microscope*
1. It is used for seeing distant objects	1. It is used for seeing very small objects lying very close to the objective.

Telescope	*Microscope*
2. It has an objective of large aperture having a long focal length and a small eye-piece of short focal length	2. It has a small objective of short focal length and a large eye-piece of large focal length

(c) Spectrometer

A spectrometer is used for a careful study of the spectra. It essentially consists of three parts: (i) a collimater (ii) a telescope and (iii) a turn-table.

(i) The Collimater

The collimater C consist of a slit the width of which can be adjusted. It is placed at the focal plane of a convex lens L_1 (Fig 10.12), so that the light starting from the slit becomes a beam of parallel rays after passing through the lens. The lens and the slit are placed at the ends of a metallic tube and the distance between them can be altered.

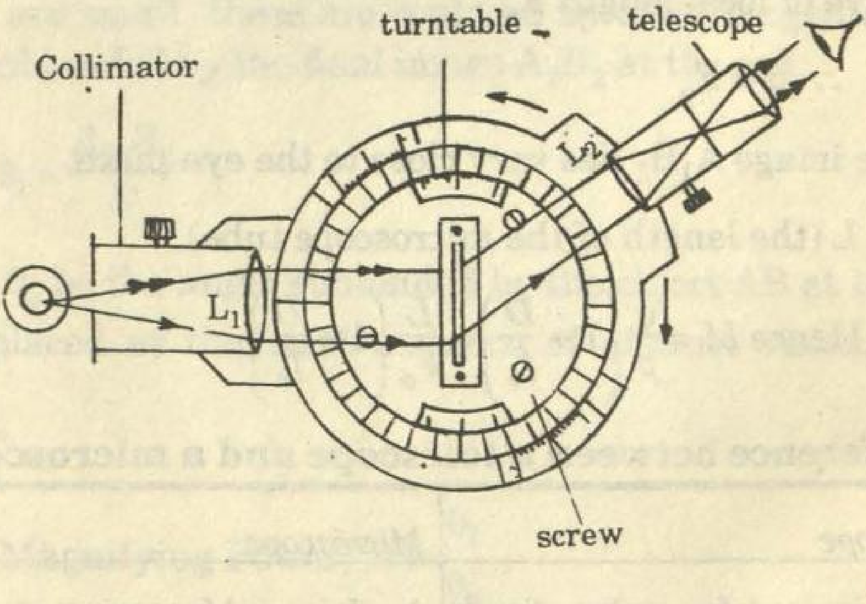


Fig.10.12 *The spectrometer*

(ii) The Telescope

It is an ordinary refracting telescope fitted with cross wires at the focal plane of the objective L_2. The telescope can be rotated about the central axis of the spectrometer. The movement of the telescope can be read on a circular scale graduated in degrees with the help of a vernier arrangement.

(iii) The prism table or turn-table.

The turn-table supporting the prism can be raised or lowered. It can also be rotated about a vertical axis through any desired angle. This is provided with three levelling screws.

Adjustments

(i) The eye-piece is focussed on the cross-wires so that they are clearly visible.

(ii) The telescope is adjusted for parallel rays so that a parallel beam falling in the objective of the telescope produces an image at the cross-wires. To achieve this, parallex is removed between the image of a far off object and the cross-wires.

(iii) The collimater is also adjusted to give a parallel beam of light. This is done by illuminating, the slit with mono-chromatic light and viewing image of the slit through the telescope. The distance of the slit from the collimater is adjusted till the slit is in focus in the telescope. The parallex is removed between the image of the slit and the cross-wires. When this is so, the collimater gives a parallel beam of light.

(iv) To make sure that the turn-table is horizontal. Place a spirit level along the line between two of the three levelling screws. The two screws are adjusted until the from the measurements taken for that line on either side of the centre. The angle of diffraction is half the difference between the two measurements. The colour of each line may be used to identify its order number. Hence the wavelength of each line can be calculated by using the diffraction grating equation.

Example 10.1

An object is placed 6 cms from a lens that has a focal length of 4 cms: Where is the image located?

Solution

Inserting $p = 6$ cm, $f = 4$ cm in the lens formula

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

we get

$$\frac{1}{6}+\frac{1}{q}=\frac{1}{4}$$

$$\frac{1}{q}=\frac{1}{4}-\frac{1}{6}=\frac{3}{12}-\frac{2}{12}=\frac{1}{12}$$

$$\therefore q = 12\,cm$$

Example 10.2

Consider the case of a diverging lens. Let $p = 6$ cm, and $f = -5$ cm. Find the location of the image.

Solution

We have in this case

$$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{1}{5}-\frac{1}{6}=-\frac{11}{30}$$

so that $q = -2.7$ cm.

The negative sign indicats that the image is virtual and located at the same side of the object.

Example 10.3

Object is viewed through a convex lens of focal length 5 cm, produces an upright image 20 cm. from the lens. What is the object distance and linear or transverse magnification?

Solution

In this case image is virtual. Therefore q = image distance $= -20$ cms. since the lens is converging $f = +5$cm. Substituting in the lens formula.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{p}+\frac{1}{-20}=\frac{1}{5}$$

$$\text{or } \frac{1}{p}=\frac{1}{5}+\frac{1}{20}=\frac{5}{20}$$

$$\text{Hence } p=\frac{20}{5}=4\,cm.$$

i.e., the object is 4 cm. from the lens. As a check the object distance is less than the focal length and so produces a virtual, upright and magnified image. The linear or transverse magnification is given numerically by

$$M=\frac{q}{p}=\frac{20}{4}=5$$

Example 10.4

A 8 cm high object is placed at 18 cm from a converging lens of 6 cm focal length. Find the position and height of the image.

Solution

Since the lens is converging, $f = 6$ cm; $p = 18$ cm. using the lens formula.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{18}+\frac{1}{q}=\frac{1}{6}$$

$$\frac{1}{q}=\frac{1}{6}-\frac{1}{18}=\frac{1}{9} \qquad \text{or } q = 9$$

The plus sign indicates that the image is real and inverted.

$$\text{Linear magnification} = \frac{\text{Size (Height) of the image}}{\text{Size (Height) of the object}} = \frac{q}{p}$$

$$\text{Height of the object} = \frac{q}{p} \times \text{Height of the image}$$

$$= \frac{9}{18} \times 8 = 4\ cm.$$

SUMMARY

Lens formula, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

Where p = Object distance, q = Image distance
f = Focal length of the lens.

Following sign conventions are used in lenses

real is positive

virtual is negative

focal length of the convex lens positive focal length of the concave lens negative

$$\text{Power of the lens in diopters} = \frac{1}{\text{Focal length in metres}}$$

The magnifying power of a telescope is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object at the unaided eye, the object and the image both lying at infinity.

$$\text{The magnifying power of a telescope} = \frac{F_o}{f_e}$$

Where F_0 = focal length of the objective.

f_e = focal length of the eye-piece

The length of the telescope in normal adjustment is $(F_0 + f_e)$

For a microscope

(a) Magnifying power =

$$\frac{\text{Angle subtended by the image}}{\text{Angle subtended by the object}} \text{ at the near point of the eye}$$

(b) For a magnifying glass

$$\text{the magnifying power} = 1 + \frac{D}{f}$$

Where D = near point distance (least distance of distinct vision)

f = focal length of the lens.

QUESTIONS

10.1 Describe with the aid of diagrams how you would use a converging lens.

a) to ignite a match in strong sunlight.

b) to produce a parallel beam of light from an electric lamp.

c) to form an image on a screen of an electric lamp.

10.2. What is the effect on the image if half of a converging lens is covered?

10.3. Why does the image of a man projected on a screen look like a man?

10.4. Does the focal length of a lens depend on the medium in which the lens is immersed?

10.5. An unsymmetrical thin lens forms an image of a point object on its axis. Is the image location changed if the lens is reversed?

10.6. If a telescope is focused on the moon and then directed toward a tree 50 *m* away, what adjustment would be necessary to focus the tree sharply? Give reasons.

10.7. Describe how would you arrange two lenses to form a microscope. Illustrate your answer by drawing a diagram to show the path of the rays by which an eye sees an image of an object not on the axis through the microscope.

10.8. What wavelength of light would you recommend for use with a microscope if the maximum detail was to be seen.

10.9. Describe with a diagram, an astronomical telescope, indicating the nature and relative focal lengths of the lenses used, their relative positions, and the nature and positions of the images formed.

10.10. Show why a pin hole placed in front of a lens leads to a good image even if the image is not quite in focus.

10.11. Explain why the lens used in expensive optical devices are composed of more than one part.

10.12. Why is a convex lens of small focal length preferred for a magnifying glass?

10.13. Under what conditions does a double convex lens acts as a diverging lens.

10.14. What are the different arrangements used for decreasing the length of terrestrial telescope.

10.15. How will you find refracture index of liquid by spectrometer?

PROBLEMS

10.1. An object is placed at 10 cm from a convex lens of focal length 18 cm. Find the position of the image.

10.2. An object is placed 8 cm in front of a concave lens of focal length 16 cm. Find the image position.

10.3. An object is placed (i) 12 cm (ii) 4 cm from a converging lens of focal length 6 cm. Calculate the image position and the magnification in each case, and draw sketches illustrating the formation of the images.

10.4. An object with a height of 2 cm is located at a distance of 8 cm from a lens with a focal length of 6 cm. Where is the image located? What is the magnification?

10.5. Suppose that the object in problem 10.4 is moved 4 cm closer to the lens. What is the new position of the image? Sketch the situation. On what side of the lens is the image? Is the image real or virtual?

10.6. An object is located 12 cm from a lens that has $f = -8$ cm. Where is the image located? Characterize the image.

10.7. An object is located 20 cm from a lens that produces a real image at a distance of 40 cm. What is the focal length of the lens? What is the type of the lens?

10.8. An object is 2 cm from a lens with f=3cm. Where is image located? Draw a ray diagram.

10.9. A lens with a focal length of 20 cm (sign of f unknown) forms an image with $q = -15$ cm. Where is the object located?

10.10. An object is located at the focal point of a diverging lens that has $f = -10$ cm. Is an image formed? Where is the image located? Characterize the image.

10.11. It is desired to produce a real image of an object that is twice as large as the object. If a lens with $f = 20$ cm is used. Where should the object be placed?

10.12. A diverging lens ($f = -12$ cm) produces an image of an object with $m = +\frac{1}{3}$. Where is the object located? Where is the image located?

10.13. A camera with a 50 mm focal length lens is used to take a picture of a person whose height is 1.8 m and located at a distance of 15 m. What is the height of the image on the film?

10.14. The image obtained with a convex lens is erect and three times the length of the object. The focal length of the lens is 10 cm. Calculate the object and image distance?

10.15. Explain which of the following appears to be larger when viewed unaided

a) the moon which subtends an angle of 0.5°

b) a 15 m high building at a distance of 800 m.

10.16. The focal length of a simple magnifier is 12.5 cm (a) What is the magnifying power if the final image is formed 25 cm in front of the eye (b) What is the magnifying power of the final image formed at infinity?

10.17. A compound microscope has an objective of focal length 1 cm and an eye-piece of focal length 5 cm. The object is located 1.05 cm from the objective and the final image is 25 cm from the eyepiece. Find the separation of the lenses and the magnifying power of the microscope.

10.18. A telescope consists of an objective lens of power 0.5 dioptre and an eyepiece of power 20 dioptre. Find its length and its magnification.

10.19. The objective of an astronomical telescope in normal adjustment has a focal length of 4.00 m and eyepiece has a focal length of 25 mm. Calculate the magnifying power of the telescope.

10.20. A simple refracting telescope is constructed from two lenses of focal lengths 50 mm and 500 mm

a) In normal adjustment what is the distance between the lenses and what is magnifying power.

b) The telescope is adjusted to form a real image of the Sun on a screen 300 mm behind the eyepiece. Given that the Sun subtends an angle of 0.01 radians to Earth, what will be the diameter of its image on the screen.

ANSWERS

CHAPTER 2

2.1 80 km, $\theta = 45^\circ$
East of North

2.3 56290 J

2.4 20.4 kg $m^2 sec^{-1}$

2.5 3

2.6 3.72 R at $32^o\ 5'$ with the x-axis

2.8 $|\vec{a} \times \vec{b}| = 48$, and $\vec{a} \times \vec{b}$ is confined to the hroizontal plane

2.9 $\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

2.10 q = 9

2.11 (a) $\frac{2}{3}\hat{i} - \frac{\hat{j}}{3} + \frac{2}{3}\hat{k}$

(b) $8\hat{i} - 4\hat{j} + 8\hat{k}$

(c) Any vector in the plane perpendicular to $\vec{a}$

2.12 34.6 Nm **2.13** 260 Nm

2.14 $(1.98 \times 10^{-15}\,\hat{i} - 2.08 \times 10^{-14}\,\hat{j})\,N$

2.15 20.4 kg $m^2 sec^{-1}$

2.16 $-5.86\,\hat{i} - 1.25\,\hat{j} + 99.1\,\hat{k}$

CHAPTER 3

3.1 8.88 m sec^{-1}

3.5 2.94 m sec^{-2}

3.6 (a) 29.25 m sec^{-2} (b) 24.5 m sec^{-2}

3.7 $V_f = 4.45$ m sec^{-1}

3.8 (a) 29.4 m sec^{-1} (b) 45.1 m

(c) 6 sec (d) 29.73 m sec^{-1}

3.9 (a) 2.94 m sec^{-1} (b) 0.441 m

3.10 (a) 120 m sec^{-1} (b) 784.0 m sec^{-1}
(c) 9.8 m sec^{-2}

3.11 $h_1 = 31360$ m $V_f = 784$ m sec^{-1}
$h_2 = 31360$ $t = 160$ sec

3.12 8 m sec^{-1}

3.13 (a) 9.8039 m sec^{-2} (b) 46.49 m sec^{-1}
(c) $h = 110.239$ m

3.15 $w = 9.8$ N **3.16** 7.5 N

3.17 400 N **3.18** 2 m sec^{-2}

CHAPTER 4

4.1 (a) 0.45 sec (b) 1.04 m

(c) 2.3 m sec^{-1} horizontal,
4.5 m sec^{-1} vertically down word

4.2 (a) 200 m sec^{-1} (b) 810 m

(c) $V_h = 160$ m sec^{-1}, $V_v = 170$ m sec^{-1}

4.4 $\theta = 45^\circ$, $V = 14.1$ m sec^{-1}, $\theta = 42^\circ$, $V = 14.0$ m sec^{-1}

4.5 $\left[x_m = \frac{V_o^2}{2g} + \frac{1}{2}\left(\frac{V_0{}^4}{g^2} - \frac{4HV_o^2}{g}\right)^{\frac{1}{2}}\right]$

4.7 $v = a\sqrt{\frac{h}{2g}}$ **4.8** 45°, 71.6°

4.11 7.8×10^{-3}

4.12 (a) $4\,\pi$ rad $\sec^{-1}$ (b) π m $\sec^{-1}$

4.13 (a) 4.2×10^{16} rad $\sec^{-1}$ (b) $2.1 \times 10^{6} m \sec^{-1}$

(c) 8.8×10^{22} m $\sec^{-2}$

4.15 0.5%

4.16 1.6 rev $\sec^{-1}$

4.17 32.52° .6250 N, 11626N

4.18 31.43m $\sec^{-1}$, 195.6N, 199.5N

CHAPTER 5

5.1 (a) 12 kg $m^2 \sec^{-1}$ out of the page

(b) 3.0 Nm out of the page

5.2 1.1×10^{-34} kg $m^2 \sec^{-1}$

5.3 (i) 0.12 kg $m^2 \sec^{-1}$ (ii) 7.1 rad $\sec^{-1}$

(iii) 4.4 rad $\sec^{-1}$

5.4 (i) $L = 2\,M_oR_oV_o$ (ii) $F = G\left(\frac{M_o^2}{4\,R_o^2}\right)$

(iii) $F = m_o\,V_o^2/R_o$ (iv) $R_o = \frac{h^2}{G\,M_o^3}$

5.5 (a) L = 6.0 J sec along positive z axis,

(b) L = 6.0 J sec along positive x-axis

(c) L = 0

5.6 2 mRV

5.8 (a) $T = W\sqrt{\frac{r^2 + L^2}{L}}$ (b) $\frac{rW}{L}$

5.9 $\frac{W \sin \theta_2}{\sin (\theta_2 - \theta_1)}, \frac{W \sin \theta_1}{\sin (\theta_2 - \theta_1)}$

5.10 (a) W, 2W (b) $\sqrt{2}W$

5.11 (a) 1050 N, 250 N (b) 4 m

5.12 $W\sqrt{\frac{h(2r-h)}{r-h}}$

5.13 74.4 g_m

5.14 (a) $\frac{Wx}{r\sin\theta}$ (b) $\frac{Wx}{r\tan\theta}$

(c) $W\left(\frac{1-x}{r}\right)$

5.15 (a) 2.71 Nm (b) 2.71 Nm

5.16 $(-0.10\,Nm)\,\hat{k}$

CHAPTER 6

6.1 2.61×10^5m

6.2 2.63×10^6m

6.3 1176 N

6.4 0 N

6.5 122000 kg

6.6 3.78 m $\sec^{-2}$

6.7 5 g_m

6.8 (a) 980 N (b) 1380 N (c) 580 N

6.9 4.9 m $\sec^{-2}$

6.10 (a) 6.67×10^{-8}N (b) 6.67×10^{-8}N

6.67×10^{-9} m $\sec^{-2}$, 6.67×10^{-10} m $\sec^{-2}$

6.11 6.25 m $\sec^{-2}$

CHAPTER 7

7.1	15.7 J	**7.2**	31.88 m, 0.27 N
7.3	2.205×10^8 watts	**7.4**	– 525 J
7.5	450 N	**7.6**	10.94 H.P
7.7	71.49 H.P	**7.8**	490 J, 980 J
7.9	2000 J, 0, 9800 J	**7.10**	5 watts
7.11	1.98 m sec^{-1}		

CHAPTER 8

8.1	0.25 hertz, 0.25 m		
8.2	4000 N m^{-1}, 5 J	**8.3**	0.628
8.4	2.25 J, 3.0 m sec^{-1}	**8.5**	0.992 m
8.6	0.0004 J, 0.04 m sec^{-1}		
8.7	986 cm sec^{-2} (approx)		
8.8	$\frac{1}{\sqrt{2}}$		
8.9	(a) $\frac{1}{5}$ sec		(b) 5 pulses sec^{-1}
8.10	(a) 40 cm sec^{-1}		(b) 10 cm
8.11	22.47 hertz		
8.12	78.4 N		
8.13	2 m, 500 m sec^{-1}		
8.14	400 hertz, 600 hertz		
8.15	26, 10, 14, 8 hertz		
8.16	75 hertz, 100 hertz, 40 m sec^{-1}		
8.17	0.533 m, 160 m sec^{-1}		

8.18	498.0 m		
8.19	16514.185 ft sec^{-1}		
8.20	345.5 m sec^{-1}, 469 m sec^{-1}, 664 m sec^{-1}		
8.21	3671 m sec^{-1}		
8.22	3560 m sec^{-1}	**8.23**	0.34 m
8.24	2142.8 hertz	**8.25**	120 : 121
8.26	260, 259	**8.27**	252 hertz

CHAPTER 9

9.1	1.44 cm		
9.2	5.75×10^{-7} m		
9.3	(a) 2.62×10^{-5}m		(b) 2.621×10^{-5}m
9.4	0.34 m		
9.5	0.156 mm		
9.6	5440 A° and 4231.1 A°		
9.7	1.923×10^{-7} m	**9.8**	5890 A°
9.9	7.91×10^{-5} m	**9.10**	654
9.11	5.93°, 4.46	**9.12**	603 lines mm^{-1}
9.13	3647 lines	**9.14**	0.984 A°

CHAPTER 10

10.1	– 225 cm		
10.2	– 5.3 cm		
10.3	(i) 12 cm, +1		(b) – 12 cm, 3
10.4	24 cm, 3		
10.5	– 12 cm, image will be virtual on the same side		

10.6	– 4.8 cm, image is virtual and erect	
10.7	13.3 cm, lens is convex	
10.8	– 6 cm, virtual	
10.9	8.6 cm, 60 cm	
10.10	∞ (infinty)	
10.11	30 cm	
10.12	24 cm, – 8 cm	
10.13	6 mm	
10.14	6.66 cm, – 20 cm	
10.15	The building	
10.16	(a) 3	(b) 2
10.17	120 cm , 25.17 cm	
10.18	40 cm , 205 cm	
10.19	160	
10.20	(a) 55 cm , 10	(b) 3 cm

Some Conversion Factors

1 radian = 57.3° = 0.159 rev

1 atomic mass unit = 1.66×10^{-27} kg

1 angstrom unit = 10^{-10} metre

1 millimicron = 10^{-9} metre

1 day = 86,400 sec

1 year = 3.16×10^{7} sec = 365 days

1 electron volt = 1.60×10^{-19} joule

1 horse power = 550 ft lb/sec = 740 watts

Some Numerical Constants

$\pi = 3.14$ $e = 2.72$

$\sqrt{2} = 1.41$ $\sqrt{3} = 1.730$

$\sin 30^\circ = \cos 60^\circ = 0.500$

$\cos 30^\circ = \sin 60^\circ = 0.866$

$\sin 45^\circ = \cos 45^\circ = 0.207$

Selected Physical Constants

Speed of light = $c = 3.00 \times 10^{8}$ m sec^{-1}

Universal Gravitational Constant G

$= 6.672 \times 10^{-11}$ N-m^2 kg^2

Planck's constant = $h = 6.63 \times 10^{-34}$ joule-sec

Electron charge = $e = 1.60 \times 10^{-19}$ coul

Electron rest mass = $m_e = 9.11 \times 10^{-31}$ kg

Electron charge to mass ratio = $e/m_e = 1.76 \times 10^{11}$ coul/kg

Proton rest mass = $m_p = 1.67 \times 10^{-27}$ kg

Mass energy relation $C^2 = E/m$ $\quad 8.99 \times 10^{16}$ jouls/kg

Speed of sound in dry air = (SPT) $v = 331$ m sec^{-1}

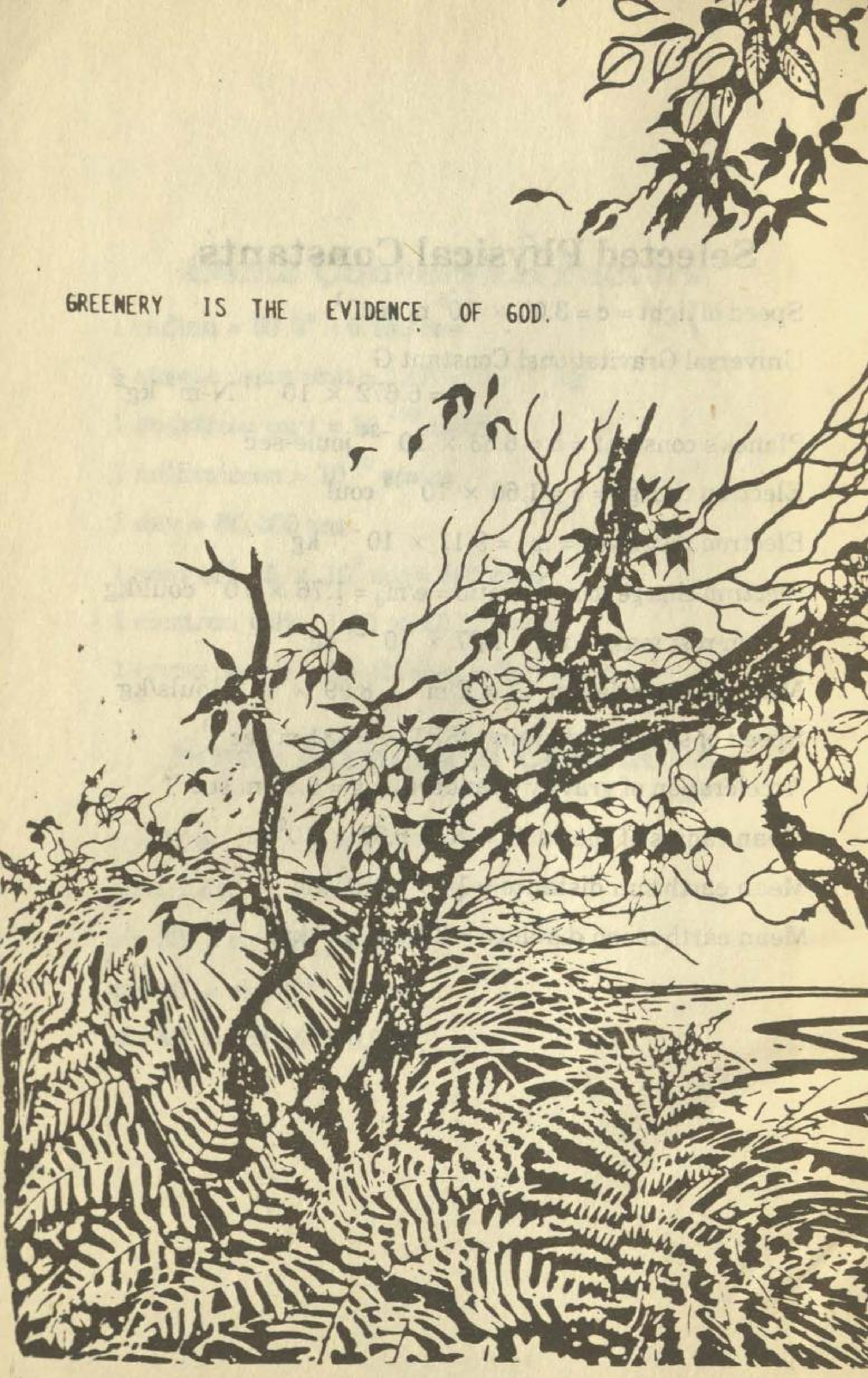
Acceleration of gravity (standard) = $g = 9.81$ m sec^{-2}

Mean radius of the earth = $R_e = 6.37 \times 10^{6}$ m

Mean earth sun distance = 1.49×10^{8} km

Mean earth moon distance = 3.80×10^{5} km

GREENERY IS THE EVIDENCE OF GOD.



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